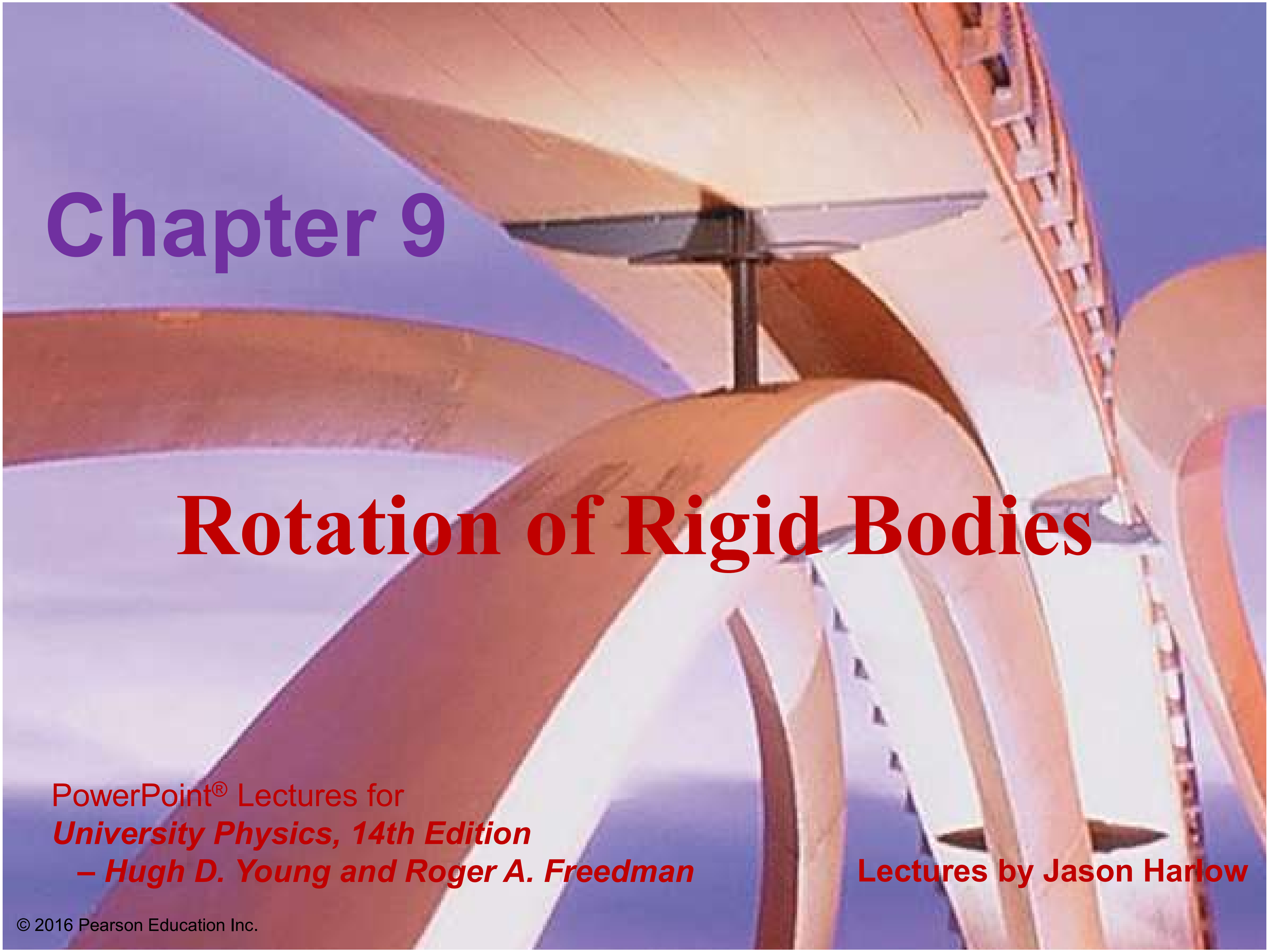


Chapter 9

Rotation of Rigid Bodies

PowerPoint® Lectures for
University Physics, 14th Edition
– Hugh D. Young and Roger A. Freedman

Lectures by Jason Harlow

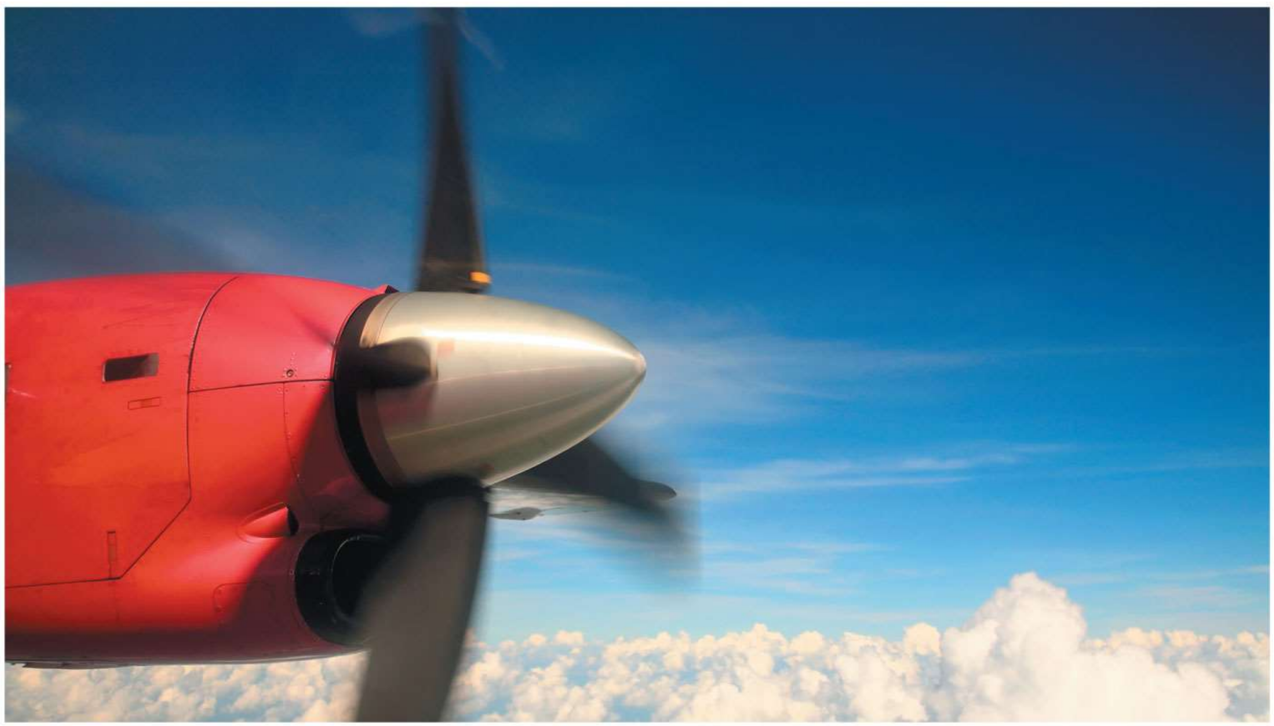
Learning Goals for Chapter 9

Looking forward at …

- how to describe the rotation of a rigid body in terms of angular coordinate, angular velocity, and angular acceleration.
- how to analyze rigid-body rotation when the angular acceleration is constant.
- the meaning of a body's moment of inertia about a rotation axis, and how it relates to rotational kinetic energy.
- how to calculate the moment of inertia of bodies with various shapes, and different rotation axes.

Introduction

- An airplane propeller, a revolving door, a ceiling fan, and a Ferris wheel all involve rotating rigid objects.
- Real-world rotations can be very complicated because of stretching and twisting of the rotating body. But for now we'll assume that the rotating body is perfectly rigid.

