Classical mechanics: conservation laws and gravity
• The homework that would ordinarily have been due today is now due **Thursday at midnight**. There will be a normal assignment due next Tuesday.

• You should have received an email regarding course evaluations; please fill them out before Oct. 2 (or go to [http://pica.tamu.edu](http://pica.tamu.edu))
Review — Newton’s laws of motion

- **speed** — the distance travelled in a certain amount of time. (For example, a car driving at 60mph.)

- **velocity** — this refers to both the speed and the direction. (For example, a car driving north at 60mph has a different velocity than a car driving east at 60mph.)

- **acceleration** — a change in velocity. (So a car that is turning but maintaining a constant speed is accelerating. So is a car that is going in a straight line but is slowing down; this is negative acceleration)
Review — Newton’s laws of motion

• An object at rest remains at rest, and an object in motion remains in motion with constant velocity

• $F=ma$

• For every force, there is an equal and opposite force
Review — momentum

- An object at rest will remain at rest, and an object in motion will remain in motion with constant velocity
- Why is this? Because of momentum!
- momentum = mass x velocity
- So we can re-state Newton’s first law as: an object’s momentum will not change unless a force is applied to it
Review — angular momentum

- There is also a “circling momentum”, or angular momentum

- A version of Newton’s first law also applies to angular momentum — an object’s angular momentum will not change unless some external force is applied

- This is why the Earth continues to orbit around the Sun, even though nothing is pushing it. And it’s why the Earth continues to spin.
Review — angular momentum

angular momentum = mass x velocity x distance

So, at fixed angular momentum, an object that is further away will move slower.
Review — angular momentum

angular momentum = mass x velocity x distance

So, at fixed angular momentum, an object that is further away will move slower.
Energy

• Energy is usually defined as “the ability to do work” or “the ability to make things move”

• There are different kinds of energy
  • kinetic energy — the energy of movement
  • thermal energy — heat is a kind of energy
  • potential energy — energy that is being stored
  • radiative energy — this is just light
  • mass-energy — this is how nuclear reactions work
Kinetic energy

• This is energy of movement. The faster a thing is moving, the more kinetic energy it has. The more mass it has, the more kinetic energy it has.

\[ KE = \frac{mv^2}{2} \]
Potential energy

- This is energy that is being stored up, and can be converted into a different kind of energy

This is an example of gravitational potential energy, which is a particularly important concept in astronomy.
Gravitational potential energy

• The potential energy that an object has due to its location in a gravitational field. If you raise it up higher, it has more potential energy.

• Near the Earth’s surface, it can be approximated as

$$PE_{\text{grav}} = mgh$$

where $g$ is the acceleration due to gravity (9.8 m/s$^2$)
Mass-energy

Mass itself can be considered a form of potential energy: a small amount of mass can be converted into a large amount of energy ($E=mc^2$)
The Sun

The Sun is an energy converting machine. Mass-energy is converted into heat by nuclear reactions in the Sun’s core. The heat escapes the Sun as radiative energy (light). The light travels to the Earth, where it is absorbed and becomes heat again.
Conservation laws in physics

Energy is always conserved. You can’t create or eliminate energy; you can only transfer it from one place to another, or convert it from one form into another.

- The total amount of energy in a system is constant, unless energy is injected or removed.
Conservation laws in physics

*Energy is always conserved.* You can’t create or eliminate energy; you can also transfer it from one place to another, or from one form into another.

- If you lift up an object in a gravitation field, you are giving it gravitational potential energy. If you drop it, will start accelerating downward, and that gravitational potential energy will convert into kinetic energy.
Conservation laws in physics

- Energy is conserved
- Momentum is conserved
- Angular moment is conserved
Conservation laws in physics

- Energy is conserved
- Momentum is conserved
- Angular moment is conserved

Conservation laws are now seen as one of the most fundamental aspects of nature, and are a tremendously important part of modern physics

Emmy Noether (1882-1935)
Q: Which of the following describes how a rocket takes off?

A. The rocket exerts a force on the ground, and then the atmosphere, causing it to accelerate upward.

B. The backward momentum of the rocket exhaust is canceled by the forward momentum of the rocket.

C. The escaping rocket exhaust changes the mass of the rocket, giving it upward momentum.

D. The burning rocket fuel heats the air beneath the rocket, causing it to move upward.
Q: Which of the following describes how a rocket takes off?

A. The rocket exerts a force on the ground, and then the atmosphere, causing it to accelerate upward.

B. The backward momentum of the rocket exhaust is canceled by the forward momentum of the rocket.

C. The escaping rocket exhaust changes the mass of the rocket, giving it upward momentum.

D. The burning rocket fuel heats the air beneath the rocket, causing it to move upward.
Gravity

- Newton came up with the “universal” law of gravitation. He realized that the force that pulls an object toward the ground is the same force that keeps the Moon in orbit around the Earth. This was revolutionary.
Gravity — an inverse square law

Gravity follows an “inverse-square” law:

\[ F = \frac{GMm}{R^2} \]

This means that if the distance between two objects doubles, the force between them is reduced by a factor of 4. If the distance triples, the force is reduced by a factor of 9.

The gravitational constant:  \[ G = 6.67 \times 10^{-11} \ \frac{m^3}{kg \times s^2} \]
Gravity — an inverse square law

Gravity follows an “inverse-square” law:

\[ F = \frac{GMm}{R^2} \]

This means that if the distance between two objects doubles, the force between them is reduced by a factor of 4. If the distance triples, the force is reduced by a factor of 9.

