Problem 1 (10 Points)
Show that the supersymmetry transformation of a chiral superfield $\Phi(x, \theta, \bar{\theta})$

$$\delta_\epsilon \Phi = (\epsilon Q + \bar{\epsilon} \bar{Q}) \Phi,$$

reproduces the supersymmetry transformations for the component fields $(A, \psi, F)$ discussed in class.

Problem 2 (10 Points)
Use the expression for a vector superfield in Wess-Zumino gauge to derive the component expansion of the superfield field strength. Show that this field strength is gauge invariant and satisfies the superspace Bianchi identity

$$D^a W_\alpha = \bar{D}_{\dot{\alpha}} W^{\dot{\alpha}}.$$

Show that the superfield strength chiral and derive its component expansion.

Problem 3 (10 Points)
Solve Wess and Bagger Chapter 5, problem 1. Use this to check that the $F$-component (i.e. the $\theta \theta$ term) of a chiral superfield transforms as a total derivative under a supersymmetry transformation.

Problem 4 (10 Points)
Solve Problem 1 of Wess and Bagger Chapter 6.

Problem 5 (10 Points each)
The most general supersymmetric hermitian and renormalizable Lagrangian in terms of chiral superfields can be written as

$$L = K(\Phi, \Phi^\dagger)|_D + (W(\Phi)|_F + hc),$$

where “hc” indicates the hermitian conjugate. Here $K$ is the Kähler potential and $W$ is the superpotential, which take the form

$$K(\Phi, \Phi^\dagger) = \Phi_i \Phi_i^\dagger,$$

$$W(\Phi) = \frac{1}{2} m_{ij} \Phi_i \Phi_j + \frac{1}{3} g_{ijk} \Phi_i \Phi_j \Phi_k,$$

respectively. Derive the component expansion of $K$ and $W$ in terms of $(A, F, \psi)$. 