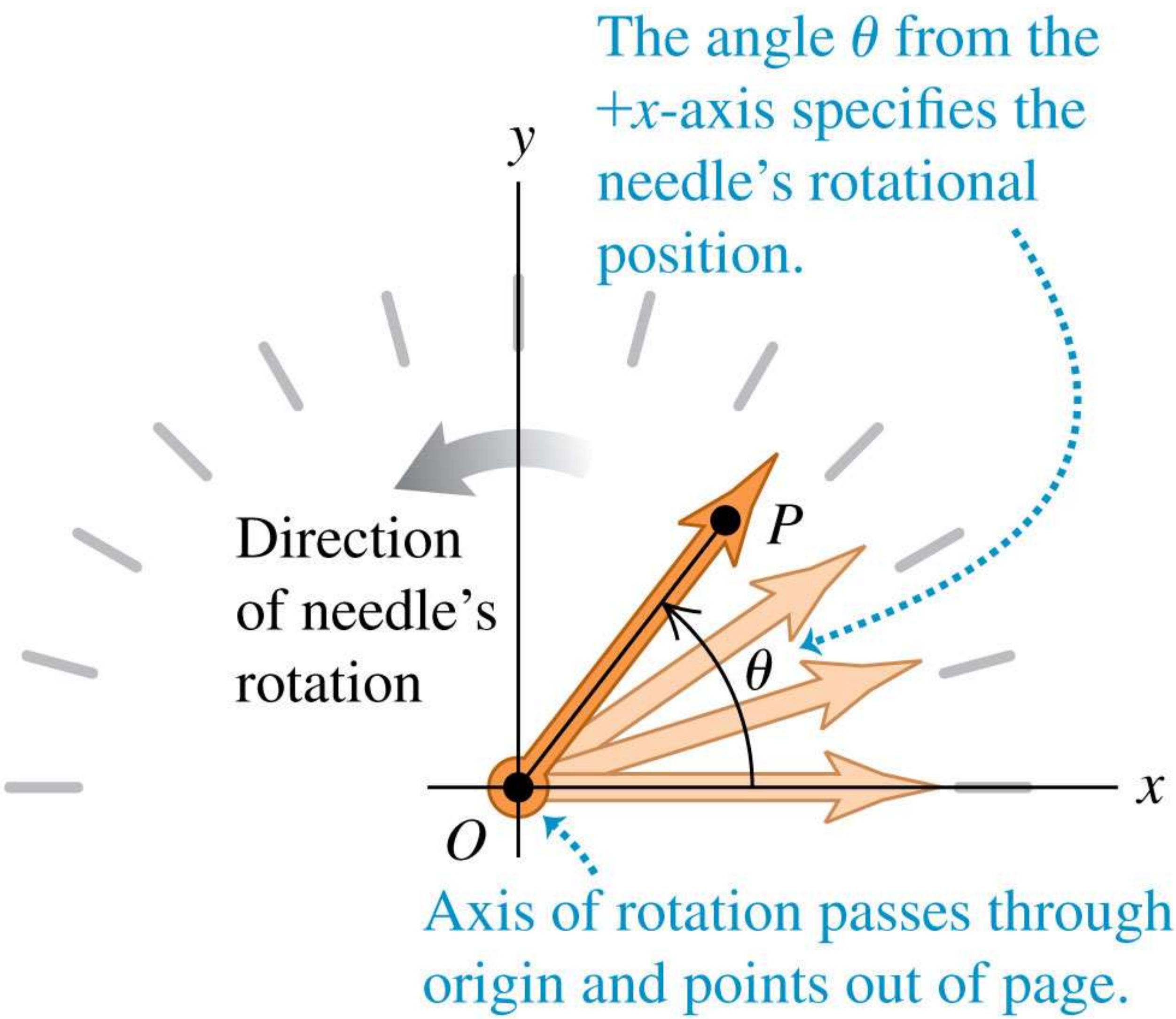


Some Jargon

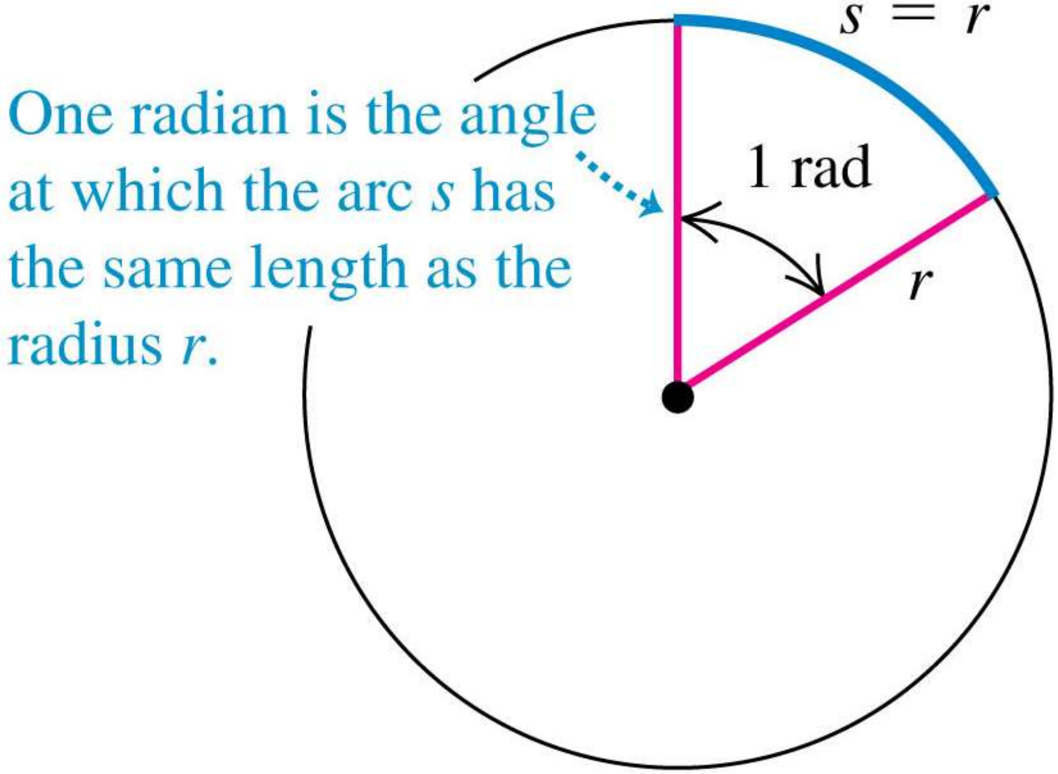
- *Fixed axis:* *I.e,* an object spins in the same place… objects on the rim of the tire go around the same place over and over again
  - –Example: Earth has a fixed axis, the sun
- *Rigid body:* *I.e,* the objects don't change as they rotate.
  - –Example: a bicycle wheel
  - –Examples of Non-rigid bodies?

Angular coordinate

- A car's speedometer needle rotates about a *fixed axis*.



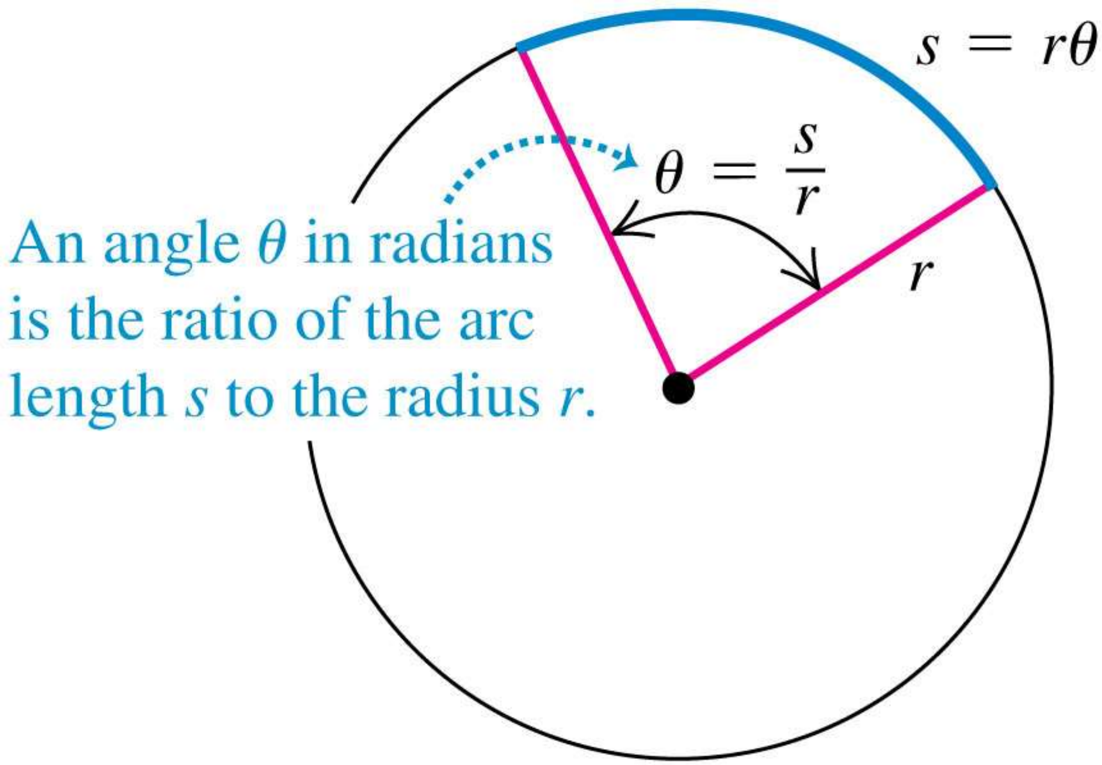
Units of angles



- One complete revolution is 360° = 2π radians.

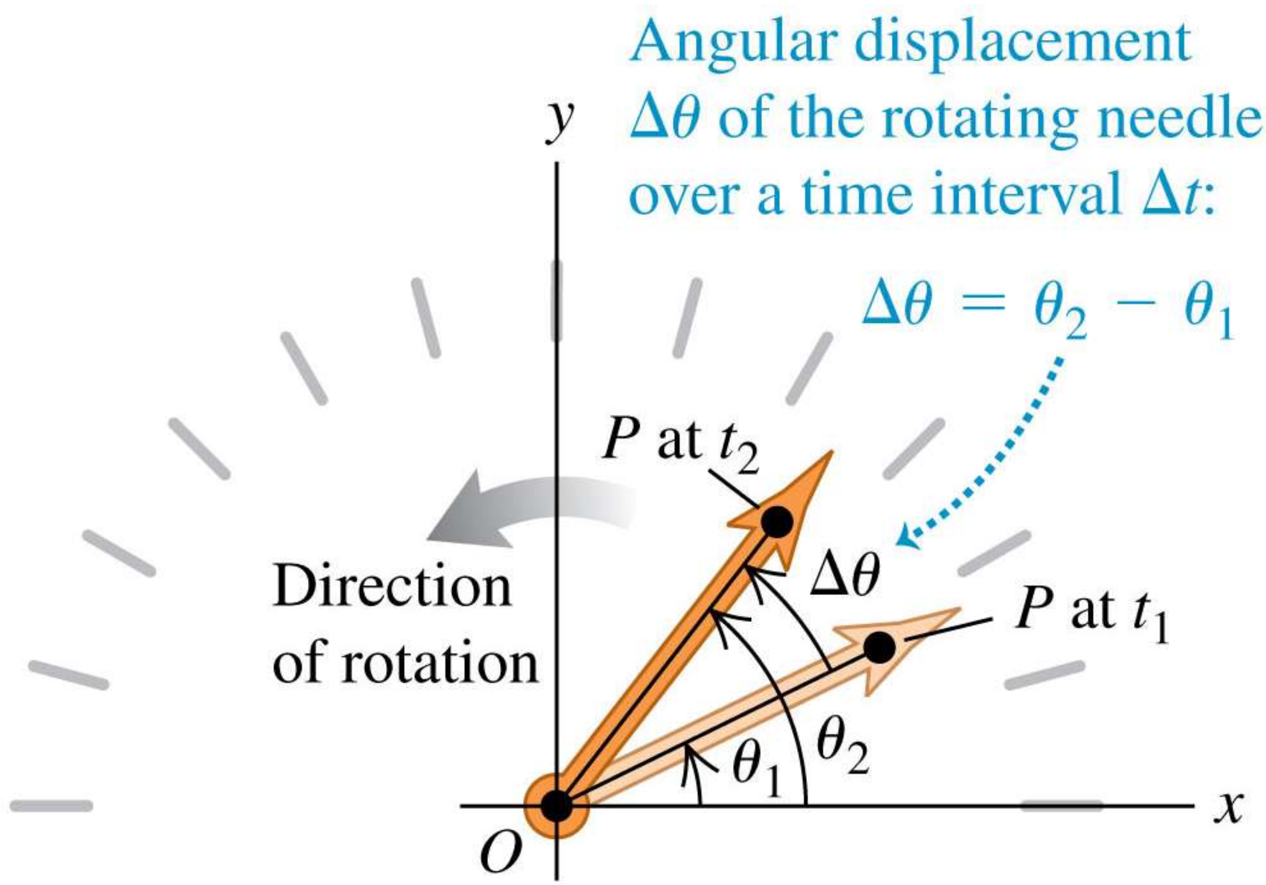
Units of angles

- An angle in radians is $\theta = s/r$, as shown in the figure.



