Clusters of Galaxies

Galaxies are not randomly strewn throughout space. Instead the majority belong to groups and clusters of galaxies. In these structures, galaxies are bound gravitationally and orbit a common center of mass.
Galaxy Distribution of North Sky in the Lick Catalog. Each pixel is the “counts” of galaxies in 10’ x 10’ bins.
Clusters of Galaxies

Groups: have less than ~50 members with a size of ~2 Mpc. They have velocity dispersions ~150 km/s and total mass $2-3 \times 10^{13} \, M_\odot$. They have mass-to-light ratios of ~300-400 $M_\odot/L_\odot$.

Clusters: have ~50 members (a poor cluster) to as many as 1000 members (a rich cluster). They have sizes of 6-8 Mpc, velocity dispersions of ~800-2000 km/s and total mass of $1-3 \times 10^{15} \, M_\odot$. They have mass-to-light ratios of >500 $M_\odot/L_\odot$.

Superclusters: clusters of clusters of galaxies, and so on.
The Local Group

There are about 35 galaxies within roughly 1 Mpc of the Milky Way and these have velocities implying they are all bound to a common center of mass (about 460 kpc in the direction of Andromeda). The most prominent members are the Milky Way (us), and the Andromeda (M31) and Triangulum (M33) galaxies.
The Local Group

There are about 35 galaxies within roughly 1 Mpc of the Milky Way and these have velocities implying they are all bound to a common center of mass (about 460 kpc in the direction of Andromeda). The most prominent members are the Milky Way (us), and the Andromeda (M31) and Triangulum (M33) galaxies.
The Local Group - Map

Figure 1. A scaled 3-D representation of the Local Group (LG). The dashed ellipsoid marks a radius of 1 Mpc around the LG barycenter (assumed to be at 462 kpc toward $l = 121.7$ and $b = -21.3$ following Courteau & van den Bergh 1999). Distances of galaxies from the arbitrarily chosen plane through the Milky Way are indicated by solid lines (above the plane) and dotted lines (below). Morphological segregation is evident: The dEs and gas-deficient dSphs (light symbols) are closely concentrated around the large spirals (open symbols). dSph/dIrr transition types (e.g. Pegasus, LGS3, Phoenix) tend to be somewhat more distant. Most dIrrs (dark symbols) are fairly isolated and located at larger distances. Also indicated are the locations of two nearby groups.
The Local Group

**Mass Estimate:** Assume M31 and Milky Way Galaxy were expanding with Universe. Their gravity caused them to halt their expansion and move toward each other.

At a time $t_{\text{max}}$ they were a distance $r_{\text{max}}$ apart.

Relative velocity and separation follow:

$$\frac{v^2}{2} = \frac{GM}{r} - C$$

$M = \text{mass of MW} + \text{M31}$.

$C$ is integration constant. Set at time $t_{\text{max}}$ when $v=0$ and $r=r_{\text{max}}$:

$$C = \frac{GM}{r_{\text{max}}}$$
The Local Group

Now rewrite conservation of energy:

\[ \frac{1}{2} \left( \frac{dr}{dt} \right)^2 = GM \left( \frac{1}{r} - \frac{1}{r_{\text{max}}} \right) \]

Solve using initial condition \( r=0 \) at \( t=0 \), solve for time:

\[ t_{\text{max}} = \int_{0}^{t_{\text{max}}} dt = \int_{0}^{r_{\text{max}}} \frac{dr}{\sqrt{2GM \sqrt{1/r - 1/r_{\text{max}}}}} \]

\[ = \frac{\pi r_{\text{max}}^{3/2}}{2\sqrt{2GM}}. \]

Since the equation is symmetric, collision will happen at \( 2t_{\text{max}} \).

Estimate the time until collision as

\[ r/v = D / v = 770 \text{ kpc} / 120 \text{ km/s} \]

Then \( 2t_{\text{max}} = t_0 + D/v \), or \( t_{\text{max}} = t_0/2 + D/2v \).

where \( t_0 \) is the current age of the Universe.
The Local Group

Therefore conservation of energy becomes:

\[ \frac{v^2}{2} = \frac{GM}{r} - \frac{GM}{r_{\text{max}}} = \frac{GM}{r} - \frac{1}{2} \left( \frac{\pi GM}{t_{\text{max}}} \right)^{2/3} \]

Inserting \( r(t_0) = D \) and \( v = v(t_0) \) we solve for mass, \( M \):

\[ M \sim 3 \times 10^{12} \text{ solar masses.} \]

Adding up light from the for the whole local group galaxies gives a mass-to-light ratio

\[ M/L \sim 70 \, \text{M}_\odot/\text{L}_\odot \]
The Virgo Cluster

First recognized by William Herschel where the constellations Virgo and Coma meet. The cluster covers 10 x 10 degrees on the sky (the Full Moon cover 0.5 x 0.5 degrees). The center of the cluster is ~18 Mpc from Earth.

