(1.) In the lectures, the Green function \( G(x, t) \) for the operator \( d^2/dx^2 + \omega^2 \), with the boundary condition that all functions vanish at \( x = 0 \) and \( x = \pi \), was obtained in two forms, namely as an infinite sum over \( \sin nx \) functions, and as a closed-form result (using the Wronskian method). These were

\[
G(x, t) = \frac{2}{\pi} \sum_{n \geq 1} \sin nx \sin nt \frac{\sin nt}{\omega^2 - n^2},
\]  

(1)

and

\[
G(x, t) = \begin{cases} 
\frac{\sin(\omega x) \sin(\omega (t - \pi))}{\omega \sin(\omega \pi)} & \text{if } x \leq t, \\
\frac{\sin(\omega (x - \pi)) \sin(\omega t)}{\omega \sin(\omega \pi)} & \text{if } x \geq t. 
\end{cases}
\]  

(2)

Show explicitly, by expanding the closed form result (2) as a Fourier series in \( \sin nx \), that it then gives rise to the infinite-series expression (1) for \( G(x, t) \).

(2.) Find the closed-form expression for the Green function (expressed in two formulae, one for \( x < t \) and the other for \( x > t \), as in the lectures) for the operator \( d^2/dx^2 + \omega^2 \), where now the boundary conditions are \( u(0) = 0, u'(\pi) = 0 \).

(3.) Construct the eigenfunctions for the operator and boundary conditions in problem 2, and hence obtain the expression for Green function as an infinite sum. Then, show explicitly that this is the same expression as one obtains by expanding the closed-form expression in problem 2 in a Fourier series.

Due Thursday 27th October in class