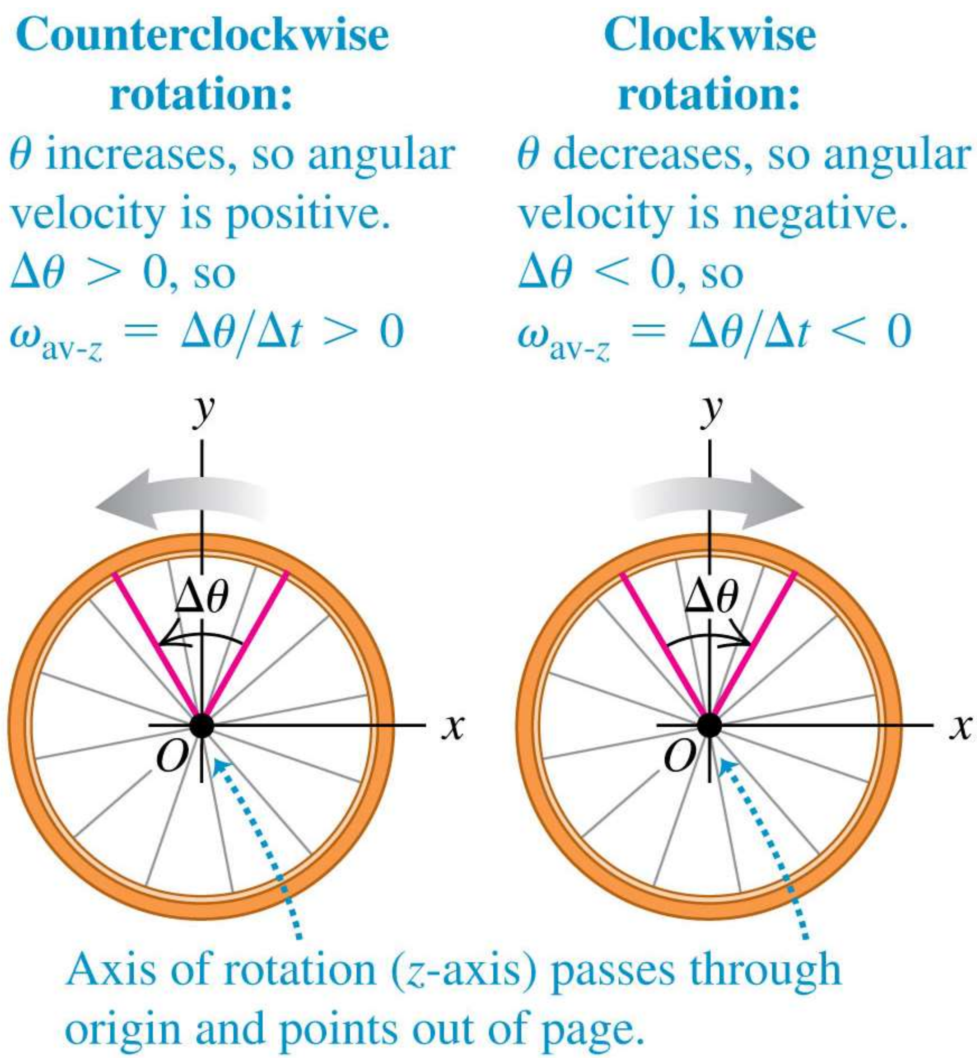
Angular velocity

- The *average angular velocity* of a body is $\omega_{\text{av-}z} = \Delta\theta/\Delta t$.
- The subscript *z* means that the rotation is about the *z*-axis.



Angular velocity

- We choose the angle θ to increase in the counterclockwise rotation.



Instantaneous angular velocity

- The **instantaneous angular velocity** is the limit of average angular velocity as $\Delta\theta$ approaches zero:

The **instantaneous angular velocity** of a rigid body rotating around the *z*-axis ...

$$\omega_z = \lim_{\Delta t \to 0} \frac{\Delta\theta}{\Delta t} = \frac{d\theta}{dt}$$

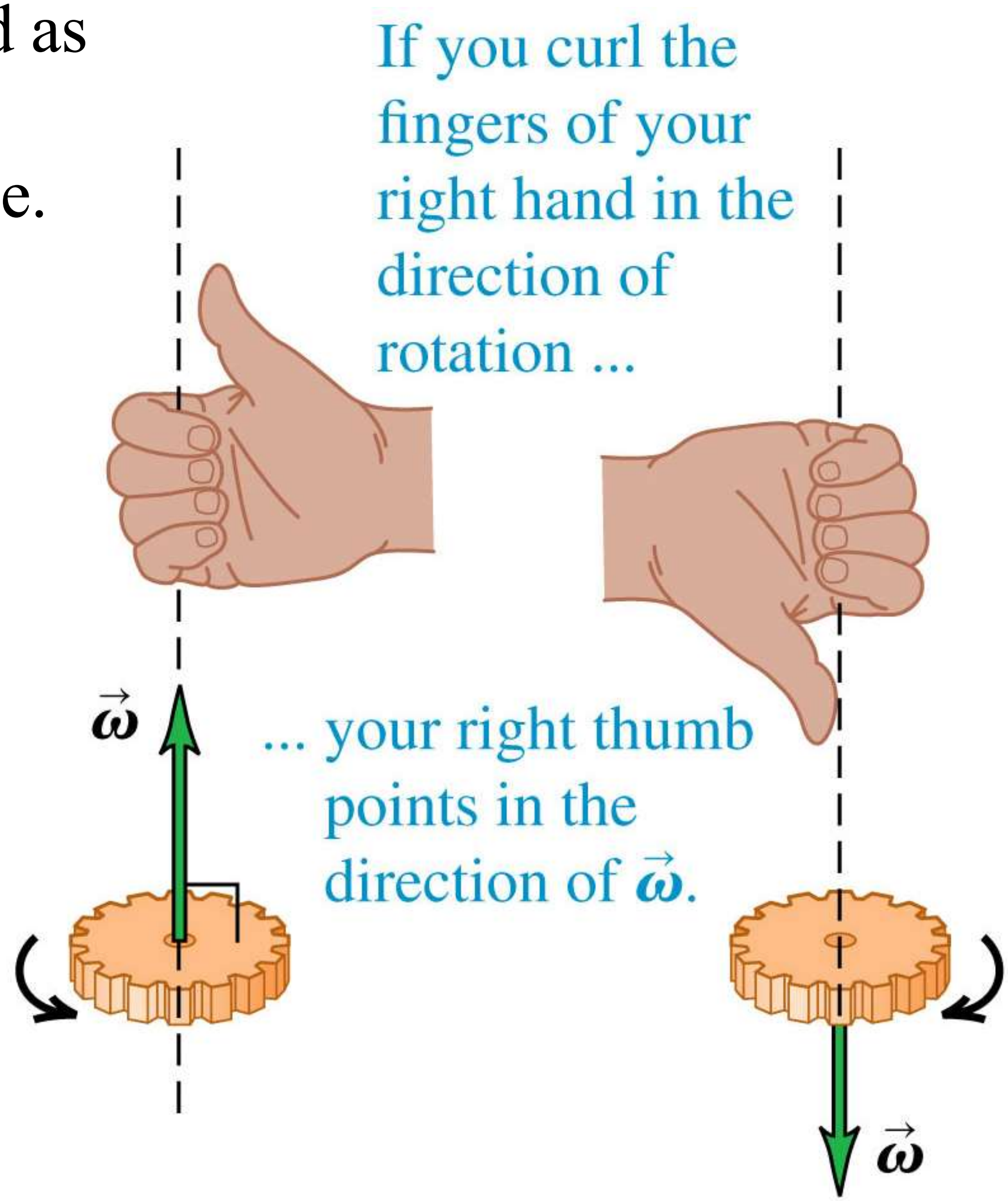
... equals the limit of the body's average angular velocity as the time interval approaches zero ...

... and equals the instantaneous rate of change of the body's angular coordinate.

- When we refer simply to "angular velocity," we mean the instantaneous angular velocity, not the average angular velocity.
- The *z*-subscript means the object is rotating around the *z*-axis.
- The angular velocity can be positive or negative, depending on the direction in which the rigid body is rotating.