The Virgo Cluster contains >250 large galaxies and more than 2000 smaller ones contained within an area 3 Mpc across.

The largest galaxies are all ellipticals (M87, M86, M84) and these have sizes equal to the distance between the Milky Way and Andromeda. These are “giant” Ellipticals (gE).
Central Part of Virgo Cluster
The Coma Cluster

The Virgo Cluster is small compared to the Coma Cluster. The Virgo Cluster contains >250 large galaxies and more than 2000 smaller ones contained within an area 3 Mpc across. Most of the large galaxies are spirals in Virgo.

The Coma Cluster is 15° of Virgo, in the constellation Coma Berenices, and is ~90 Mpc away. It has an angular diameter of ~4° which at 90 Mpc away is a linear diameter of 6 Mpc.

Coma contains possibly more than 10,000 galaxies. Of the >1000 large galaxies, only 15% are spirals. The majority are ellipticals (and some S0’s).
Abell Catalog: 1958 from George Abell, used POSS plates of the northern >30°.

Selected clusters as overdensities of galaxies with the following criteria.

≥50 galaxies in the interval m_3 ≤ m ≤ m_3 + 2 within θ_A

where m_3 is the magnitude of the 3rd brightest galaxy.

θ_A = \frac{1.7'}{z}
Galaxies in Clusters and Groups

$\theta_A$ is the Abell radius, and corresponds to $R_A \sim 1.5 \, h^{-1} \, Mpc$. Redshift range for Abell Clusters should be $0.02 < z < 0.2$.

**Abell Catalog** contains 1682 clusters + 1030 clusters that do not fulfill all criteria.

**ACO (Abell Corwin & Olowin) Catalog** contains 4076 clusters (from 1989).

Abell redshift estimates are surprisingly good to 30%...
### Galaxies in Clusters and Groups

<table>
<thead>
<tr>
<th>Richness Class</th>
<th>( N ) ((m_3 &lt; m &lt; m_3 + 2))</th>
<th>Number in Abell’s Catalog</th>
</tr>
</thead>
<tbody>
<tr>
<td>0</td>
<td>30-49</td>
<td>&gt;1000</td>
</tr>
<tr>
<td>1</td>
<td>50-79</td>
<td>1224</td>
</tr>
<tr>
<td>2</td>
<td>80-129</td>
<td>383</td>
</tr>
<tr>
<td>3</td>
<td>130-199</td>
<td>68</td>
</tr>
<tr>
<td>4</td>
<td>200-299</td>
<td>6</td>
</tr>
<tr>
<td>5</td>
<td>&gt;300</td>
<td>1</td>
</tr>
</tbody>
</table>
### Galaxies in Clusters and Groups

<table>
<thead>
<tr>
<th>Distance Class</th>
<th>$m_{10}$ (mag of 10th brightest galaxy)</th>
<th>Estimated Redshift</th>
<th>Number in Abell’s Catalog</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>13.3-14.0</td>
<td>0.0282</td>
<td>9</td>
</tr>
<tr>
<td>2</td>
<td>14.1-14.8</td>
<td>0.0400</td>
<td>2</td>
</tr>
<tr>
<td>3</td>
<td>14.9-15.6</td>
<td>0.0577</td>
<td>33</td>
</tr>
<tr>
<td>4</td>
<td>15.7-16.4</td>
<td>0.0787</td>
<td>60</td>
</tr>
<tr>
<td>5</td>
<td>16.5-17.2</td>
<td>0.131</td>
<td>657</td>
</tr>
<tr>
<td>6</td>
<td>17.3-18.0</td>
<td>0.198</td>
<td>921</td>
</tr>
</tbody>
</table>

Abell redshift estimates are surprisingly good to 30%...
Morphological Classification

![Diagram of morphological classification with symbols and categories L, F, C, I, B, and cD]
Spatial Distribution of Galaxies

We only measure projected density distribution, \( N(R) \). This is related to the 3D distribution, \( n(r) \).

