(1) Starting from the Lorentz force equation \( m \frac{d^2 x^\mu}{d\tau^2} = e F^\mu_{\nu} \frac{dx^\nu}{d\tau} \), and taking the special case where \( \vec{E} = (E, 0, 0) \) and \( \vec{B} = 0 \), where \( E \) is a constant, solve directly for the components of \( x^\mu \) as functions of proper time \( \tau \). Show that by choosing constants of integration appropriately, the results reproduce the solution for the particle motion obtained in section (3.2.1) of the lectures.

(2a) Show that the Bianchi identity \( \partial_\mu F^\nu_{\rho \mu} + \partial_\nu F^\rho_{\mu \nu} + \partial_\rho F^\mu_{\nu \rho} = 0 \) can be written as
\[
\partial^\mu \ast F^\mu_{\nu \rho} = 0,
\]
where \( \ast F_{\mu \nu} \) is the Hodge dual of \( F_{\mu \nu} \), defined by \( \ast F^\mu_{\nu \rho} \equiv \frac{1}{2} \varepsilon^{\mu \nu \rho \sigma} F^\sigma_{\mu \nu} \).

(2b) Show that the vector \( V^\mu \equiv \varepsilon^{\mu \nu \rho \sigma} A^\nu F^\rho_{\mu \sigma} \) has the property that
\[
\partial_\mu V^\mu = \frac{1}{2} \varepsilon^{\mu \nu \rho \sigma} F^\mu_{\nu \rho} F^\sigma_{\mu \nu}.
\]

(3a) Show that
\[
\varepsilon^{\mu \nu \rho \sigma} \varepsilon_{\alpha \beta \gamma \delta} = -\delta^\rho_{\alpha} \delta^\sigma_{\beta} \delta^\mu_{\gamma} - \delta^\rho_{\alpha} \delta^\mu_{\beta} \delta^\nu_{\gamma} - \delta^\nu_{\alpha} \delta^\mu_{\beta} \delta^\rho_{\gamma} + \delta^\nu_{\alpha} \delta^\rho_{\beta} \delta^\mu_{\gamma} + \delta^\mu_{\alpha} \delta^\rho_{\beta} \delta^\nu_{\gamma} + \delta^\mu_{\alpha} \delta^\nu_{\beta} \delta^\rho_{\gamma}.
\]
(Hint: Note that the left-hand side is antisymmetric in \( \mu \nu \rho \) and antisymmetric in \( \alpha \beta \gamma \). Show that the right-hand side has the same antisymmetry properties. This means that one can prove the identity by checking only a very small number of non-trivial explicit index assignments (i.e. choices for the free indices that give a non-zero expression on each side of the equations).)

(3b) Using the result in part (3a), show that \( \varepsilon^{\mu \nu \rho \sigma} \varepsilon_{\alpha \beta \rho \sigma} = -2 \delta^\rho_{\alpha} \delta^\nu_{\beta} + 2 \delta^\mu_{\beta} \delta^\nu_{\alpha} \).

(3c) Use the result in part (3b) to show that \( \ast (\ast F^\mu_{\nu}) = -F^\mu_{\nu} \). (i.e. that the Hodge dual of the Hodge dual of a 2-index antisymmetric tensor is equal to minus the original tensor.)