<table>
<thead>
<tr>
<th>App. Mag</th>
<th>Stars</th>
<th>E/S0 (red)</th>
<th>E/S0 (blue)</th>
<th>Sabc</th>
<th>Sd/Irr</th>
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</thead>
<tbody>
<tr>
<td>15.25</td>
<td><img src="image1.png" alt="Image" /></td>
<td><img src="image2.png" alt="Image" /></td>
<td><img src="image3.png" alt="Image" /></td>
<td><img src="image4.png" alt="Image" /></td>
<td><img src="image5.png" alt="Image" /></td>
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<tr>
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<td><img src="image7.png" alt="Image" /></td>
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<td><img src="image9.png" alt="Image" /></td>
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<tr>
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<td><img src="image24.png" alt="Image" /></td>
<td><img src="image25.png" alt="Image" /></td>
</tr>
</tbody>
</table>

Driver et al. 2006
Galaxy Luminosity Functions

• Luminosity Function is the number density (# per volume) of some population of objects (e.g., galaxies) of a specific luminosity.

• Here we take the number density of galaxies with absolute magnitude $M$ and $M + dM$.

$$\Phi(M)\,dM$$

• The Total Density of Galaxies is then the integral of the luminosity function:

$$n = \int_{-\infty}^{\infty} \Phi(M)\,dM$$

• Rewrite in terms of the # density of galaxies between luminosity $L$ and $dL$:

$$\Phi(L)\,dL$$
Galaxy Luminosity Functions

- Measuring Luminosity Functions seems easy, but hard in practice.
  - Need *Luminosities*, requires measuring fluxes (magnitudes) and distances.
  - Need large and representative sample of galaxies. Sufficiently large Volume required (galaxies clustered in structures on ~100 Mpc scales). But, need depth to get low-luminosity galaxies at any given distance. (Always a trade off between depth and area in galaxy surveys.)
  - Photometric errors are problematic. A well known bias, *Malmquist bias*. One will always measure an increase in the average luminosity of a sample with distance because the less-luminous objects are missed. (Difference between *Flux-limited and Volume Limited* samples.)
Malmquist bias

Introduce the star-count function (star=galaxy=x-ray source=..., in this case), \( A(m) \), which is the differential number of stars with apparent magnitude, \((m+dm, m)\).

\[
A(m) \equiv \frac{dN}{dm}
\]

In practice, there is some limiting magnitude \( m_{\text{lim}} \) such that \( A(m) \) is only measurable for \( m < m_{\text{lim}} \).

A sample that consists of all objects \( m < m_{\text{lim}} \) is said to be a **Magnitude limited sample**.

For a magnitude limited sample, the **observed** mean Absolute magnitude will be brighter than the **true** mean Absolute magnitude.

The **Volume** (distance) out to which you can see the most luminous object will be larger than for the mean object.
Redshift distribution of galaxies in Hubble Deep Field North
Dickinson et al. 2003
Malmquist bias

For a magnitude limited sample, the \textit{observed} mean Absolute magnitude will be brighter than the \textit{true} mean Absolute magnitude.

Consider the simplest case where stars have the same distance (then $m = M + \text{constant}$).
Malmquist bias

Derive effect of Malmquist bias given that you can observe a sample with measured mean \( <m_m> \) and variance \( \sigma_m \) and we will take that the sample has a Gaussian distribution (for Malmquist bias discussion here, not change in notation: \( s=\)distance, and \( \nu(s) \) and \( n(s) \) are defined below)

Calculate the number of stars in a magnitude-limited sample what have **Absolute** magnitude in \((M+dM, M)\) and **Apparent** magnitude \((m+dm, m)\).

\[
\frac{d^2 N}{dm \, dM} = \Phi(M) \frac{dn}{ds} \left( \frac{ds}{dm} \right)_M
\]

\(s(m,M)\) is the distance to an object in the sample

dn is the number of stars that lie within a volume \(dV = (\omega \, s^2 \, ds)\), where \(\nu(s) = \) number density of objects (number per unit volume).

\[
\frac{dn}{ds} = \omega \, s^2 \, \nu(s)
\]
Malmquist bias

\[
\frac{dn}{ds} = \omega s^2 \nu(s) \quad \Longrightarrow \quad \frac{d^2 N}{dm \ dm \ dM} = \Phi(M) \frac{dn}{ds} \left( \frac{ds}{dm} \right)_M
\]

yields

\[
A(m) = \int_{-\infty}^{\infty} dM \ \frac{d^2 N}{dm \ dm \ dM} = \omega \int_{-\infty}^{\infty} dM \ \Phi(M) \left( \frac{\partial s}{\partial m} \right)_M \ s^2 \nu(s)
\]

change integration variable from \( M \) to \( s \), because \( m - M = 5 \log s + 5 \), and so ,

\[
(\partial M / \partial s)_m = -(\partial m / \partial s)_M
\]

this gives

\[
A(m) = \omega \int_0^{\infty} ds \ \Phi(M) \ s^2 \nu(s)
\]
Malmquist bias

We can now compute the Mean Absolute magnitude of objects in a sample that have apparent magnitude $m$

$$\langle M \rangle_m = \frac{\int_{-\infty}^{\infty} dM \frac{d^2 N}{dm dM}}{\int_{-\infty}^{\infty} dM}$$