If cluster is not very elliptical, assume spherical symmetry to 1st approximation.

\[
N(R) = \int_{-\infty}^{\infty} dz \, n \left( \sqrt{R^2 + z^2} \right) = 2 \int_{R}^{\infty} \frac{dr \, r \, n(r)}{\sqrt{r^2 - R^2}}
\]

second step is a change of variable from \( r \) to light-of-sight coordinate \( z \), \( r = (R^2 + z^2)^{1/2} \)
Isothermal Distributions

If clusters are in some kind of equilibrium, the pressure gradient must balance gravity:

\[ \frac{dP}{dr} = -\rho \frac{GM(r)}{r^2} \]

The mass density and number density are related by the average mass:

\[ \rho(r) = \langle m \rangle n(r) \]

And the total mass within a radius \( r \) is given by

\[ M(r) = 4\pi \int_0^r dr \, r^2 \rho(r) \]
Isothermal Distributions

Differentiation of pressure equation gives:

\[
\frac{d}{dr} \left( \frac{r^2}{\rho} \frac{dP}{dr} \right) + 4\pi Gr^2 \rho = 0
\]

Using Ideal Gas Law, \( P = nkT \), and the fact that \( T \) is related to velocity dispersion:

\[
\frac{3}{2} k_B T = \frac{\langle m \rangle}{2} \langle v^2 \rangle ,
\]

The derivative of the pressure then becomes:

\[
\frac{dP}{dr} = \frac{k_B T}{\langle m \rangle} \frac{d\rho}{dr} = \frac{\langle v^2 \rangle}{3} \frac{d\rho}{dr} = \sigma_v^2 \frac{d\rho}{dr}
\]
Isothermal Distributions

The derivative of the pressure then becomes:

\[
\frac{dP}{dr} = k_B T \frac{d\rho}{\langle m \rangle \, dr} = \frac{\langle v^2 \rangle \, d\rho}{3 \, dr} = \sigma_v^2 \frac{d\rho}{dr}
\]

Recall that \( \langle v^2 \rangle = \sigma_x^2 + \sigma_y^2 + \sigma_z^2 = 3\sigma^2 \)

Therefore combining the above with our previous equation:

\[
\frac{d}{dr} \left( \frac{r^2 \, dP}{\rho \, dr} \right) + 4\pi Gr^2 \rho = 0
\]

we have then:

\[
\frac{d}{dr} \left( \frac{\sigma_v^2 \, r^2}{\rho} \, d\rho \right) + 4\pi Gr^2 \rho = 0
\]
Isothermal Distributions

\[ \frac{d}{dr} \left( \frac{\sigma_v^2 r^2}{\rho} \frac{d\rho}{dr} \right) + 4\pi G r^2 \rho = 0 \]

To solve this requires reasonable boundary conditions. Choose \( \rho(0) = \rho_0 \) to be the central density and \( (d\rho/dr)_{|r=0} = 0 \)

Solving the above gives:

\[ \rho(r) = \frac{\sigma_v^2}{2\pi G r^2} \]

This is the density distribution for a singular isothermal sphere. Numerical simulations give the central density to be related to the core radius:

\[ \rho_0 = \frac{9\sigma_v^2}{4\pi G r_c^2} \]
King Models

King models relieve the problem of diverging mass in isothermal spheres and have upper cut-off. Analytic approximation is

\[ \rho(r) = \rho_0 \left[ 1 + \left( \frac{r}{r_c} \right)^2 \right]^{-3/2} \]

This leads to a projected density using

\[ N(R) = \int_{-\infty}^{\infty} dz \, n \left( \sqrt{R^2 + z^2} \right) = 2 \int_{R}^{\infty} \frac{dr \, r \, n(r)}{\sqrt{r^2 - R^2}} \]

which gives

\[ \Sigma(R) = \Sigma_0 \left[ 1 + \left( \frac{R}{r_c} \right)^2 \right]^{-1} \quad \text{with} \quad \Sigma_0 = 2 \rho_0 r_c \]
King Models

King models relieve the problem of diverging mass in isothermal spheres and have upper cut-off. Analytic approximation is

\[ \rho(r) = \rho_0 \left[ 1 + \left( \frac{r}{r_c} \right)^2 \right]^{-3/2} \]

