Physics 218 Honors
Exam 1

1. An astronaut (mass 80 kg) is tethered to a spacecraft (mass $10^5$ kg) by a cord. Both are traveling initially in uniform motion. The astronaut pulls on the cord with a force of 100 N. Calculate a) the acceleration of the astronaut; b) the acceleration of the spacecraft; c) the tension in the cord.

A = acceleration of spacecraft
a = acceleration of astronaut
M = mass of spacecraft
M = mass of astronaut
T = tension in cord = 100 N

Newton #2:

T = MA  \quad A = \frac{T}{M} = \frac{(100 \text{ N})}{(10^5 \text{ kg})} = 10^{-3} \text{ m/s}^2 \quad \text{towards the astronaut}

T = ma  \quad a = \frac{T}{m} = \frac{(100 \text{ N})}{(80 \text{ kg})} = 1.125 \text{ m/s}^2 \quad \text{towards the spacecraft}
2. A clever physics student (mass 60 kg) decides that she can descend from a height of 10 m above ground by grasping a rope attached to a pulley, mounted from a beam above her, and jumping horizontally off the building. The other end of the rope is attached to a sandbag of mass 50 kg sitting on the ground. What is the velocity with which the student lands on the ground?

M = mass of sandbag
M = mass of student
T = tension in rope

Take up to be + for force and acceleration

Newton #2 for student:

\[ +T - mg = -ma \]

Newton #2 for sandbag:

\[ +T - Mg = +Ma \]

Note: if the acceleration of the student is down with value -a, the acceleration of the sandbag will be up with the value +a.

Eliminate T by subtracting the two equations:

\[ (M+m)a = (m-M)g \]

\[ a = \frac{(m-M)}{m+M} g = \frac{10}{110} \cdot 9.8 = 0.89 m/s^2 \]

Now calculate the velocity when she hits ground. Her equations of motion are

\[ y = y_0 - \frac{1}{2} at^2 \]
\[ v = -at \quad t = -v/a \]

\[ 0 = 10 - \frac{1}{2} a\left(\frac{v}{a}\right)^2 \]
\[ v = \sqrt{2 \cdot 20 \cdot 0.89} = 4.2 m/s \]
3. A chain consists of 5 links, each succeeding link having twice the mass of the last. Starting with the lowest link, the link masses are 1 kg, 2 kg, 4 kg, 8 kg, and 16 kg. The heaviest link is suspended from a beam and the chain hangs freely. Calculate the force acting between each pair of adjacent links.

Let

\[ F_1 = \text{force between 1 kg link and 2 kg link} \]
\[ F_2 = \text{force between 2 kg link and 4 kg link} \]
\[ F_3 = \text{force between 4 kg link and 8 kg link} \]
\[ F_4 = \text{force between 8 kg link and 16 kg link}. \]

\[ F_1 = (1 \text{ kg})(9.8 \text{ m/s}^2) = 9.8 \text{ N} \]
\[ F_2 = (1 \text{ kg} + 2 \text{ kg})(9.8 \text{ m/s}^2) = 29.4 \text{ N} \]
\[ F_3 = (1 \text{ kg} + 2 \text{ kg} + 4 \text{ kg})(9.8 \text{ m/s}^2) = 68.6 \text{ N} \]
\[ F_4 = (1 \text{ kg} + 2 \text{ kg} + 4 \text{ kg} + 8 \text{ kg})(9.8 \text{ m/s}^2) = 147 \text{ N} \]
4. Using unit vectors, write expressions for the four body diagonals of a cube (straight line segments from one corner to another through the center), assuming one corner to be located at the origin and 3 edges to be aligned along the x, y, and z axes. Determine the angles that the body diagonals make with the adjacent edges. Determine the length of a body diagonal in terms of the edge length $a$.

\[ b_1 = a (\hat{i} + \hat{j} + \hat{k}) \]
\[ b_2 = a (\hat{i} + \hat{j} - \hat{k}) \]
\[ b_3 = a (\hat{i} - \hat{j} + \hat{k}) \]
\[ b_4 = a (-\hat{i} + \hat{j} + \hat{k}) \]
\[ \vec{b}_1 \cdot \hat{i} = |\vec{b}_1| \cos \vartheta = a((\hat{i} + \hat{j} + \hat{k}) \cdot \hat{i} \]
\[ |\vec{b}_1|^2 = a^2 (\hat{i} + \hat{j} + \hat{k}) \cdot (\hat{i} + \hat{j} + \hat{k}) = 3a^2 \]
\[ b_1 = \frac{a}{\sqrt{3}} \]
\[ \cos \vartheta = \frac{a}{\sqrt{3}a} = 1/\sqrt{3} \]
\[ \vartheta = 55^\circ \]
5. An exciting amusement park ride consists of a car that travels with the acceleration shown in the graph. The car is at rest at t=0. How far does the car travel in the time from t=0 to t=20 s? What is its maximum velocity? What is the average velocity of the car during the first 16 seconds of motion?

\[ a(t) = \begin{cases} 0 & \text{for } t<6 \text{ s} \\ 25(t-6)/10 & \text{for } 6<t<16 \text{ s} \\ 25 & \text{for } t>16 \text{ s} \end{cases} \]

- For \( t<6 \) s, \( v = 0 \)
- For \( 6 \leq t < 16 \) s, \( v(t) = \int_0^t a(t) \, dt = \int_6^t \frac{25}{10} (t-6) \, dt = \frac{25}{10} \int_6^t t \, dt = \frac{25}{20} t^2 \)
  \( v(16 \text{ s}) = 125 \text{ m/s} \)
- For \( t > 16 \) s, \( v = 125 \text{ m/s} \) This is the maximum velocity.

Distance traveled in first 20 s:

\[ x = \int_0^{20} v(t) \, dt = \int_0^6 \frac{25}{20} (t-6)^2 \, dt + \int_6^{16} 125 \, dt = \frac{25}{60} (16-6)^3 + 125(20-16) = 916 \text{ m} \]

Average velocity on the interval \( 0 < t < 16 \) s:

\[ \langle v \rangle = \frac{x(16 \text{ s}) - x(0)}{16 \text{ s}} = \frac{25}{60} (16-6)^3}{16} = 26 \text{ m/s} \]
6. A stone is thrown vertically upward. On its way up, it passes point A with speed \(v_0\), and it passes point B (3.00 m higher than point A) with speed 0.70 \(v_0\). Calculate the speed \(v_0\) and the maximum height reached by the stone above point B.

Choose y-axis vertical, origin at point A, \(t=0\) when stone passes point A.

Equations of motion:
\[
y(t) = v_0 t - \frac{1}{2} gt^2
\]
\[
v(t) = v_0 - gt
\]

Now the stone has velocity 0.7 \(v_0\) when it passes point B, \(y = 3\) m:
\[
0.7v_0 = v_0 - gt_B
\]
\[
t_B = 0.3v_0 / g
\]

\[
3 = v_0 t_B - \frac{1}{2} gt_B^2 = (0.3 - \frac{0.3^2}{2}) \frac{v_0^2}{g} = 0.255 \frac{v_0^2}{g}
\]

\[
v_0 = 10.7 \text{ m/s}
\]

Maximum height above B:
\[
v(t) = 0: \quad t = \frac{v_0}{g} = 1.09 \text{ s}
\]
\[
y = (10.7 \text{ m/s})(1.09s) - \frac{1}{2}(9.8 \text{ m/s}^2)(1.09s)^2 = 5.88m
\]

So max height above point B is 2.88 m.