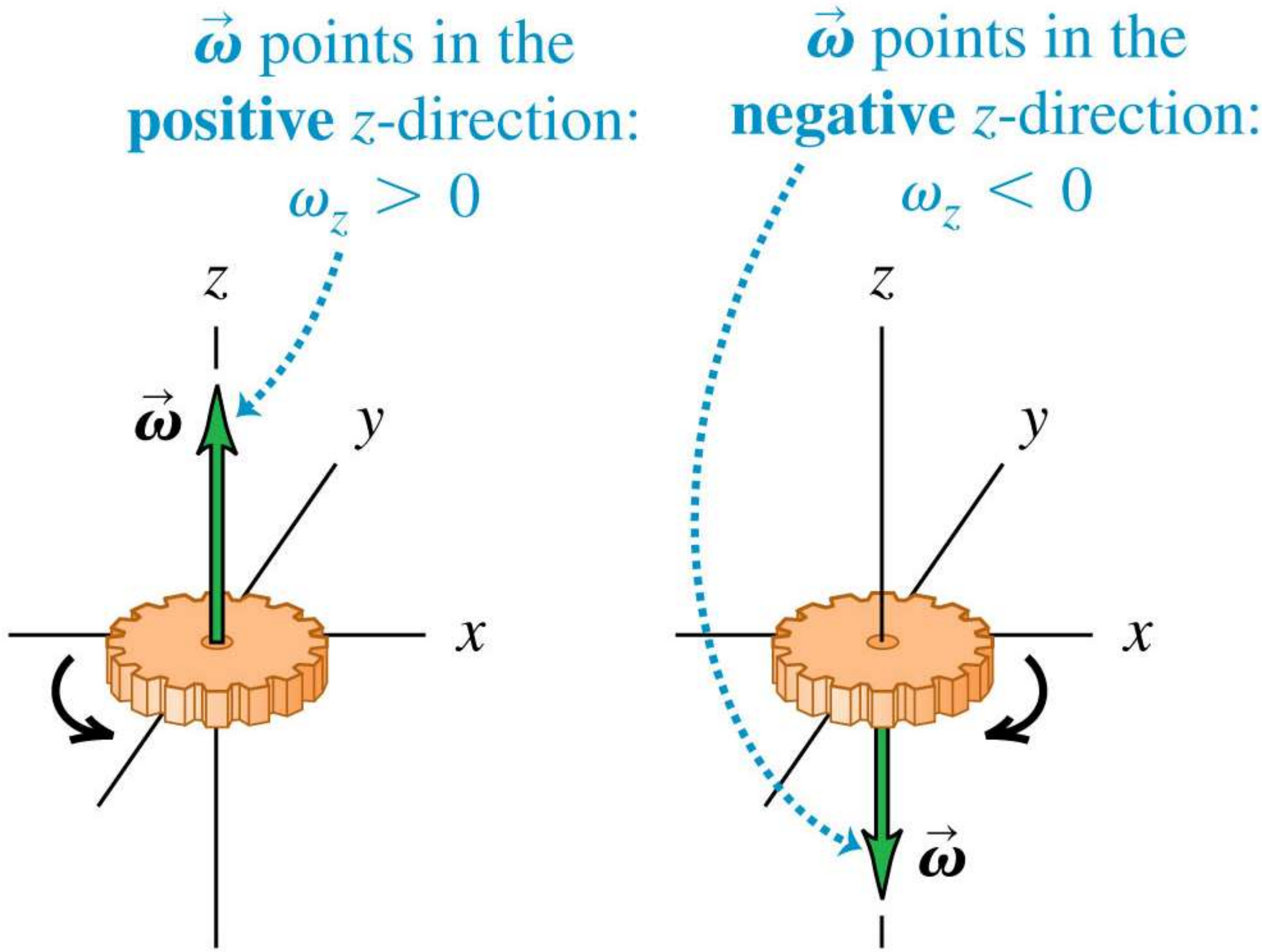
Angular velocity is a vector

- Angular velocity is defined as a vector whose direction is given by the right-hand rule.



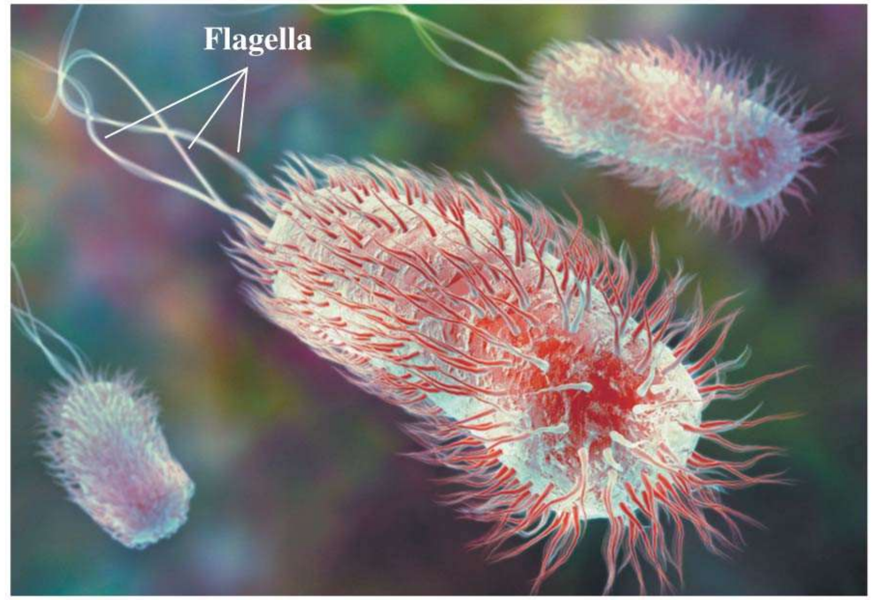
Angular velocity is a vector

- The sign of ω_z for rotation along the z-axis



Rotational motion in bacteria

- *Escherichia coli* bacteria are found in the lower intestines of humans and other warm-blooded animals.
- The bacteria swim by rotating their long, corkscrew-shaped flagella, which act like the blades of a propeller.

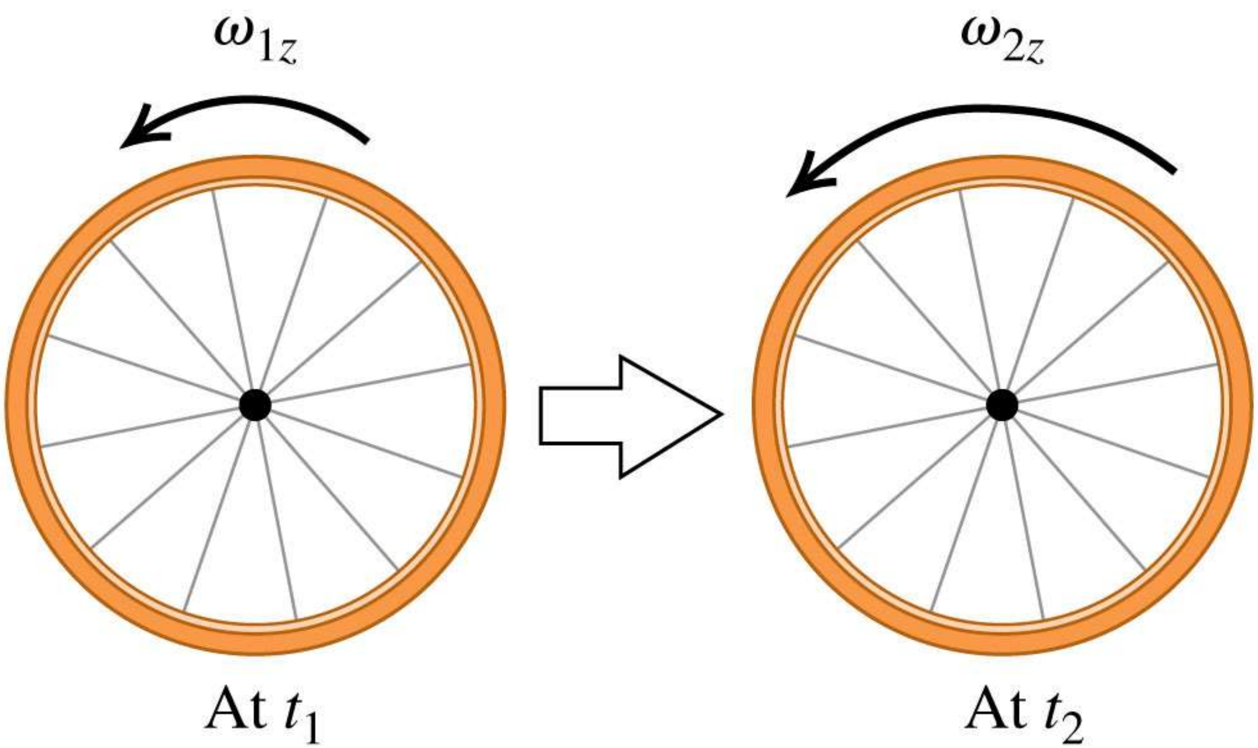


- Each flagellum is rotated at angular speeds from 200 to 1000 rev/min (about 20 to 100 rad/s) and can vary its speed to give the flagellum an angular acceleration.

Angular acceleration

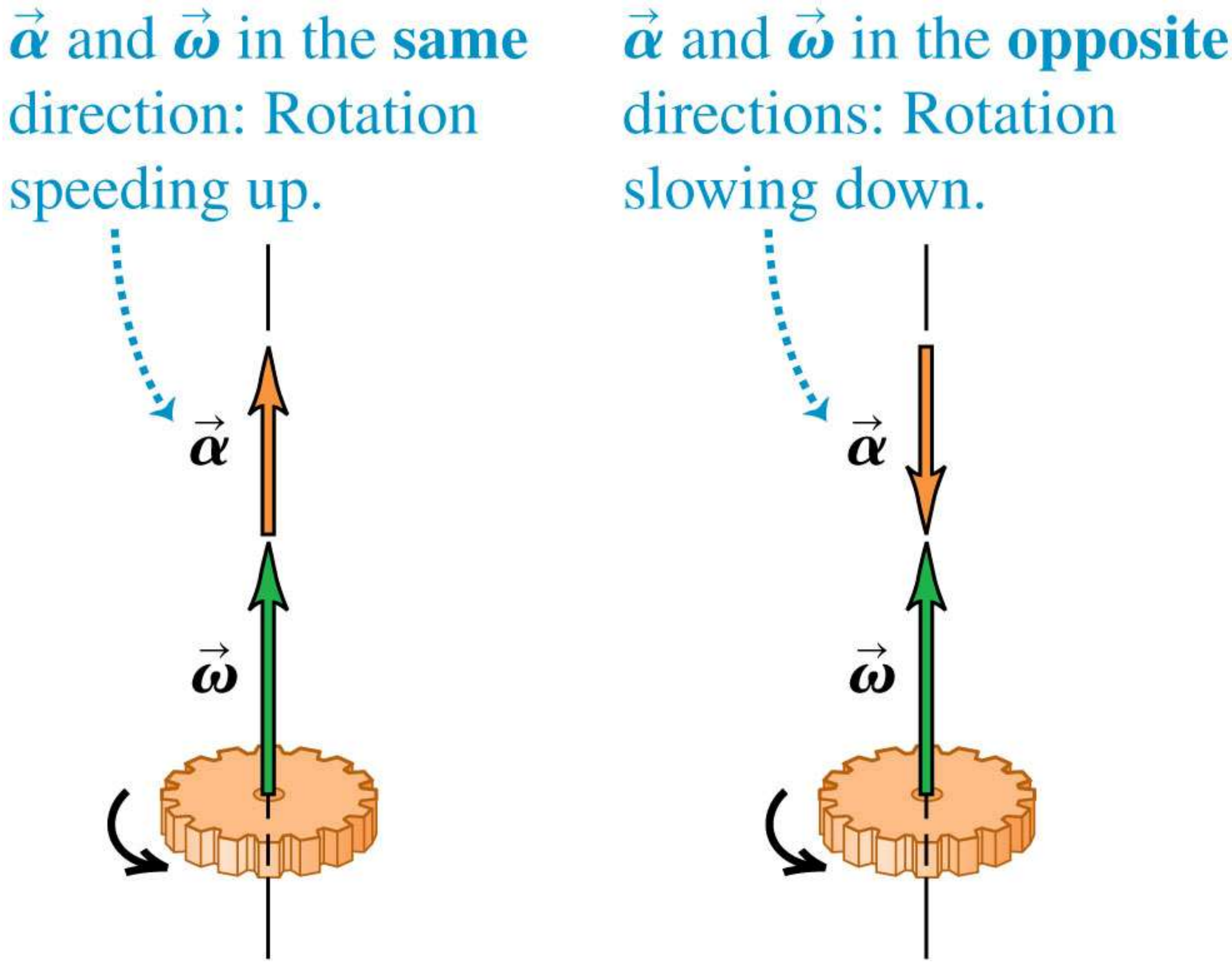
The average angular acceleration is the change in angular velocity divided by the time interval:

$$\alpha_{av\text{-}z} = \frac{\omega_{2z} - \omega_{1z}}{t_2 - t_1} = \frac{\Delta\omega_z}{\Delta t}$$



The instantaneous angular acceleration is $\alpha_z = d\omega_z/dt$.