The gravitational constant: \( G = 6.67 \times 10^{-11} \frac{m^3}{kg \times s^2} \)
Gravity — the difference between mass and weight

- Mass is an intrinsic quantity, and doesn’t depend on where you are (F=ma)
- Weight refers the force exerted on you by gravity. An object weighs less on the Moon than on the Earth, and weighs nothing in the absence of a gravitation field.
Gravity — acceleration doesn't depend on mass

- We’ve all heard the (perhaps even true?) story of Galileo experimenting with gravity by dropping objects with different weight from the leaning tower of Pisa.

- What happens if you drop a hammer and a feather in the absence of air resistance? Which hits the ground first?
Gravity — acceleration doesn’t depend on mass

\[ F = \frac{G M m}{R^2} \]

\[ F = m a \]

\[ a = \frac{G M}{R^2} \]

So the acceleration due to gravity doesn’t depend on the object’s mass!
Gravity — acceleration on Earth

The acceleration due to gravity is:

\[ a = \frac{GM}{R^2} \]

Plug in some numbers to get the acceleration due to gravity on Earth's surface:

\[ G = 6.67 \times 10^{-11} \frac{m^3}{kg \times s^2} \]  

(gravitational constant)

\[ M = 6.0 \times 10^{24} kg \]  

(mass of earth)

\[ R = 6.37 \times 10^6 m \]  

(radius of Earth)
Gravity — acceleration on Earth

The acceleration due to gravity is:

\[ a = \frac{GM}{R^2} \]

Plug in some numbers to get the acceleration due to gravity on Earth’s surface:

\[ a = 9.8 \frac{\text{m}}{\text{s}^2} \]
Gravity — acceleration on Earth

The acceleration due to gravity is:

\[ a = \frac{GM}{R^2} \]

Plug in some numbers to get the acceleration due to gravity on Earth’s surface:

\[ a = 9.8 \, \text{m/s}^2 \]

This means that an object’s speed will increase by 9.8 m/s with each passing second. At t=0s, it will fall at 0 m/s; at t=1s it will fall at 9.8 m/s; at t=2s it will fall at 19.6 m/s; etc.
Gravity — acceleration on Earth

The acceleration due to gravity is:

\[ a = \frac{GM}{R^2} \]

Plug in some numbers to get the acceleration due to gravity on Earth’s surface:

\[ a = 9.8 \frac{\text{m}}{\text{s}^2} \]

This means that an object’s speed will increase by 9.8 m/s with each passing second. At \( t=0 \) s, it will fall at 0 m/s; at \( t=1 \) s it will fall at 9.8 m/s; at \( t=2 \) s it will fall at 19.6 m/s; etc.
Gravity — orbits

From Newton’s first law, we know that objects do not want to move on curved trajectories; they want to move in a straight line. So if they’re moving in an orbit, they are undergoing a constant acceleration due to gravity.
Gravity — orbits

• The inward acceleration required to keep an object in a circular orbit is

\[ a_c = \frac{v^2}{R} \]

• The acceleration due to gravity is:

\[ a_g = \frac{F_g}{m} = \frac{GM}{R^2} \]
Gravity — orbits

\[ \frac{v^2}{R} = \frac{GM}{R^2} \]

\[ v = \sqrt{\frac{GM}{R}} \]
Gravity — orbits

\[
\frac{v^2}{R} = \frac{GM}{R^2}
\]

\[v = \sqrt{\frac{GM}{R}}\]

This gives the relationship between orbital speed, orbital radius, and the mass of the central object (for circular orbits only!)
Gravity — orbits

How fast does the Moon orbit the Earth?

\[ v = \sqrt{ \frac{GM}{R} } \]

\( G = 6.67 \times 10^{-11} \, \text{m}^3 \text{kg}^{-1} \text{s}^{-2} \) (gravitational constant)

\( M = 6.0 \times 10^{24} \text{kg} \) (mass of earth)

\( R = 3.85 \times 10^8 \text{m} \) (radius of Moon’s orbit)

\( v = 1000 \frac{\text{m}}{\text{s}} \)
Gravity — orbits

The previous derivation only works for circular orbits; the derivation becomes more complicated in the case of elliptical orbit. And we know from Kepler’s first law that orbits are, in general, ellipses.
Gravity — orbits

We also know from Kepler’s second law (equal area in equal time) that objects on elliptical orbits speed up when they are on the inner parts of the orbit, and slow down when they are on the outer part.
Gravity — orbits

Two of the most important (and useful) “laws” in astronomy are the conservation of energy, and the conservation of angular momentum. So if you want to know how an object is moving, realizing that the energy and the angular momentum are conserved will help you figure it out!
Conservation laws — angular momentum

\[ \text{angular momentum} = \text{mass} \times \text{velocity} \times \text{radius} \]

The orbital radius is large, so the velocity is small.

The orbital radius is small, so the velocity is large.
Conservation laws — energy

orbital energy = kinetic energy + potential energy

The gravitational potential energy is large, so the kinetic energy is small.

The gravitational potential energy is small, so the kinetic energy is large.
Gravity — tides

It is a common misconception that the high tide is caused by the Moon “pulling” the oceans toward it. But this can’t be quite true — if it were, how come there’s a high tide on Earth both in the direction of the Moon, but also in the opposite direction?
Gravity — tides

• The force of gravity decreases with distance. This means that the Earth’s gravity is *very slightly* stronger at your feet than at your head.

• But if you were *really* tall, then the gravity at your head would be significantly weaker than at your head.

• A **tidal force** is the stretching felt by an object due to the change in the gravitational pull from another object.
Gravity — tides

- Gravity from the Moon pulls harder on the near side of the Earth than on the far side, squishing the Earth and creating a bulge on either side. This causes a high tide.
- The Earth rotates, but the locations of the bulges are always approximately in-line with the Moon.
Gravity — tidal locking

The tidal force from the Moon causes a kind of friction on the Earth’s rotation, slowing it down. And the Earth does the same thing to the Moon, which has caused it to become tidally locked.