This leads to a projected density using

\[ N(R) = \int_{-\infty}^{\infty} dz \, n \left( \sqrt{R^2 + z^2} \right) = 2 \int_{R}^{\infty} \frac{dr \, r \, n(r)}{\sqrt{r^2 - R^2}} \]

which gives

\[ \Sigma(R) = \Sigma_0 \left[ 1 + \left( \frac{R}{r_c} \right)^2 \right]^{-1} \quad \text{with} \quad \Sigma_0 = 2 \rho_0 r_c \]

Typical core radius is \( r_c \sim 0.25 \, h^{-1} \, \text{Mpc} \).
Dynamical Mass of Clusters

Dynamical timescale is the crossing time:

\[ t_{\text{cross}} \sim \frac{R_A}{\sigma_v} \sim 1.5 h^{-1} \times 10^9 \text{ yr} \]

Where typical velocity dispersion is \( \sim 1000 \text{ km s}^{-1} \).

Because the dynamical time is \(<<\) than the age of the Universe, clusters must bound systems. Else they would have dissolved.

The Virial Theorem applies, so

\[ 2K + U = 0 \]

where

\[ K = \frac{1}{2} \sum_i m_i v_i^2 \quad \quad U = -\frac{1}{2} \sum_{i \neq j} \frac{G m_i m_j}{r_{ij}} \]
Dynamical Mass of Clusters

The total mass of the cluster is \( M = \sum m_i \)

the velocity dispersion is \( \langle v^2 \rangle = \frac{1}{M} \sum_i m_i v_i^2 \)

Define the gravitational radius: \( r_G = 2M^2 \left( \sum_{i \neq j} \frac{m_i m_j}{r_{ij}} \right)^{-1} \)

Then we can rewrite \( K \) and \( U \) as

\[
K = \frac{M}{2} \langle v^2 \rangle \quad U = -\frac{G M^2}{r_G}
\]

Solving for the mass yields

\[
M = \frac{r_G \langle v^2 \rangle}{G} = \frac{3r_G \sigma^2}{G}
\]
Dynamical Mass of Clusters

Transitioning to projected quantities, we have

\[ r_G = \frac{\pi}{2} R_G \]

where

\[ R_G = 2M^2 \left( \sum_{i \neq j} \frac{m_i m_j}{R_{ij}} \right)^{-1} \]

Therefore

\[ M = \frac{3\pi R_G \sigma_v^2}{2G} \]

\[ = 1.1 \times 10^{15} M_\odot \left( \frac{\sigma_v}{1000 \text{ km/s}} \right)^2 \left( \frac{R_G}{1 \text{ Mpc}} \right) \]

Monday, August 27, 2012
Missing Mass Problem in Clusters

In 1933 Fritz Zwicky measured the doppler shift velocities of galaxies in the Coma Cluster.

He measured the velocity dispersion (average velocity) of cluster galaxies to be $\sigma = 977$ km/s.

This gives a Virial Mass of $M = 5\sigma^2 \frac{R}{G} = 3.3 \times 10^{15} M_\odot$.

Comparing this to all the luminosity from the galaxies in the cluster, $L_{\text{tot}} = 5 \times 10^{12} L_\odot$ gives a mass-to-light ratio of

$$M/L \approx 660 \frac{M_\odot}{L_\odot}.$$ 

The Luminous matter in Coma accounts for $1/660 = 0.1\%$ of the mass! Zwicky argued in 1933 that Dark Matter must dominate clusters. Turns out it does, but at the time no one believed Zwicky.....
Cluster Galaxy Dynamics

Two-body collisions of galaxies yield a “relaxation” timescale,

\[ t_{\text{relax}} = t_{\text{cross}} \frac{N}{\ln N} \]

For clusters this is \(>>\) age of Universe.

Therefore, motion of galaxies is dominated by gravitational potential of cluster.

Galaxies in a cluster are not “thermalized”. If they were they would all have same mean kinetic energy, \(\sigma \sim m^{-1/2}\).

Rather other processes define velocity distribution of galaxies.
Violent Relaxation

This quickly establishes virial equilibrium in gravitational collapse.

Small inhomogeneities cause gravitational excesses.

These scatter infalling particles (galaxies) and density inhomogeneities are amplified.

Fluctuations act on matter like scattering centers. They change over time yielding an effective exchange of energy between particles.

