(1) Three charges are located along the $z$ axis; a charge $+2q$ fixed at $z = 0$, and two charges $-q$, at $z = \alpha \cos \omega t$ and $-z = \alpha \cos \omega t$ respectively. Determine the lowest non-vanishing multipole moment. Assuming that $ka << 1$ (where $k = \omega$ as usual), determine the angular power distribution in the radiation zone, and find the total power radiated.

(2) Two electric dipoles, each of constant strength $p$, are located in the $xy$ plane at opposite ends of a diameter of a circle of radius $a$ centred on the origin. One dipole points in the positive $z$ direction, while the other points in the negative $z$ direction. They rotate around the circle with angular frequency $\omega$. (i.e. one dipole is located at $(x, y, z) = (a \sin \omega t, a \cos \omega t, 0)$ and the other at $(x, y, z) = (-a \sin \omega t, -a \cos \omega t, 0)$.)

It is assumed that $\omega a << 1$.

(2a) Show that in the multipole expansion discussed in chapter 8 of the lectures, the first non-zero contribution comes from the electric quadrupole.

(2b) Using equation (8.78) in the lectures, calculate the angular power distribution $dP/d\Omega$, and show that the angular dependence is proportional to $(1 - 3 \cos^2 \theta + 4 \cos^4 \theta)$.

(2c) Integrate over all solid angles to obtain an expression for the total radiated power $P$.

[Hint: You may find it useful in this question to note that the charge density for a point dipole of strength $\vec{p}$ at the location $\vec{r}_0$ can be written as $\rho(\vec{r}) = -\vec{p} \cdot \nabla \delta^3(\vec{r} - \vec{r}_0)$.

(3) Optional question for masochists:

Starting from the expression for the magnetic field for the electric quadrupole term in the multipole expansion, given in equation (8.60) of the notes, calculate the electric field directly using $\vec{E} = i/k \vec{\nabla} \times \vec{B}$, and show that it is equal to (8.64) in the notes.

Due on Tuesday 30’th November