Angular acceleration as a vector



Velocity and Acceleration

Define the angular velocity ω:

$$\bar{\omega} = \frac{\Delta\theta}{\Delta t} \text{ or } \omega = \frac{d\theta}{dt} \quad \text{radians/ sec}$$

Define α as the angular acceleration

$$\alpha = \frac{d\omega}{dt} \text{ or } \alpha = \frac{d^2\theta}{dt^2} \quad \text{radians/ sec}^2$$

Uniform Angular Acceleration

Derive the angular equations of motion for *constant* angular acceleration

$$\Theta = \Theta_0 + \omega_0 t + \frac{1}{2}\alpha t^2$$

$$\omega = \omega_0 + \alpha t$$

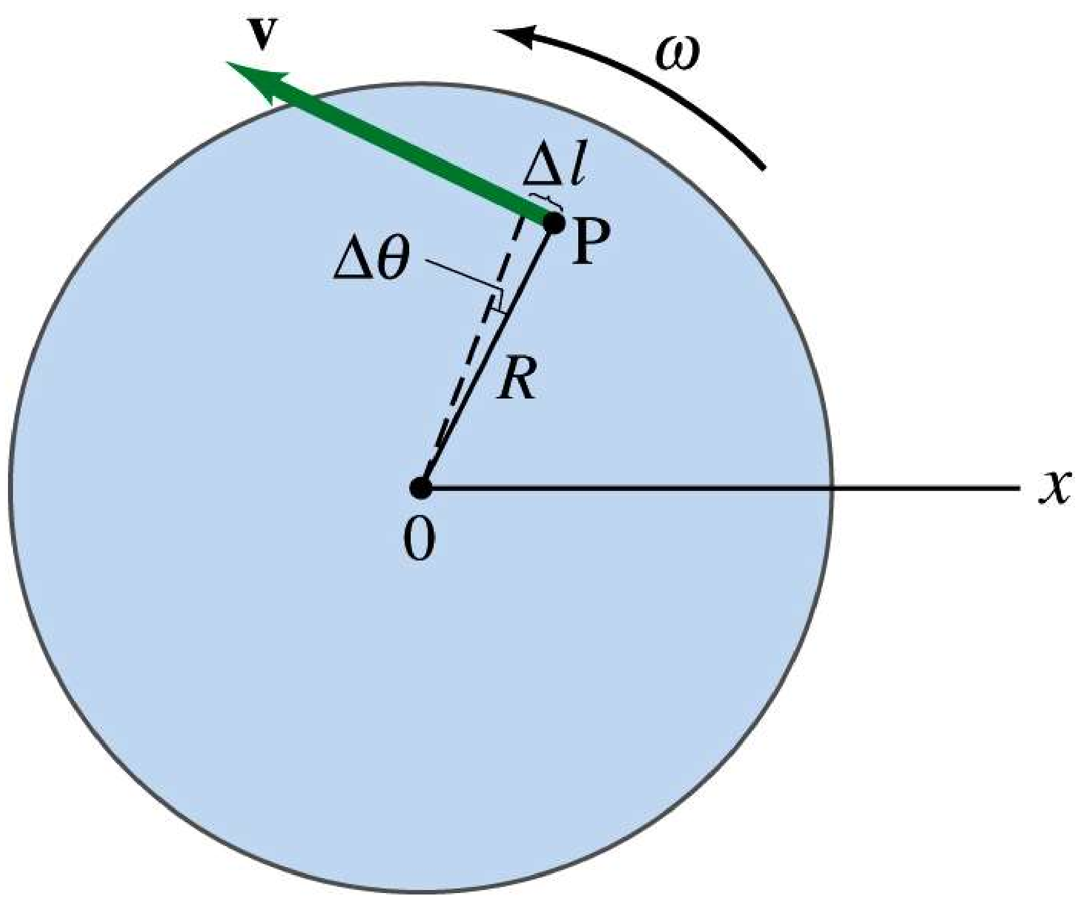
Rotation with constant angular acceleration

- The rotational formulas have the same form as the straight-line formulas, as shown in Table 9.1 below.

Straight-Line Motion with Constant Linear Acceleration	Fixed-Axis Rotation with Constant Angular Acceleration
$a_x = \text{constant}$	$\alpha_z = \text{constant}$
$v_x = v_{0x} + a_x t$	$\omega_z = \omega_{0z} + \alpha_z t$
$x = x_0 + v_{0x}t + \frac{1}{2}a_x t^2$	$\theta = \theta_0 + \omega_{0z}t + \frac{1}{2}\alpha_z t^2$
$v_x^2 = v_{0x}^2 + 2a_x(x - x_0)$	$\omega_z^2 = \omega_{0z}^2 + 2\alpha_z(\theta - \theta_0)$
$x - x_0 = \frac{1}{2}(v_{0x} + v_x)t$	$\theta - \theta_0 = \frac{1}{2}(\omega_{0z} + \omega_z)t$

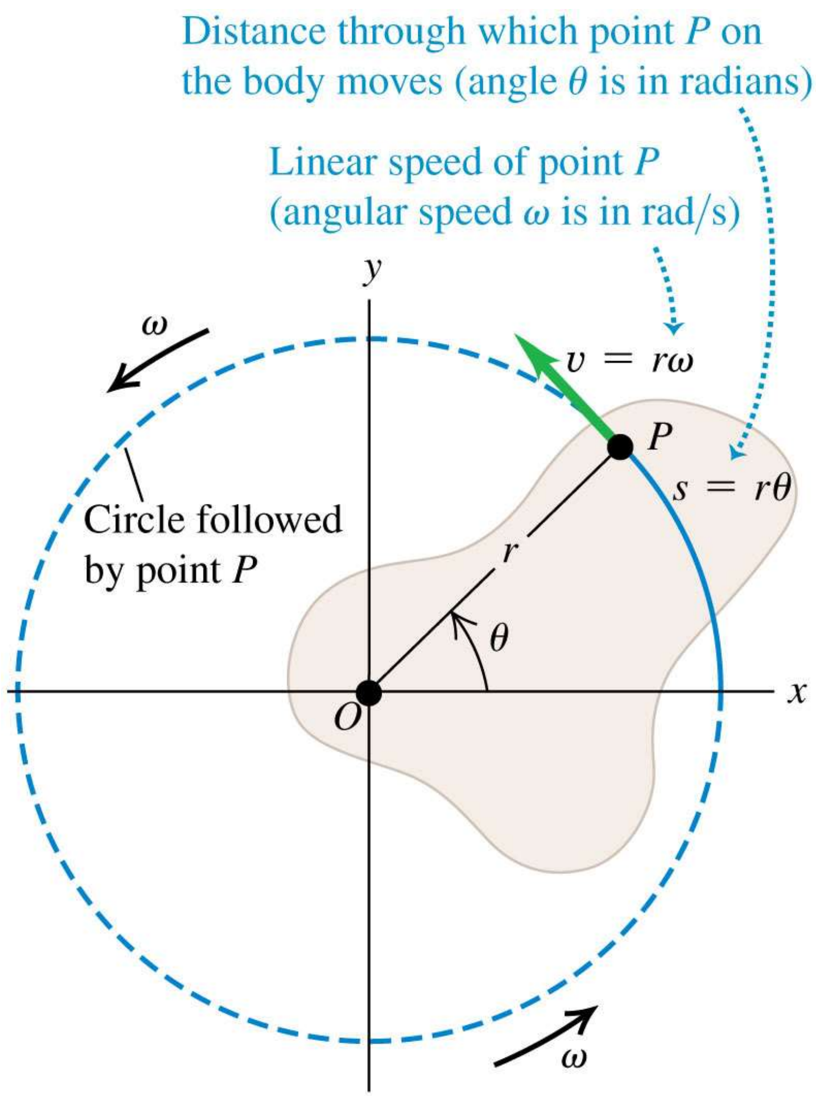
Motion on a Wheel

What is the *linear* speed of a point rotating around in a circle with angular speed ω, and constant radius R?



