(1) Evaluate
\[ \int_{-\infty}^{\infty} \frac{x^2 dx}{1 + x^4} \]
using the calculus of residues.

(2) Show, using the calculus of residues, that
\[ \int_{0}^{\pi} d\theta (\cos \theta)^{2n} = \frac{\pi (2n)!}{2^{2n} (n!)^2} \]

(3) Using the calculus of residues, show that
\[ \int_{0}^{\infty} \frac{\log x dx}{1 + x^4} = -\frac{\pi^2}{8\sqrt{2}} \]

(4) Evaluate, by using the calculus of residues,
\[ \int_{-\infty}^{\infty} \frac{dx \cos(ax)}{x^2 + b^2} \]
where \( a \) and \( b \) are real numbers that may have either sign. (Be careful about how you close off the contour!)

(5) Prove that
\[ \sum_{n=-\infty}^{\infty} \frac{1}{(n^2 + a^2)(n^2 + b^2)} = \frac{\pi}{b^2 - a^2} \left( \coth \frac{\pi a}{a} - \frac{\coth \pi b}{b} \right). \]

Due Thursday 17th November in class