In a statistical average all galaxies obtain the same velocity distribution.

Numerical simulations show this process takes as long as the crossing timescale -- it happens as fast as the collapse itself.
Dynamical Friction.

What happens as galaxy passes through cluster?

Effects are gravitational.

As a mass $M$ passes through medium, it feels the gravity and produces a “wake” of higher density (because masses in medium have been compressed along the path of the moving object. This is dynamical friction and is a net gravitational force on $M$ that opposes the galaxy’s motion. Kinetic energy was transferred from $M$ to the surrounding material as $M$’s speed is reduced.
Dynamical Friction.

Depends on:

Gravity, so must be proportional to $m$ (mass of moving object) and $\rho$, density of medium.

Inversely to velocity because faster particle feels gravity for shorter period.

Inversely to velocity (again) because particle has moved a greater distance from “wake”. Therefore:

$$\frac{dv}{dt} \propto -\frac{m \rho v}{|v|^3}$$
Contours show the density enhancement of stars due to the motion of a mass $M$ in the positive $z$ direction.
Dynamical Friction

Derivation of Force of dynamical friction is complicated. It depends on the speed, $v_M$, the density of the surrounding material, $\rho$, and mass, $M$, squared.

$$f_d \simeq C \frac{G^2 M^2 \rho}{v_M^2}$$

One can estimate the timescale for dynamical friction to halt two colliding objects. Consider the dynamical friction force on Globular clusters within the Milky Way. The dark matter distribution is

$$\rho(r) = \frac{v_M^2}{4\pi G r^2}$$

Insert this into the dynamical friction force equation above, which gives

$$f_d = C \frac{G^2 M^2 \rho(r)}{v_M^2} = C \frac{GM^2}{4\pi r^2}$$
Dynamical Friction

\[ f_d = C \frac{G^2 M^2 \rho(r)}{v_M^2} = C \frac{GM^2}{4\pi r^2} \]

Assume orbits are circular, then the angular momentum is just \( L = M \, v_M \, r \). And, the torque is \( \tau = r \, f_d = (dL / dt) \).

For a flat rotation curve, \( v_M \) is constant at large radii. Thus, the derivative of \( L \) is \( (dL/dt) = M v_M \, (dr/dt) \). This gives:

\[ M v_M \frac{dr}{dt} = -r C \frac{GM^2}{4\pi r^2} \]

Integrating this equation gives an expression describing the time required for the object to spiral into the center of the host medium from an initial radius, \( r_1 \).

\[
\int_{r_1}^{0} r \, dr = -\frac{CGM}{4\pi v_M} \int_{0}^{t_c} dt \quad \text{Or:} \quad t_C = \frac{2\pi v_M r_i^2}{CGM}
\]

Or, you can calculate the maximum distance an object could have traveled in a time \( t_{\text{max}} \):

\[ r_{\text{max}} = \sqrt{\frac{t_{\text{max}} CGM}{2\pi v_M}} \]
Consider a globular cluster that orbits in the Andromeda galaxy (M31). Assume the cluster’s mass is $5 \times 10^6$ solar masses with $v_M = 250$ km s$^{-1}$. Age of M31 is approximately $\sim 13$ Gyr.

The maximum radius at which a GC could have spiraled into the center of M31 is $r_{\text{max}} = 3.7$ kpc, whereas the “halo” of M31 is more like 100 kpc.

Note that $r_{\text{max}} \sim M^{1/2}$. Clusters with greater masses have higher maximum radii, so this likely explains the lack of massive GCs in M31 today.

NOTE ! Dynamical friction affects globular clusters, and satellite galaxies around larger galaxies, and other galaxies in clusters of galaxies.
Morphology-Density Relation

Strong Morphology-Density Relation. Whereas 70% of field galaxies are spirals, clusters are dominated by ellipticals.

The morphology–density relation in the Sloan Digital Sky Survey

Tomotsugu Goto,1,2*† Chisato Yamauchi,3 Yutaka Fujita,3 Sadanori Okamura,2 Maki Sekiguchi,1 Ian Smail,4 Mariangela Bernardi5 and Percy L. Gomez5

Color-Density Relation

E+A Galaxies

Also referred to as K+A or “post-starburst” galaxies.

First found in Galaxy clusters, spectra look like K-stars superimposed with A-stars (showing strong Balmer lines) with no other star-formation indication.

Presence of A-stars indicates star formation within the last ~1 Gyr.

