Part (a). Define $r_{EM}$ and $r_{ES}$ to be the Earth-Moon and Earth-Sun distances, respectively. Eqn. 19.3 gives the x-component of the tidal force. The ratio of the tidal force from the Moon on the Earth to that from the Sun on the Earth is:

$$\frac{\text{Tidal Force from Moon}}{\text{Tidal Force from Sun}} = \frac{M_{\text{Moon}}/R_{EM}^3}{M_{\odot}/R_{ES}^3} = 2.2$$

Part (b). When the Sun-Earth-Moon all lie on a straight line, the tidal force from the Sun and Moon add, producing stronger tides (these are “Spring” tides). When the Sun-Earth-Moon make a 90 degree angle, the differential force vectors are perpendicular, so they partially negate, producing weaker tides (these are “Neap” tides).

**Tidal Disruption of Jupiter**

Assuming that both the Sun and Jupiter have constant density and spherical volumes, their average densities are:

- Sun: $\rho_{\odot} = \frac{3M_{\odot}}{4\pi R_{\odot}^3} = 1.41 \text{ kg m}^{-3}$
- Jupiter: $\rho_J = \frac{3M_J}{4\pi R_J^3} = 1.25 \text{ kg m}^{-3}$

The Roche limit gives the minimum distance that Jupiter can approach the Sun before the tidal force of the Sun on Jupiter exceeds Jupiter’s gravitation binding force. The Roche limit is given by equation 19.4. This gives the minimum distance before tidal disruption:

$$r < f_R \left(\frac{\rho_{\odot}}{\rho_J}\right)^{1/3} R_{\odot} = f_R \times 1.04 \times R_{\odot}.$$  

Under the assumption $f_R = 2.44$ we have $r_{\text{min}} = 2.54R_{\odot} = 1.77 \times 10^8 \text{ m}.$

**Synchronization of the Earth-Moon System**

Part (a). Assuming $dP/dt$ is constant, we calculate the length-of-day $= P(t) = P(t_0) - (dP/dt) \times \Delta t$. Where $P(t_0)$ is the length of the day today (24 hrs) and $\Delta t = 542$ million years. We rewrite $dP/dt = 0.0016 \text{ s / century}$ as

$$\frac{dP}{dt} = 0.0016 \text{ s/century} \times \left(\frac{1 \text{ century}}{100 \text{ years}}\right) \times \left(\frac{10^6 \text{ years}}{\text{Million years}}\right) \times \left(\frac{1 \text{ hour}}{3600 \text{ s}}\right) = 4.4 \times 10^{-3} \text{ hrs/million years}$$

This gives the time 542 mya (millions of years ago) as:

$$P(t = 542 \text{ mya}) = 24 \text{ hrs} \left(4.4 \times 10^{-3} \text{ hrs/million years}\right) \times (542 \text{ million years}) = 21.6 \text{ hrs}$$
Therefore, assuming the rate that the Earth’s period is declining is constant the length of the day during
the Cambrian period would have been 21 hours and 36 minutes.

Part (b). This problem involves Kepler’s 3rd law. The Earth-Moon system will be synchronized when the
Earth’s rotational period equals the Moon’s orbital period (and both equal 47 days). Kepler’s 3rd law gives:

\[ P^2 = \frac{4\pi^2}{GM} a^3 \]

and solving for \( a \) gives

\[ a = \left[ \frac{P^2 GM}{4\pi^2} \right]^{1/3} = 5.5 \times 10^8 \text{ m} = 550,000 \text{ km} \]

In comparison, the current distance to the Moon is 384,400 km.

Part (c). Inverting the equation from Part (a), we have

\[ \Delta t = \frac{P - P(t_0)}{(dP/dt)} \]

where \( P = 47 \text{ days} \times (24 \text{ hrs / day}) = 1128 \text{ hrs}, P(t_0) = 24 \text{ hrs} \) and \( dP/dt = 4.4 \times 10^{-9} \text{ hrs/years} \). This
gives \( \Delta t = 2.51 \times 10^{11} \text{ years} \) (about 250 *billion* years). Given that the lifetime of our Sun is only another
5 billion years, it is unlikely any lifeforms remaining on Earth would witness this event.

