603: Electromagnetic Theory I

Problem Sheet 10

(1) Consider a solenoid of radius $a$ wound with $N$ turns per unit length, carrying a current $I$. Suppose the $z$-axis runs along the axis of the cylinder, and that it is of length $L$, with $-\frac{1}{2}L \leq z \leq \frac{1}{2}L$. Show that when $N$ is large, the magnetic field on the axis is in the $z$ direction, with

$$B_z = \frac{2\pi NI}{c} (\cos \theta_1 + \cos \theta_2),$$

where $\theta_1$ and $\theta_2$ are indicated in the diagram below:

(2) For a long solenoid where $L >> a$, show that near the axis and near the midpoint of the solenoid (i.e. $\rho << a$ and $|z| << L/2$, in cylindrical polar coordinates), the magnetic field is mainly parallel to the $z$ axis, but has a small radial component

$$B_\rho \approx \frac{96\pi NI a^2 z \rho}{c L^4}.$$  

(Hint: Use $\hat{\nabla} \cdot \vec{B} = 0$.)

(3) Show that near the end of a long solenoid, the magnetic field near the axis has

$$B_z \approx \frac{2\pi NI}{c}, \quad B_\rho \approx \pm \frac{\pi NI}{c} \frac{\rho}{a}.$$  

(4) a circular current loop of radius $a$ carrying a current $I$ lies in the $x - y$ plane, centred on the origin. Show that the only non-vanishing coponent of the vector potential, in cylindrical polar coordinates, is

$$A_\phi(\rho, z) = \frac{4Ia}{c} \int_0^\infty dk \, \cos kz \, I_1(k\rho_<) \, K_1(k\rho_>)$$

Due on Tuesday 30th April