Dressler & Gunn 1983
X-ray Radiation from Clusters of Galaxies

Starting with UHURU satellite (1970s) and then Einstein and ROSAT X-ray satellites, discovery that Clusters emit copious X-ray emission.
X-ray Radiation from Clusters of Galaxies

Emission process is optically thin thermal bremsstrahlung (free-free radiation) from a hot, low-density gas in *Intracluster Medium (ICM)*. Because emission depends on relative speed of electrons, it depends on gas Temperature. Depends on cluster mass. Clusters with mass $\sim 10^{14} - 10^{15} \, M_\odot$ have temperatures of $10^7$-$10^8 \, K$ (1-10 keV).

$\epsilon_{ff} = \frac{32\pi Z^2 e^6 n_e n_i}{3 m_e c^3} \sqrt{\frac{2\pi}{3 k_B T m_e}} e^{-h\nu/k_B T} g_{ff}(T, \nu)$

“gaunt” factor

$g_{ff} \approx \frac{3}{\sqrt{\pi}} \ln \left( \frac{9k_B T}{4h\nu} \right)$

For Solar Abundance:

$\epsilon_{ff} \approx 3.0 \times 10^{-27} \sqrt{\frac{T}{1 \, K}} \left( \frac{n_e}{1 \, \text{cm}^{-3}} \right)^2 \text{erg cm}^{-3} \text{s}^{-1}$
X-ray Radiation from Clusters of Galaxies

Bremsstrahlung (ff-emission)

$k_B T_e = 1 \text{ keV, 3 keV, 9 keV; } N_H = 0 \text{ cm}^{-2}$
X-ray Radiation from Clusters of Galaxies

**Line Emission**: Hot diffuse gas hypothesis was confirmed by presence of highly ionized atomic species in ICM emission. Most prominent is Hydrogen-like iron line, Fe XXVI, just below 7 keV.

At lower temperatures, < 2 keV, dominated by line emission from C, N, O, Mg, Ne, Si, S, Ar, Ca, etc.

\[ \epsilon \approx 6.2 \times 10^{-19} \left( \frac{T}{1 \text{ K}} \right)^{-0.6} \left( \frac{n_e}{1 \text{ cm}^{-3}} \right)^2 \]

Including continuum and lines, emissivity is the above (for Solar Abundance for \( T \sim 10^5 - 10^7 \) K)
X-ray Radiation from Clusters of Galaxies

Including photo-absorption of neutral hydrogen with given column density

$\text{ff+fb+bb-\text{emission}}$

$k_B T_e = 3 \text{ keV}, \quad A = 0.4; \quad N_H = 0, \quad 3 \times 10^{-20}, \quad 10^{21} \text{ cm}^{-2}$

---

Monday, August 27, 2012
**Models for X-ray emission**

**Hydrostatic Assumption.** Consider speed of sound,

\[ c_s \approx \sqrt{\frac{P}{\rho_g}} = \sqrt{\frac{n k_B T}{\rho_g}} = \sqrt{\frac{k_B T}{\mu m_p}} \sim 1000 \text{ km s}^{-1} \]

where \( \mu \) is the average molecular mass:

\[ \mu := \frac{\langle m \rangle}{m_p} \]

For a fully ionized gas, \( \mu=1/2 \). For gas with heavier elements, \( \mu\sim0.63 \). The sound-crossing time for the cluster is

\[ t_{sc} = \frac{2 R_A}{c_s} \sim 7 \times 10^8 \text{ yr} \]

The sound crossing timescale is the time for “pressure” equilibrium to occur.
Models for X-ray emission

Hydrostatic Equilibrium:

\[ \nabla P = -\rho_g \nabla \Phi \]

For a spherically symmetric case,

\[ \frac{1}{\rho_g} \frac{dP}{dr} = -\frac{d\Phi}{dr} = -\frac{GM(r)}{r^2} \]

Inserting \( P = nk_B T = \rho_g k_B T / (\mu m) \) and differentiating \( \rho_g \) and \( T \) need to be determined from the X-ray observations, which is a challenge. But, if you do this, you get the cluster Mass.

