GOAL: So far we have described charges at rest. Our goal in this chapter is to describe charges in motion.
A current (electric current) is any motion of (positive!) charges from one region of a conductor to the other.

This motion is caused by an electric field within the conductor, so that the charges experience a force

\[ \vec{F} = q \vec{E} \]

**Def. of current**

\[ I = \frac{\Delta Q}{\Delta t} \]

\( \Delta Q \): amount of charge passing through cross section \( A \) of the wire

\( \Delta t \): per unit of time
Units: \[ [I] = \frac{1C}{1S} = 1A \quad A = \text{Ampere} \]

Notice: current describes by convention the flow of positive charges.

Current parallel to moving charges

Electrons moving to the left result in a motion of positive charges (current) to the right!

Charge Conservation

As discussed in lecture 1, charge (and thus current) is conserved in an electrical circuit.
RESISTANCE AND OHM'S LAW

For some conductors the current I is proportional to the potential drop in the conductor

\[ I \propto V \quad \text{or} \quad V = I \cdot R \]

The proportionality constant "R" is called resistance

Units: \( [R] = \frac{V}{A} = \Omega = \text{Ohm} \)

The relation \( V = IR \) is called Ohm's law and it describes an idealized situation, that is only
Satisfied to a degree of approximation.

**Resistivity**

Consider a conductor with the form of a cylinder

\[ A = \text{cross-section} \]
\[ L = \text{length} \]

The resistance \( R \) will naturally increase with the length \( L \)

\[ R \propto L \]

and decrease with the surface cross section \( A \)

\[ R \propto \frac{L}{A} \quad \text{or} \quad R = \rho \frac{L}{A} \]
The proportionality constant $\rho$ (rho) is called resistivity

\[ \rho = \text{resistivity} \]

Just like e.g. the density of materials, the resistivity is a property of the material (and temperature) that does not depend on the shape and size.

Units: \([\rho] = \Omega \times m\)

IMPORTANT: It is important to distinguish between resistance and resistivity. $R$ depends on $A, L$ while $\rho$ does not!
Perfect conductor $\Rightarrow$ zero resistivity
Perfect insulator $\Rightarrow$ infinite resistivity

**TEMPERATURE DEPENDENCE**

The resistance of every conductor varies with the temperature and for a small temperature range (up to 100°C) this is described by the eqn:

$$R_T = R_0 \left(1 + \alpha (T - T_0)\right)$$

- $R_T$: reference resistance
- $R_0$: temperature coefficient
- $T$: reference temperature
- $T_0$: temperature
SUPERCONDUCTIVITY

Some materials (metallic alloys, oxides) show a phenomenon called superconductivity. From $R_T = R_0 (1 + \alpha (T-T_0))$, we see that the resistance of ordinary conductors decreases if the temperature decreases. Superconductors behave in this way until the temperature drops to a value where a "phase transition" takes place.

$\rho \rightarrow 0$ superconductor

The current will flow indefinitely as there is no resistance.
This can be used in computer chips, power distribution.

Superconductivity was found in 1911 by Kamerlingh-Onnes and is nowadays explained by the BCS theory and Landau-Ginzburg theory.

**EXERCISE**

Two wires made of pure copper have different resistances. These wires may differ in:

A. length
B. cross section
C. resistivity
D. temperature
Answer: All choices are correct! (A), (B) is clear. If the wires are of the same material then $R$ is the same. However, $R$ depends on $T$, so if the wires are at a different temperature, then $R$ changes $\Rightarrow$ (C) is correct. (D) is correct for a similar reason.

**EXERCISE**

A copper wire with diameter 1.02 mm and a cross section of $8.20 \cdot 10^{-7} \text{m}^2$ has a resistance of 1.02 $\Omega$ at a temperature of 20°C.
Find the resistance at 0°C and 100°C. The temperature coefficient of resistivity of copper is 0.0039 °C⁻¹.

Answer:

At T = 0°C

\[ R = R_0 (1 + \alpha (T - T_0)) = \]
\[ = 1.02 \, \Omega \left( 1 + (0.0039 \, \text{°C}^{-1}) (0^\circ \text{C} - 20^\circ \text{C}) \right) \]
\[ = 0.945 \, \Omega \]

At T = 100°C gives \( R = 1.34 \, \Omega \)
ELECTROMOTIVE FORCE + CIRCUITS

For a current to circulate through a circuit we need a source of energy called emf (e.g. battery)

\[ \text{emf} = \text{electromotive force} \]

An ideal emf maintains a constant potential difference between its two terminals, independent of the current!

\[ \text{V}_{\text{ab}} \]

Battery

Chemical process results in \( \text{V}_{\text{ab}} \)
IMPORTANT! \( \text{Emf is not a force but has units } \frac{\text{energy}}{\text{charge}} \)

The value of the emf is the energy per unit charge that the source provides to move charges from the \( \Theta \) terminal to the \( + \) terminal.

For an ideal \( \text{Emf} \) this is stored as potential between terminal \( i.e. \)

\[
\mathcal{E} = V_{ab}
\]
If we add a resistor and build a circuit

The voltage drop across the lamp is

\[ V = IR \quad \text{Ohm's law} \]

and thus

\[ \Delta V_{ab} = IR \]
EXERCISE
Current in a circuit

A circuit contains a battery and two identical light bulbs, which are resistors. The battery maintains a constant potential difference between its terminals independently of the current through it.
The bulbs are equally bright when the switch is closed, so an equal amount of current flows through them.

When the switch is opened: what happens to the brightness of bulb A?

A) It increases (because bulb A now receives the current that formerly went through B)

B) It stays the same (the current through bulb A is unchanged)

Answer:
Answer: The key is to notice that a battery is a source of constant potential but not constant current!

A 9V battery produces whatever current is determined by the resistance of the external circuit but the Emf is always 9V!

The current in bulb A depends only on its resistance R and the voltage across it.

Since bulbs A, B are in parallel the voltage across them is the same
The presence (or absence) of B does not influence A at all and the brightness remains the same ⇒ \( \text{(B)} \) is correct.

What changes in the process is the current provided by the battery. When the switch is closed we need twice as much current from the battery!

\[
\text{INTERNAL RESISTANCE IN A SOURCE OF EMF}
\]

Real sources do not behave as ideal sources because that charge encounters resistance “r” (called internal resistance).
The total emf is then
\[ \text{Emf} = V_{ab} + IR \]  
\[ \text{non-idealized situation} \]

from Ohm's law

The current \( I \) is still determined by
\[ V_{ab} = IR \]

From (1, 2) we obtain the current for a source with internal resistance
\[ I = \frac{\text{Emf} \neq}{R + r} \]