Using our expression now for $(d^2 N/dmdM)$, we have

$$\langle M \rangle_m = \frac{\int_{0}^{\infty} ds \Phi(M) s^2 \nu(s)}{\int_{0}^{\infty} ds \Phi(M) s^2 \nu(s)}$$

Note that $(\partial M/\partial m)_s = 1$, and that so differentiating our expression for $A(m)$ w.r.t. $m$ gives

$$\frac{dA}{dm} = \omega \int_{0}^{\infty} ds \frac{d\Phi}{dM} s^2 \nu(s)$$
Malmquist bias

Comparing

\[ \langle M \rangle_m = \frac{\int_0^\infty ds \ M \Phi(M) s^2 \nu(s)}{\int_0^\infty ds \ \Phi(M) s^2 \nu(s)} \]

with

\[ \frac{dA}{dm} = \omega \int_0^\infty ds \frac{d\Phi}{dM} s^2 \nu(s) \]

we can write:

\[ \frac{1}{A} \frac{dA}{dm} = \left\langle \frac{1}{\Phi} \frac{d\Phi}{dM} \right\rangle_m \]

and also that

\[ \frac{1}{A} \frac{d^2A}{dm^2} = \left\langle \frac{1}{\Phi} \frac{d^2\Phi}{dM^2} \right\rangle_m \]
Malmquist bias

\[ \frac{1}{A} \frac{dA}{dm} = \left\langle \frac{1}{\Phi} \frac{d\Phi}{dM} \right\rangle_m \]

Now must have some form for \( \Phi(M) \), take as a Gaussian:

\[ \Phi(M) = \frac{1}{\sqrt{2\pi\sigma^2}} \exp\left(-\frac{(M - M_0)^2}{2\sigma^2}\right) \]

In which case \( \Phi^{-1} (d\Phi/dM)_m \) yields that

\[ \frac{1}{A} \frac{dA}{dm} = -\left\langle \frac{M - M_0}{\sigma^2} \right\rangle_m \]

Solving for \( \langle M \rangle \) gives:

\[ \langle M \rangle_m = M_0 - \sigma^2 \frac{d \ln A}{dm} \]

Therefore, \( \langle M \rangle \) is always lower than \( M_0 \): \( \langle M \rangle \) is Brighter.
Malmquist bias

Since $\langle M \rangle$ is measurable, one can correct for the Malmquist bias if $\sigma$ is known. In principle, this must be estimated as well. For this, insert

$$ \Phi(M) = \frac{1}{\sqrt{2\pi\sigma^2}} \exp \left( -\frac{(M - M_0)^2}{2\sigma^2} \right) $$

into

$$ \frac{1}{A} \frac{d^2 A}{dm^2} = \left\langle \frac{1}{\Phi} \frac{d^2 \Phi}{dM^2} \right\rangle_m $$

which yields

$$ \frac{1}{A} \frac{d^2 A}{dm^2} = \left\langle \left( \frac{M - M_0}{\sigma^2} \right)^2 - \frac{1}{\sigma^2} \right\rangle $$

using $\sigma^2_m \equiv \langle M^2 \rangle_m - \langle M \rangle_m^2$ with a little algebra....

$$ \sigma^2_m = \sigma^2 + \sigma^4 \frac{d^2 \ln A}{dm^2} $$
Malmquist bias

Now, consider the simple case that the sample is distributed homogeneously through space, so $\nu(s) = \text{constant}$

$$A(m) \propto \exp [0.6 \log(m - M_0)]$$

Taking the derivatives, we can determine the effect on

$$\langle M \rangle_m = M_0 - \sigma^2 \frac{d \ln A}{dm} \quad \sigma_m^2 = \sigma^2 + \sigma^4 \frac{d^2 \ln A}{dm^2}$$

For a homogeneously distributed population with a Gaussian distribution of magnitudes, the mean is shifted by $0.6 \sigma^2$ and the variance is unaffected.
AN ANALYTIC EXPRESSION FOR THE LUMINOSITY FUNCTION FOR GALAXIES*

PAUL SCHECHTER
California Institute of Technology and the Institute for Advanced Study
Received 1975 April 29; revised 1975 June 30

ABSTRACT

A new analytic approximation for the luminosity function for galaxies is proposed, which shows good agreement with both a luminosity distribution for bright nearby galaxies and a composite luminosity distribution for cluster galaxies. The analytic expression is proportional to \( L^{-5/4} e^{-L/L^*} \), where \( L^* \) is a characteristic luminosity corresponding to a characteristic absolute magnitude \( M^*_{B(0)} = -20.6 \). For an individual cluster, the characteristic magnitude may be determined with an accuracy of \( \sim 0.25 \) mag, suggesting its use as a standard candle. The analytic expression is used to compute an expected richness–absolute magnitude correlation for first ranked cluster galaxies and an expected dispersion, which are compared with the data of Sandage and Hardy.