Assuming a radially constant \( T \) simplifies the equation, but normally clusters show temperature gradients.
Models for X-ray emission

**The β-model**

Assuming density profile of matter is isothermal, we have

\[
\frac{d \ln \rho}{dr} = -\frac{1}{\sigma_v^2} \frac{GM}{r^2}
\]

And isothermal, then

\[
\frac{d \ln \rho_g}{dr} = -\frac{\mu m_p}{k_B T_g} \frac{GM}{r^2}
\]

This yields:

\[
\rho_g(r) \propto [\rho(r)]^\beta \quad \text{with} \quad \beta := \frac{\mu m_p \sigma_v^2}{k_B T_g}
\]
Models for X-ray emission

**The β-model**

For the King model formula, this gives.

\[
\rho_g(r) = \rho_{g0} \left[ 1 + \left( \frac{r}{r_c} \right)^2 \right]^{-3\beta/2}
\]

For a surface brightness profile of:

\[
I(R) \propto \left[ 1 + \left( \frac{R}{r_c} \right)^2 \right]^{-3\beta+1/2}
\]

Fits to clusters give \( r_c = 0.1 - 0.3 \ \text{h}^{-1} \ \text{Mpc} \) and \( \beta \sim 0.65 \).

Alternatively \( \beta \) is measured from velocity dispersion, which gives \( \beta \sim 1 \).

This is the \( \beta \)-discrepancy. Possibly owing to the fact that the gas is not isothermal.
Cooling Flows in Clusters

We have assumed ICM is in hydrostatic equilibrium, but as gas cools via its emission, it should lose internal energy in absence of some other heating mechanism.

The cooling timescale is

\[ t_{\text{cool}} := \frac{u}{\epsilon_{\text{ff}}} \]

\[ \approx 8.5 \times 10^{10} \text{ yr} \left( \frac{n_e}{10^{-3} \text{ cm}^{-3}} \right)^{-1} \left( \frac{T_g}{10^8 \text{ K}} \right)^{1/2} \]

where \( u \) is the energy density, \( u = (3/2) \, n k T \). At the centers of clusters, the density gets large, and \( t_{\text{cool}} < t_H \) (the hubble time). Then hydrostatic equilibrium should no longer apply.

Some clusters show cooling flows at the level of 100 solar masses per year.
Chandra image of Centaurus cluster. Colors indicate cool inner region possibly from a cooling flow.
Radio jets. X-ray emission suppressed.
Possible holes in X-ray-emitting gas
The **Bullet Cluster.** This object appears to be two galaxy clusters that have merged.
Evidence for Dark Matter

The **Bullet Cluster**. This object appears to be two galaxy clusters that have merged. Most of the galaxies passed through each other, but the hot X-ray-emitting gas smashed into each other and stopped in its tracks.

The gravitational lensing analysis of background galaxies shows that all the mass (the dark matter) has followed the galaxies. The Dark Matter is acting solely as point sources that interact only by gravity. No other known model for gravity can explain this except Dark Matter.
The Bullet Cluster

- Orange: stars
- Red: X-ray gas
- Blue: Mass from lensing measurements

See Clowe et al. 2006
The Bullet Cluster

Orange: stars
Red: X-ray gas
Blue: Mass from lensing measurements

See Clowe et al. 2006
The Bullet Cluster

Orange: stars
Red: X-ray gas
Blue: Mass from lensing measurements

See Clowe et al. 2006
The Bullet Cluster

Orange: stars
Red: X-ray gas
Blue: Mass from lensing measurements

See Clowe et al. 2006

Monday, August 27, 2012
The Sunyaev-Zeldovich Effect

Free Electrons in ICM upscatter CMB photons

\[
\frac{\Delta I^\text{RJ}_\nu}{I^\text{RJ}_\nu} = -2y
\]

where

\[
y = \int dl \frac{k_B T_g}{m_e c^2} \sigma_T n_e
\]

with

\[
\sigma_T = \frac{8\pi}{3} \left( \frac{e^2}{m_e c^2} \right)^2
\]

where \( y \) is the Compton-\( \gamma \) parameter. \( y \) is proportional to the Thompson cross section, \( \sigma_T \), times the electron density \( n_e \). This is the Thompson optical depth: \( n_e \sigma_T \).

Overall \( y \) is proportional to the gas pressure, \( P = n kT \) along the line of sight.
The Sunyaev-Zeldovich Effect
The Sunyaev-Zeldovich Effect

SZ maps of three clusters at $0.37 < z < 0.55$. Since SZ is proportional to electron density, mass fraction of baryons can be measured if one knows the total mass of the cluster.