(1) Derive the condition on the constant 4-vector $k_\mu$ such that

$$\phi \equiv e^{ik_\mu x^\mu}$$

solves the wave equation $\square \phi = 0$.

(2a) Consider two successive Lorentz boosts along the $x$ axis, one with velocity $v_1$ and the other with velocity $v_2$. Show that the two boosts commute; i.e. the final result is the same whether one makes the $v_1$ boost first followed by the $v_2$ boost, or instead one makes the $v_2$ boost first followed by the $v_1$ boost.

(2b) Show that the result of making the two successive boosts is again a Lorentz boost along the $x$ axis, and obtain the expression for the net boost velocity $v_3$ for this transformation.

(2c) Show that for successive boosts with $\vec{v}_1 = (v_1, 0, 0)$ along the $x$ axis and $\vec{v}_2 = (0, v_2, 0)$ along the $y$ axis, the transformations do not commute.

(2d) Show that the resulting transformations in part (2c) are not pure boosts, and so they must involve spatial rotations also. (You are not asked to derive the explicit decomposition into boost times rotation.)

(3) Complete the derivation of the discussion in section 2.4 of the lecture notes, by deriving the Lorentz transformation that gives $\vec{B}'$ in terms of $\vec{E}$ and $\vec{B}$, for an arbitrary Lorentz boost with velocity $\vec{v}$.

(4a) Define the scalar quantity $R$ by $R^2 = \eta_{\mu\nu} x^\mu x^\nu$. Show that $\partial_\mu R = \eta_{\mu\nu} x^\nu / R$.

(4b) Hence show that

$$\square \frac{1}{R^2} = 0$$

Due in class on Monday 16th September