Subject headings: galaxies: clusters of — galaxies: photometry

I. INTRODUCTION

For a wide range of extragalactic problems one needs to know the luminosity function for galaxies. For example, the spatial covariance function for galaxies can be obtained from the projected angular...
Galaxy Luminosity Functions

- The Schecter Luminosity Function

\[ \Phi(L) = \left( \frac{\Phi^*}{L^*} \right) \left( \frac{L}{L^*} \right)^\alpha \exp\left(-\frac{L}{L^*}\right) \]
Galaxy Luminosity Functions

• The Schecter Luminosity Function

\[ \Phi(L) = \left( \frac{\Phi^*}{L^*} \right)^\alpha \left( \frac{L}{L^*} \right) \exp \left( -L/L^* \right) \]

• Rewrite in terms of Absolute Magnitude. Recall that

\[ \Phi(L) \, dL = \Phi(M) \, dM \]

• Because \( dL/dM \sim d(10^{0.4M}) / dM = 0.4 \log(10) \, L \):

\[ \Phi(M) = \Phi(L) \left| \frac{dL}{dM} \right| = \Phi(L) \, 0.4 \ln 10 \, L \]

\[ = (0.4 \ln 10) \Phi^* 10^{0.4(\alpha+1)(M^*-M)} \times \exp \left( -10^{0.4(M^*-M)} \right). \]
Galaxy Luminosity Functions

• The Schecter Luminosity Function

\[
\Phi(L) = \left( \frac{\Phi^*}{L^*} \right) \left( \frac{L}{L^*} \right)^\alpha \exp\left(-\frac{L}{L^*}\right)
\]

\[
\Phi(M) = \Phi(L) \left| \frac{dL}{dM} \right| = \Phi(L) 0.4 \ln 10 L
\]

\[
= (0.4 \ln 10) \Phi^* 10^{0.4(\alpha+1)(M^*-M)}
\]

\[
\times \exp\left(-10^{0.4(M^*-M)}\right)
\]

\[
\Phi^* = 1.6 \times 10^{-2} \ h^3 \ Mpc^{-3}, \quad \Phi^* = 1.6 \times 10^{-2} \ h^3 \ Mpc^{-3},
\]

\[
M_B^* = -19.7 + 5 \log h, \quad \text{or} \quad M_K^* = -23.1 + 5 \log h,
\]

\[
L_B^* = 1.2 \times 10^{10} \ h^{-2} \ L_\odot, \quad \alpha = -1.07.
\]

\[
\alpha = -0.9.
\]
Galaxy Luminosity Functions

• The total Luminosity Density of a galaxy population described by a Schecter Function is

$$ l_{tot} = \int_{0}^{\infty} dL \ L \ \Phi(L) = \Phi^* \ L^* \ \Gamma(2 + \alpha) $$

• This is formally only defined for $\alpha > -2$. What happens at “lower” power law slopes?
Galaxy Luminosity Functions

- Different galaxy populations have different luminosity functions.

Driver et al. 2006
Galaxy Luminosity Functions

- Different galaxy populations have different luminosity functions.
- Are slopes, $\alpha > -1$ OK?

Rudnick et al. 2009
Eddington Bias

Eddington Bias arises from photometric (flux) errors. In general, there are many, many more fainter objects in your sample than brighter ones. Random, photometric errors will cause relatively few bright sources to enter the fainter bin, but will cause relatively more faint sources to enter your high mass bins.


A good example of a variant of this is Fontanot et al. (2009, MNRAS, 397, 1776)
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Galaxy Luminosity Functions

- Different galaxy populations have different luminosity functions.
Galaxy Luminosity Functions, Color Dependence

- Different galaxy populations have different luminosity functions.

Driver et al. 2006
Galaxy Luminosity Functions, Color Dependence

Driver et al. 2006
Galaxy Color Distributions

See Baldry et al. 2004
See Baldry et al. 2004
Generalized Surface Brightness Distributions

Generalized surface brightness profile proposed by Sérsic (1963)

\[ I(R) = I_0 \exp(-kR^{1/n}) \]

Where “n” is the Sersic Index. Can choose n and k to reproduce familiar results for galaxy disks and spheroids:

\[ I(R) = I_0 \exp(-kR), \quad n = 1 \]

\[ I(R) = I_0 \exp(-kR^{1/4}), \quad n = 4 \]

Higher n, more compact is surface-brightness profile. Strong relation between sersic indexes and other galaxy properties.
Generalized Surface Brightness Distributions

![Graph showing generalized surface brightness distributions with logarithmic scales for brightness and radius. The graph includes curves for different values of n, specifically n=10 and n=1.](image-url)
Galaxy Color Distributions

Blanton et al. 2003
Galaxy Color Distributions

Hogg et al. 2004

Monday, August 20, 2012