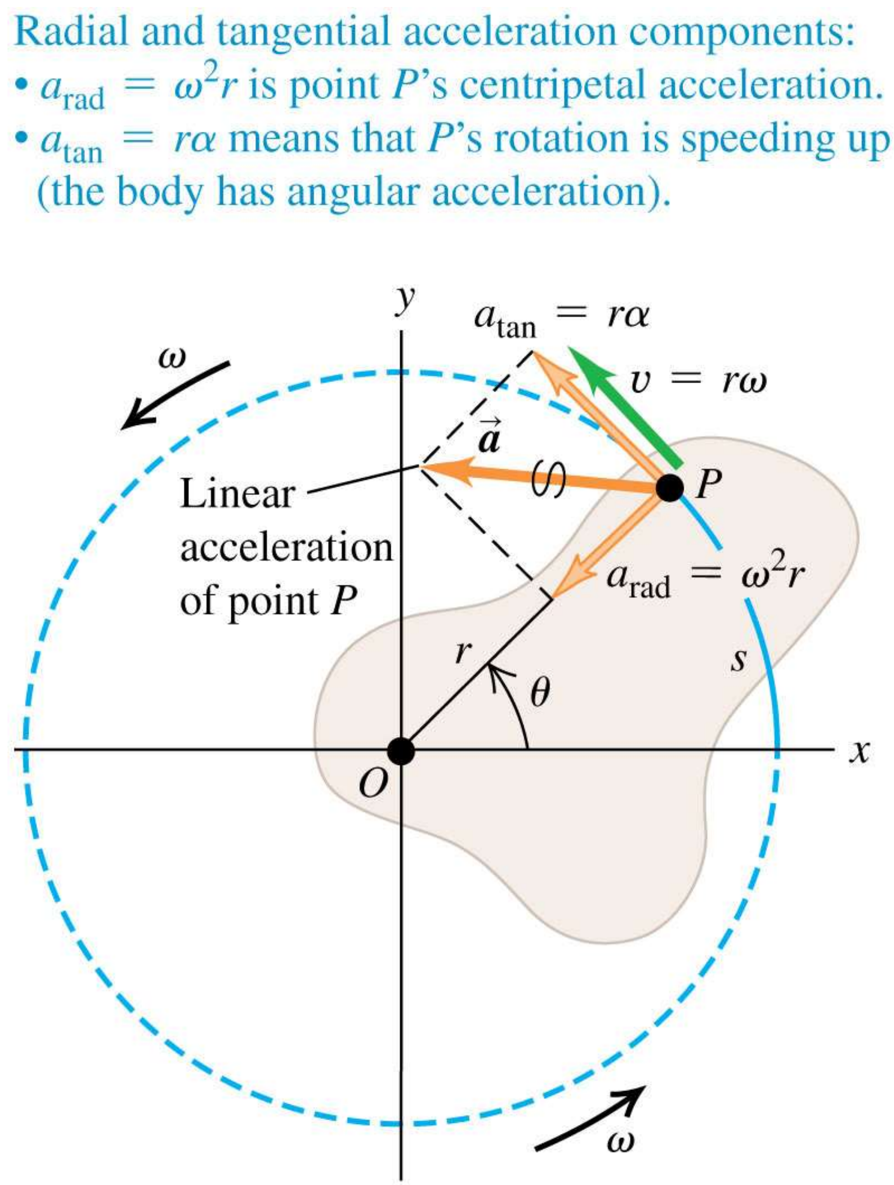
Relating linear and angular kinematics

- A point at a distance r from the axis of rotation has a linear speed of $v = r\omega$.



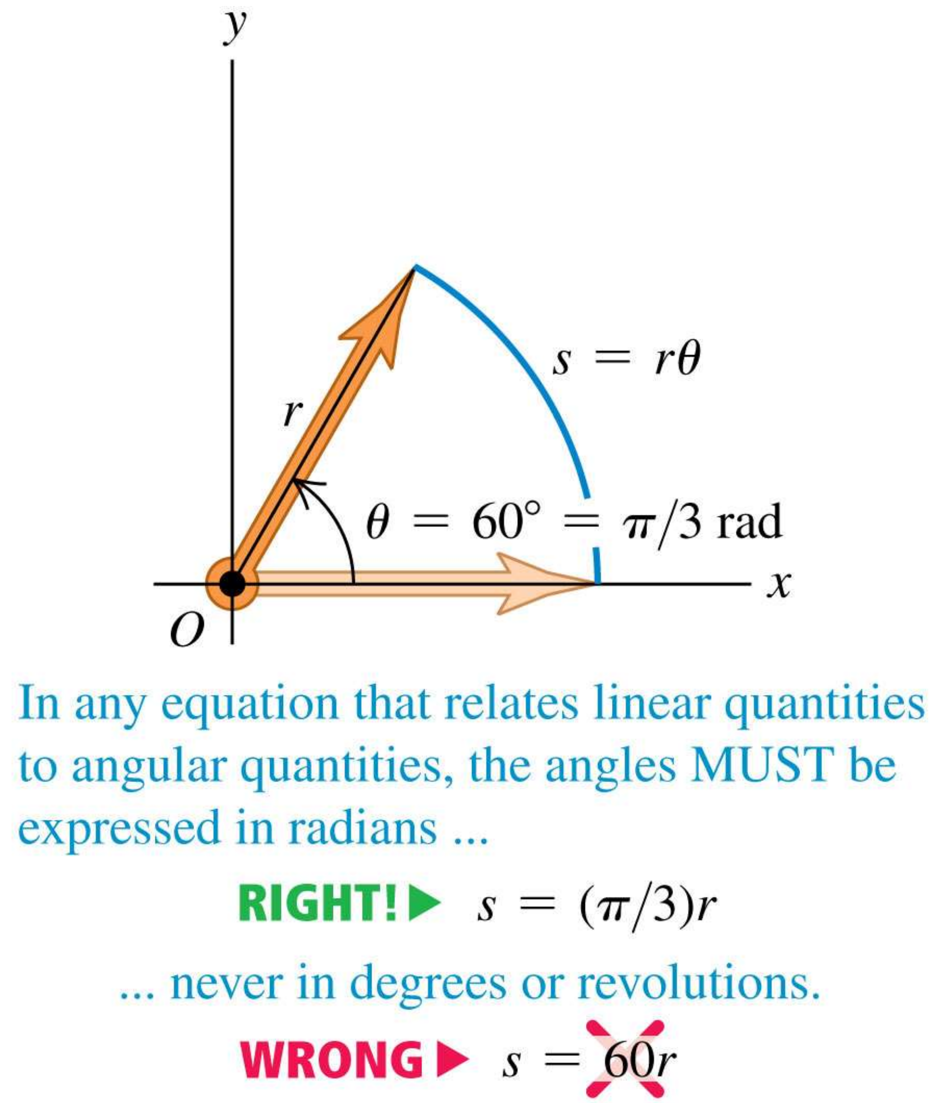
Relating linear and angular kinematics

- For a point at a distance r from the axis of rotation:
  - its tangential acceleration is $a_{\tan} = r\alpha$;
  - its centripetal (radial) acceleration is $a_{rad} = v^2/r = r\omega$.



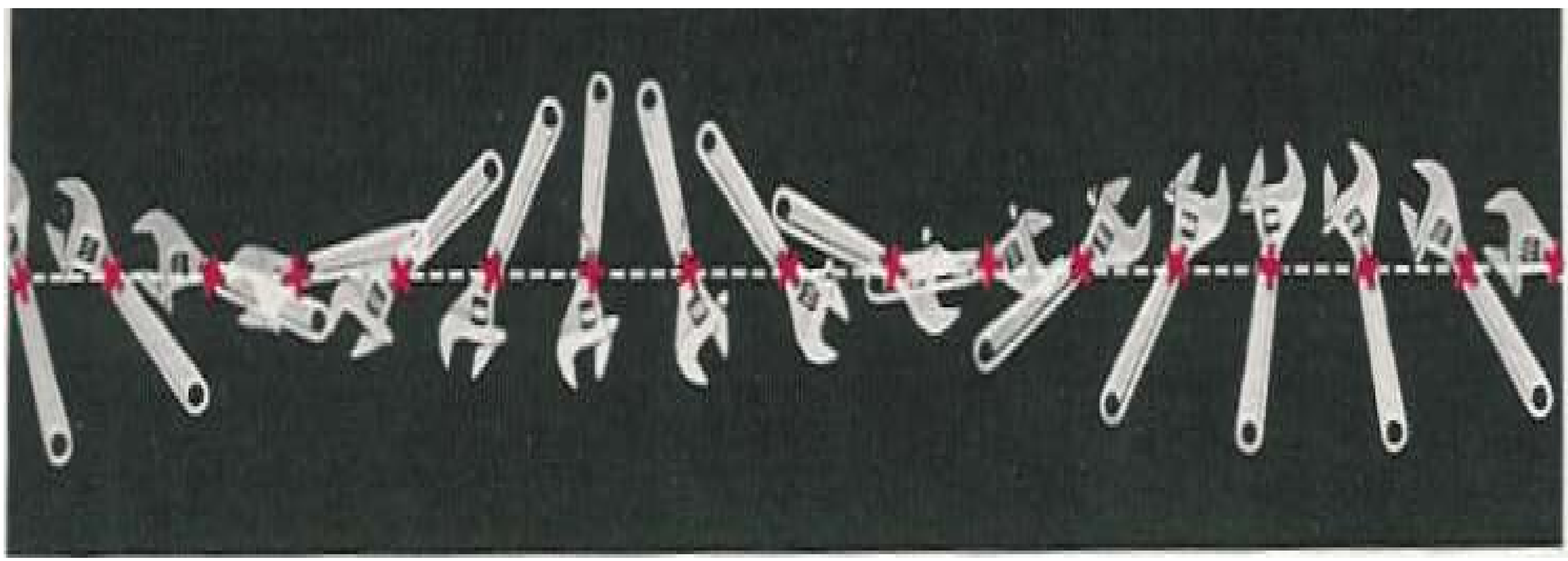
The importance of using radians, not degrees!

- Always use radians when relating linear and angular quantities.



Rotation *and* Translation

Objects can both translate and rotate at the same time. They do both around their *center of mass*.



Rolling without Slipping

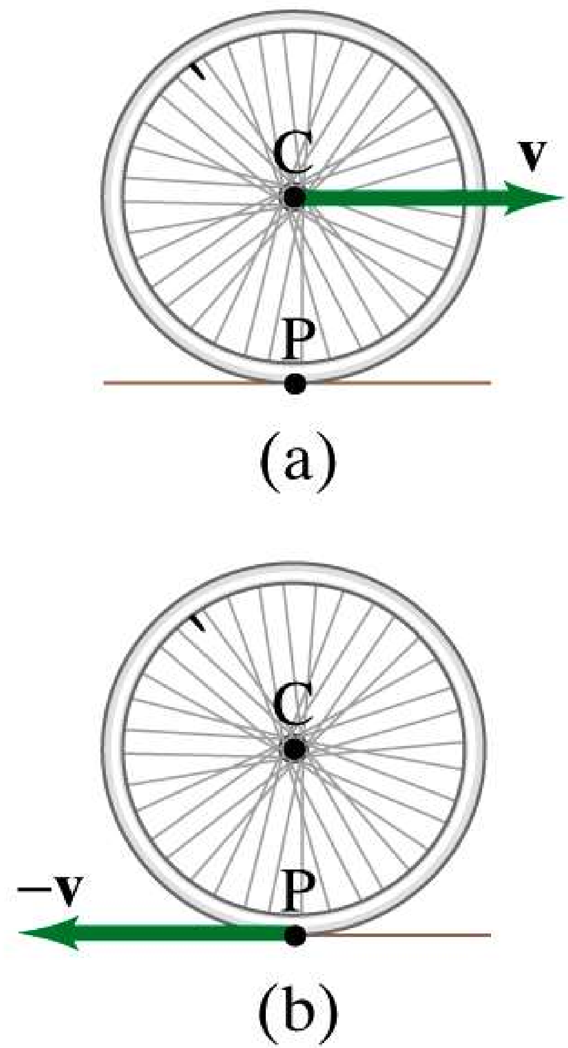
- In reality, car tires both rotate and translate
- They are a good example of something which *rolls* (translates, moves forward, rotates) *without slipping*
- Is there friction? What kind?

