(1) Suppose

\[ \frac{dy}{dx} = g(y/x) \]

where \( g \) is an arbitrary function of the ratio \( y/x \). Show that the substitution \( u = y/x \) leads to a separable equation for \( u \) as a function of \( x \), and present the general solution. (\( i.e. \) reduce to quadratures.)

(2) Determine which of the following are exact differentials, and which are not:

(a) \( \omega = ydx + xdy \), \hspace{1cm} (b) \( \omega = ydx - xdy \),
(b) \( \omega = ydx - xdy \),
(c) \( \omega = \sin y dx + \cos y dy \), \hspace{1cm} (d) \( (x + y)dx + \tan x dy \)

In each case that is not exact, find an integrating factor that renders it exact. (Hint: It is sometimes useful to try integrating factors that are functions of \( x \) only or \( y \) only.)

(3) By finding an appropriate integrating factor, solve

\[ \frac{dy}{dx} = - \frac{2x^2 + y^2 + x}{xy} \]

(4) Construct the two independent solutions of the the second-order ODE \( y'' + y = 0 \) by applying Frobenius’s method of seeking series solutions of the form

\[ y(x) = \sum_{n \geq 0} a_n x^n \]

(Obtain the recursion relation for the coefficients \( a_n \), and then solve it to obtain the two solutions as infinite series, with explicit expressions for the coefficients.)

Due Tuesday 13th September in class