**Coriolis effect on sports**

Part (a) The Coriolis force is given by \( \vec{F} = -2\vec{\Omega} \times \vec{v} \) where \( \vec{\Omega} \) is the rotation vector of the Earth and \( \vec{v} \) is the
velocity of the football. The Coriolis acceleration is therefore

\[ \vec{a} = 2\vec{v} \times \vec{\Omega} . \]

By the right-hand rule, a ball thrown North from a location in the Northern hemisphere will feel an accel-
eration to the East (it will deflect to the right as viewed by an observer facing North).

Part (b)

The angular rotation rate of the Earth is \( |\vec{\Omega}| = 2\pi \text{ radians / day} \times 1 \text{ day} / 86,400 \text{ s} = 7.27 \times 10^{-5} \text{ rad/s} \) and we are given that \( |\vec{v}| = 25 \text{ m/s} \).

The cross product above gives \( a = 2v\Omega \sin \theta \), where \( \theta = 90^\circ - \ell \) where \( \ell \) is the latitude (i.e., at
the equator, \( \ell = 0^\circ \), the coriolis force of an object moving directly N would be zero). The acceleration is
therefore \( a = 2v\Omega \sin(90^\circ - \ell) \). The deflection distance depends on the flight time, \( \Delta t \),

\[ \Delta d = \frac{1}{2} a (\Delta t)^2 \]

and \( \Delta t = D/v \) where \( D \) is the distance the ball travels and \( v \) is the velocity of the ball. This gives

\[ d = \frac{1}{2} \left[ 2v\Omega \sin(90^\circ - \ell) \right] (D/v)^2 = \frac{D^2 \Omega \sin(90^\circ - \ell)}{v} \]

Plugging in the numbers (using \( \ell = 30.6278^\circ \) for College Station) gives \( d = 0.0035 \text{ m} = 3.5 \text{ mm} \). This is not
nearly as significant as other factors (the receiver does not need to correct for this!).
Problem 3.3

Part (a). The distance to Sirius is given by the parallax formula (equation 3.1).

\[ d = \frac{1}{p'} \text{pc} = 2.64 \text{pc} = 8.61 \text{lyr} = 5.44 \times 10^5 \text{AU} = 8.14 \times 10^{16} \text{m} \]

Part (b). The distance modulus is then given by 
\[ 5 \log_{10}(d/10 \text{pc}) = 5 \log_{10}(d) - 5 = -2.89. \]

Problem 3.5

Part (a). We want to solve for one side of a right triangle, \( d \), knowing that the other side is \( h = 1.9 \text{ cm} \), and that the subtended angle is \( \theta = 0.001'' \). We can relate \( d \) and \( h \) to \( \theta \) through the tangent, \( h/d = \tan \theta \), which for the small angle approximation is \( d/h \approx \theta \) for \( \theta \) measured in radians. This makes

\[ \theta = 0.001'' \times \frac{1 \text{ degree}}{3600 \text{ arcseconds}} \times \frac{\pi \text{ radians}}{180 \text{ degrees}} = 4.85 \times 10^{-9} \text{ radians}. \]

Therefore, \( d = h/\theta = 0.019 \text{ m} /4.85 \times 10^{-9} = 3.92 \times 10^6 \text{ m} = 3920 \text{ km} \). In other words, you (in College Station) could resolve a dime held by someone in Seattle, WA. (Actually, quite a bit further. The distance from CS to Seattle is only \( \approx 3000 \text{ km} \).)

Part (b). The growth rate is given as 5 cm per week (1 week = 7 days), so the rate is \( 8 \times 10^{-8} \text{ m s}^{-1} \). Using the same calculation as for part (a), the angle \( \theta = 0.000004'' = 1.9 \times 10^{-11} \text{ radians} \). For SIM to resolve the change in a length of a blade of grass in 1 second, then the maximum distance it could be from the grass would be, \( d = (8 \times 10^{-8})/1.9 \times 10^{-11} = 4125 \text{ m} \), or approximately 4 km!