(1) Show by explicit direct substitution, making use of the result that \( \nabla^2 |\vec{r} - \vec{r}'|^{-1} = -4\pi \delta^3(\vec{r} - \vec{r}') \), that if 
\[
\phi(\vec{r}) = \int \frac{\rho(\vec{r}') \, d^3\vec{r}'}{|\vec{r} - \vec{r}'|},
\]  
then \( \phi \) satisfies \( \nabla^2 \phi(\vec{r}) = -4\pi \rho(\vec{r}) \).

(2a) Show that \( \nabla^2 (1/r) = 0 \) for \( r > 0 \). Hence show that 
\[
\psi_1 = \partial_x \left( \frac{1}{r} \right), \quad \psi_2 = \partial_y \left( \frac{1}{r} \right), \quad \psi_3 = \partial_z \left( \frac{1}{r} \right)
\]
each satisfy \( \nabla^2 \psi_I = 0 \) (for \( r > 0 \)), for \( I = 1, I = 2 \) and \( I = 3 \). (Here, \( \partial_x \) means \( \partial/\partial x \), etc.)

(2b) Hence deduce that 
\[
\Phi_1 = \frac{x}{r^3}, \quad \Phi_2 = \frac{xy}{r^5}, \quad \Phi_3 = \frac{xyz}{r^7}
\]
each satisfy \( \nabla^2 \Phi_I = 0 \) for \( r > 0 \).

(3a) Substitute the expressions \( \vec{E} = -\vec{\nabla} \phi - (1/c)\partial \vec{A}/\partial t \) and \( \vec{B} = \vec{\nabla} \times \vec{A} \) into the Maxwell field equations in vacuum,
\[
\vec{\nabla} \cdot \vec{E} = 4\pi \rho, \quad \vec{\nabla} \times \vec{B} - \frac{1}{c} \frac{\partial \vec{E}}{\partial t} = \frac{4\pi}{c} \vec{J},
\]
and hence obtain the equations satisfied by the potentials \( \phi \) and \( \vec{A} \).

(3b) Suppose now that the gauge potentials are in Lorenz gauge, which means they obey \( \vec{\nabla} \cdot \vec{A} + (1/c)\partial \phi/\partial t = 0 \). What do the equations you obtained in part (3a) now become?

(3c) Suppose we begin with gauge potentials \( \phi \) and \( \vec{A} \) that are not in Lorenz gauge. Derive a differential equation for the gauge transformation parameter \( \lambda \) such that \( \phi' \) and \( \vec{A}' \), defined by 
\[
\phi' = \phi - \frac{1}{c} \frac{\partial \lambda}{\partial t}, \quad \vec{A}' = \vec{A} + \vec{\nabla} \lambda,
\]
are in Lorenz gauge.

Due in class on Wednesday 3rd February