1. According to the virial theorem, the central radial-velocity dispersion is related to the mass of the galaxy by \( \sigma_r^2 \propto M/R \). Use arguments similar to those for the Tully-Fisher relation (see class notes) to show that \( L \propto \sigma_r^4 \), which is the Faber Jackson relation.

2. The IMF has the form \( \phi(m) \propto m^\gamma \). Calculate the ratio in the mass in stars between two IMFs, the Salpeter IMF, with \( \gamma = -2.35 \) over the range \( m=0.1 - 100 \ M_\odot \), and a Milky-way type IMF with \( \gamma = -2.35 \) for \( m=1 - 100 \ M_\odot \) and \( \gamma = -1 \) for \( m=0.1 - 1 \ M_\odot \). Assume both IMFs have upper and lower cutoffs \( m_\ell=0.1 \ M_\odot \) and \( m_u=100 \ M_\odot \) and that they have the same amount of mass in stars when integrated from \( 1 - 100 \ M_\odot \), that is

\[
\int_{1 \ M_\odot}^{100 \ M_\odot} m \phi(m)_{\text{Salpeter}} \, dm = \int_{1 \ M_\odot}^{100 \ M_\odot} m \phi(m)_{\text{Milky Way}} \, dm
\]

3. The number of galaxies per unit redshift and per unit apparent magnitude is given by

\[
\frac{d^2 N}{dm \, dz} = \frac{\omega \, dV(z)}{4\pi \, dz} \frac{d \Phi(L)}{dL} \frac{dL}{dm}
\]

where \( \omega \) is the solid angle covered by the sample, \( V(z) \) is the comoving volume in redshift \( z \), \( L \) is the luminosity, and \( \Phi(L) \) is the luminosity function (assumed to be a Schecter function and assumed to be constant with redshift).

a. Neglecting galaxy K-corrections and neglecting the expansion of the Universe, show that, \( N(<m) \), the integrated number counts of galaxies brighter than apparent magnitude \( m \) is given by

\[
N(< m) = \frac{\omega}{3} (4\pi f_m)^{-3/2} \phi^* L_*^{3/2} \Gamma \left( \alpha + \frac{5}{2} \right)
\]
where $f_m$ is the flux corresponding to apparent magnitude $m$, and $\Phi^*$, $L^*$, and $\alpha$ are the Schecter function parameters.

b. Plot $N(<m)$ as a function of $m$ for the luminosity function for red galaxies ($M^* = -21.5$, $\alpha = -0.8$, $\Phi = 2.3 \times 10^{-3} \text{ Mpc}^{-3}$), and blue galaxies ($M^* = -21.3$, $\alpha = -1.2$, $\Phi = 2.9 \times 10^{-3} \text{ Mpc}^{-3}$) using the data from Baldry et al. (2004). Assume $\omega = 1 \text{ deg.}$

4. Using the models provided on the course website, calculate and plot the $V - K$ color for the instantaneous burst stellar population synthesis model as a function of time for ages=10 Myr, 30 Myr, 100 Myr, 300 Myr, 1 Gyr, and 3 Gyr. Repeat your calculation for the constant star-formation rate population synthesis model and compare it to the instantaneous burst model.

5. Explain the qualitative effect and the difference between the Malmquist bias and the Eddington bias.

6. The radio galaxy 8C 1435+63 briefly held the distinction of being the highest redshift galaxy, with redshift $z=4.25$ (Spinrad, Dey, & Graham 1995). Assuming cosmological parameters, $h=0.70$, $\Omega_M = 0.27$, and $\Lambda = 0.73$, calculate the following for this galaxy. You will likely require numerical integration using a computer to calculate these.

   a. How old was the Universe at this redshift? Express your answer both in terms of years and as a fraction of the present age of the Universe.

   b. What is the present proper distance (comoving distance) in Mpc to this galaxy?

   c. What is the luminosity distance to this galaxy?

   d. What is the angular diameter distance to this galaxy?

   e. The angular diameter of the nucleus of 8C 1435+63 is measured to be 5". What linear diameter does this correspond to in units of kpc?

   f. Given the linear diameter you measured in part (e), what would the angular diameter of 8C 1435+63 be if it were placed at a redshift $z=1.5$? Give your answer in arcseconds. Comment on how this compares to the angular diameter you derived in part (e) (i.e., would the galaxy appear on the sky with a smaller or bigger angular size if it were at $z=1.5$ rather than at $z=4.25$?)