✓ A. Static

✗ B. Kinetic

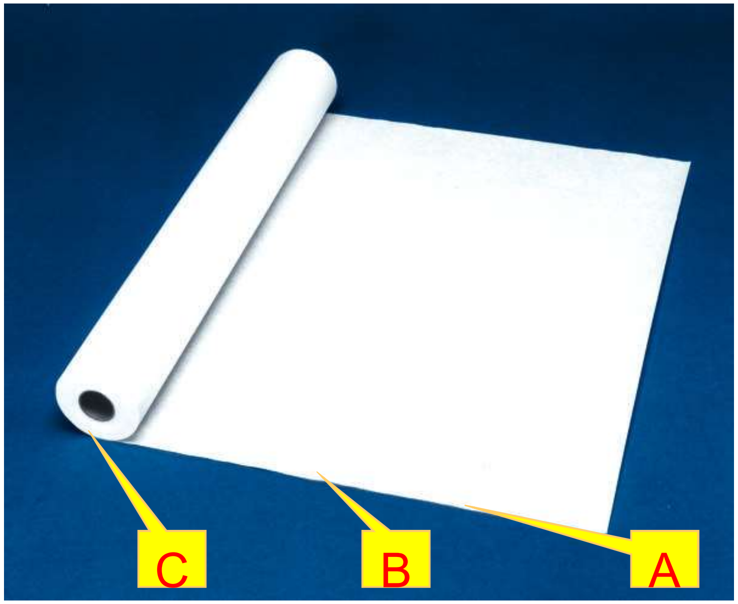
A Rolling Wheel

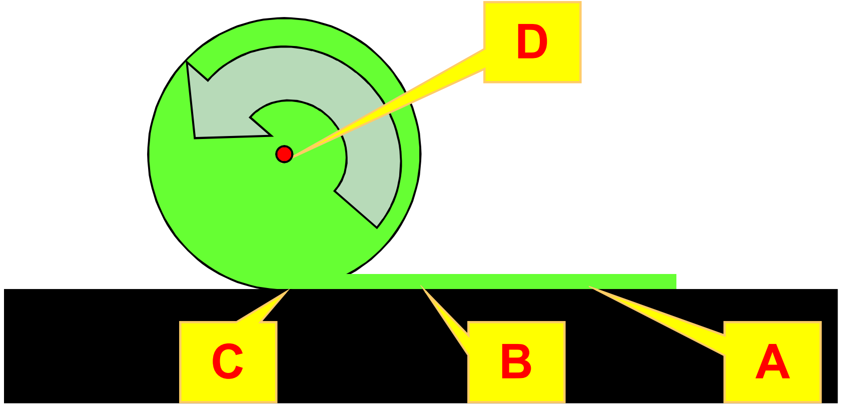
- A wheel rolls on the surface without slipping with velocity V (your speedometer)
- What is the velocity of the center of the wheel (point C)?
- What is the velocity of the lowest point (point P) w.r.t. the ground?
  - Does it make sense to you?



Try Differently: Paper Roll

- A paper towel unrolls with velocity V
  - Conceptually same thing as the wheel
  - What's the velocity of points:
  - A? B? C? D?

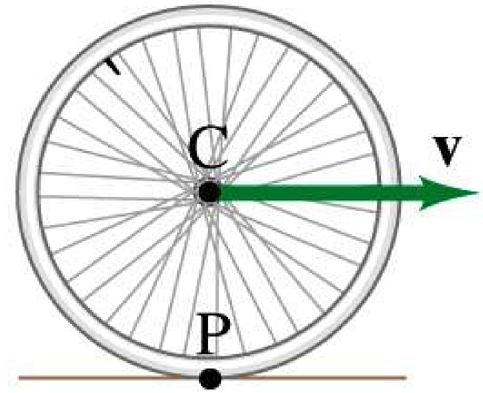




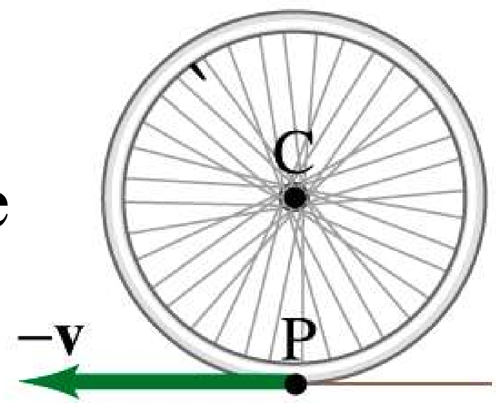
- Point C is where rolling part separates from the unrolled portion
  - Both have same velocity there

Back to the Wheel

- Pick reference point C
  - Wheel is rotating but not moving, ground moves with speed *V*
  - *Use angular velocity ω:*
- Velocity of P w.r.t. C = $-\omega R$
  - Same as velocity of ground w.r.t. bike
- Then velocity of C (and bike) w.r.t. the ground = $+\omega R = V$
- *Also, velocity of P w.r.t. ground is zero:* $= -\omega R + \omega R = 0$



(a)



(b)

Bicycle comes to Rest

A bicycle with initial linear velocity V_0 decelerates uniformly (without slipping) to rest over a distance *d*. For a wheel of radius *R*:

a) What is the angular velocity at $t_0=0$?

b) Total revolutions before it stops?

c) Total angular distance traversed wheel?

d) The angular acceleration?

e) The total time until it stops?

Rotational kinetic energy

- The rotational kinetic energy of a rigid body is:

Rotational kinetic energy of a rigid body rotating around an axis → $K = \frac{1}{2} I \omega^2$

(I: Moment of inertia of body for given rotation axis; ω: Angular speed of body)

- The moment of inertia, I, is obtained by multiplying the mass of each particle by the square of its distance from the axis of rotation and adding these products:

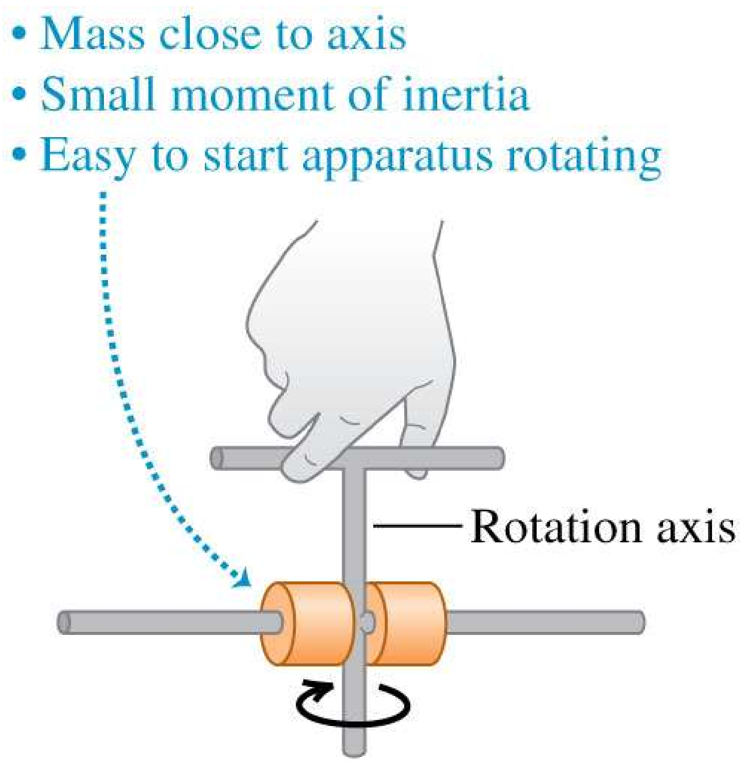
Moment of inertia of a body for a given rotation axis → $I = m_1 r_1^2 + m_2 r_2^2 + \cdots = \sum_i m_i r_i^2$

(m_1, m_2, m_i: Masses of the particles that make up the body; r_1, r_2, r_i: Perpendicular distances of the particles from rotation axis)

- The SI unit of I is the kilogram-meter^2 ($kg \cdot m^2$).

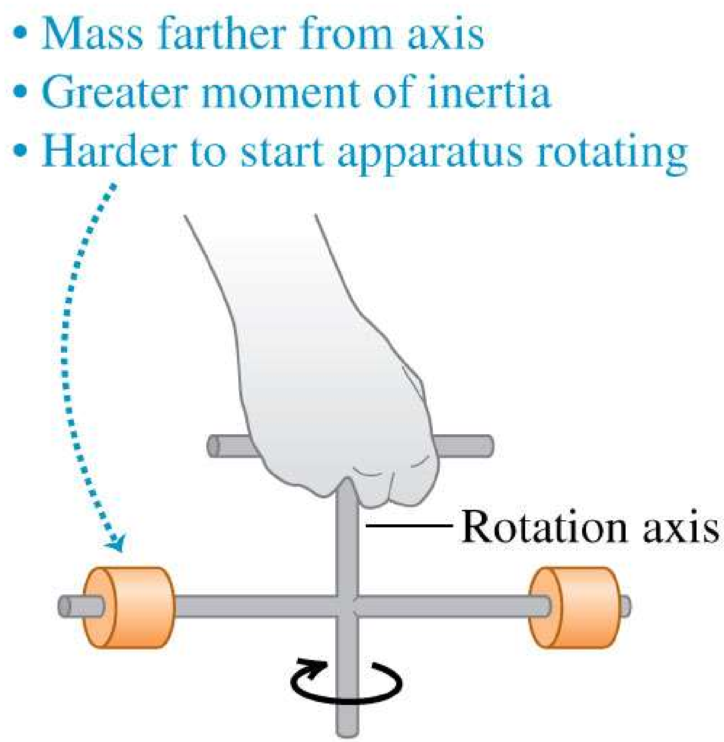
Moment of inertia

- Here is an apparatus free to rotate around a vertical axis.
- To reduce the moment of inertia, lock the two equal-mass cylinders close to the center of the horizontal shaft.



Moment of inertia

- Here is an apparatus free to rotate around a vertical axis.
- To increase the moment of inertia, lock the two equal-mass cylinders far from the center of the horizontal shaft.



Moment of inertia of a bird's wing

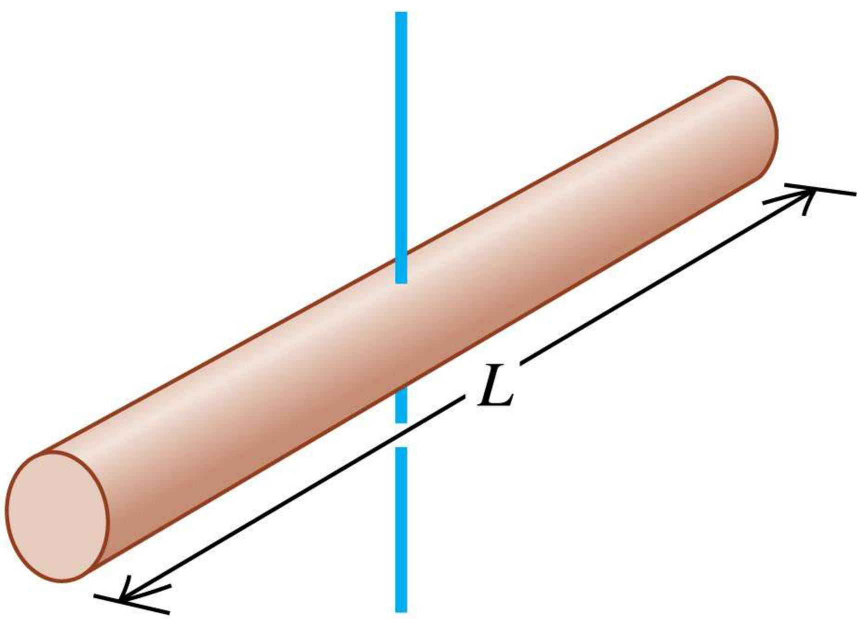
- When a bird flaps its wings, it rotates the wings up and down around the shoulder.
- A hummingbird has small wings with a small moment of inertia, so the bird can move its wings rapidly (up to 70 beats per second).
- By contrast, the Andean condor has immense wings with a large moment of inertia, and flaps its wings at about one beat per second.

Moments of inertia of some common bodies: Slide 1 of 4

- Table 9.2

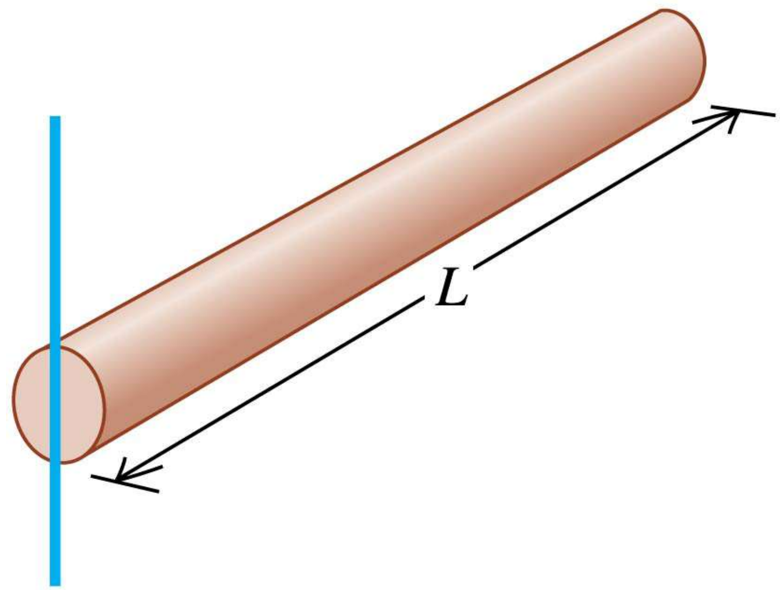
(a) Slender rod,
axis through center

$$I = \frac{1}{12}ML^2$$



(b) Slender rod,
axis through one end

$$I = \frac{1}{3}ML^2$$

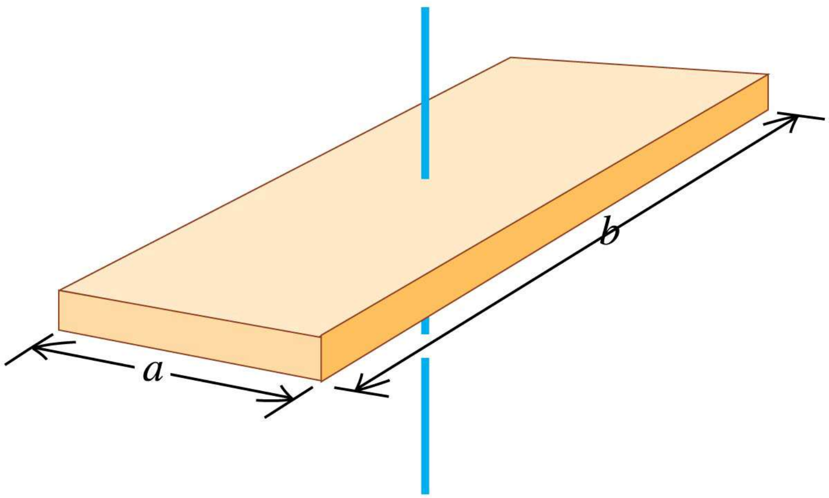


Moments of inertia of some common bodies: Slide 2 of 4

- Table 9.2

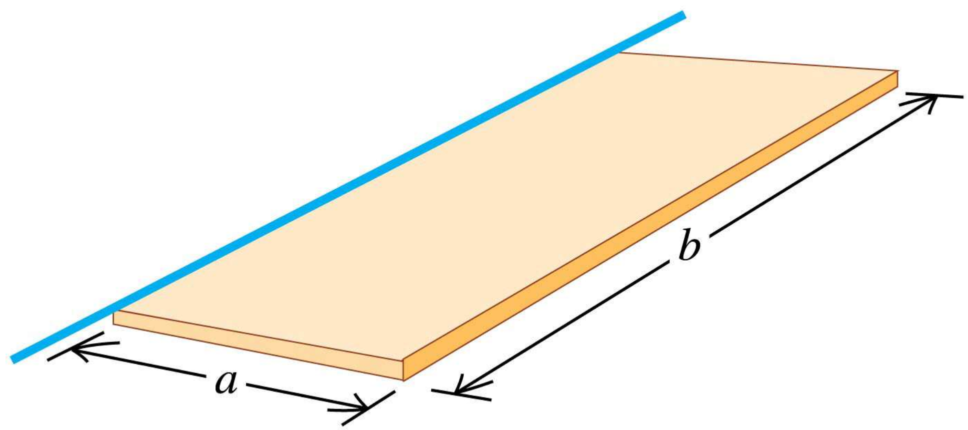
(c) Rectangular plate, axis through center

$$I = \frac{1}{12}M\left(a^2 + b^2\right)$$



(d) Thin rectangular plate, axis along edge

$$I = \frac{1}{3}Ma^2$$



Moments of inertia of some common bodies: Slide 3 of 4

- Table 9.2

(e) Hollow cylinder

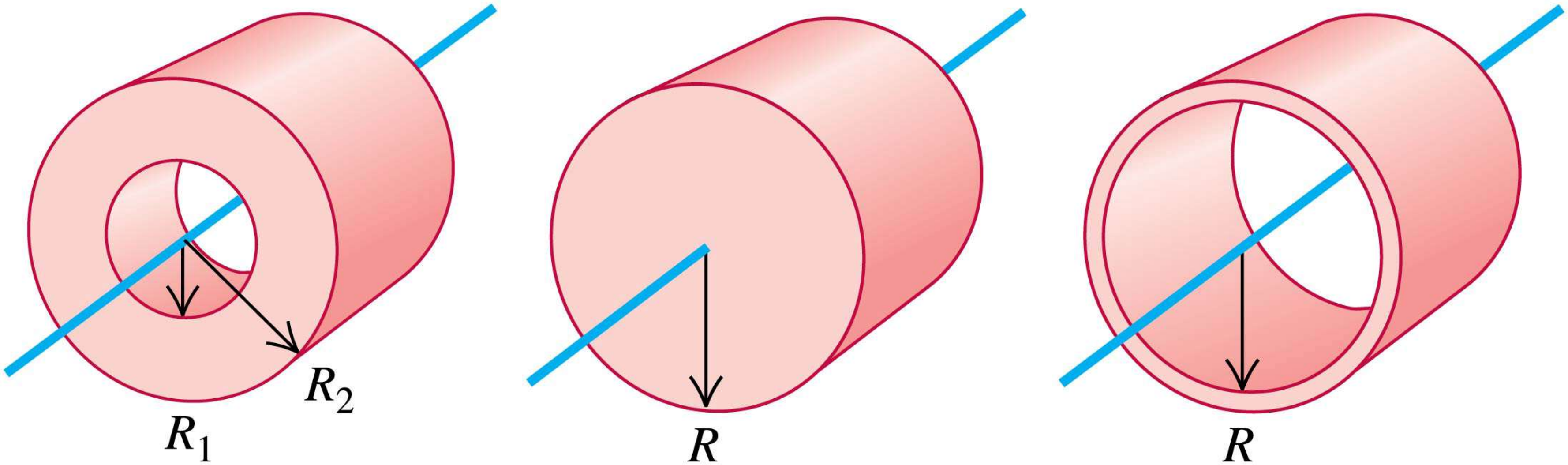
$$I = \frac{1}{2}M(R_1^2 + R_2^2)$$

(f) Solid cylinder

$$I = \frac{1}{2}MR^2$$

(g) Thin-walled hollow cylinder

$$I = MR^2$$

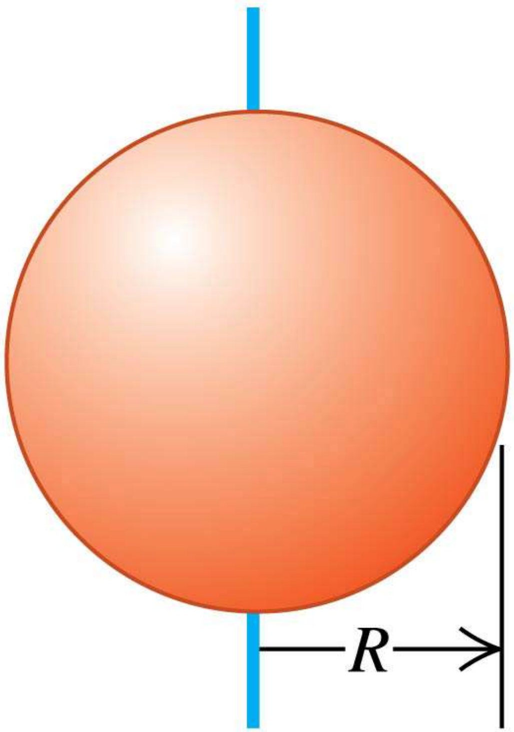


Moments of inertia of some common bodies: Slide 4 of 4

- Table 9.2

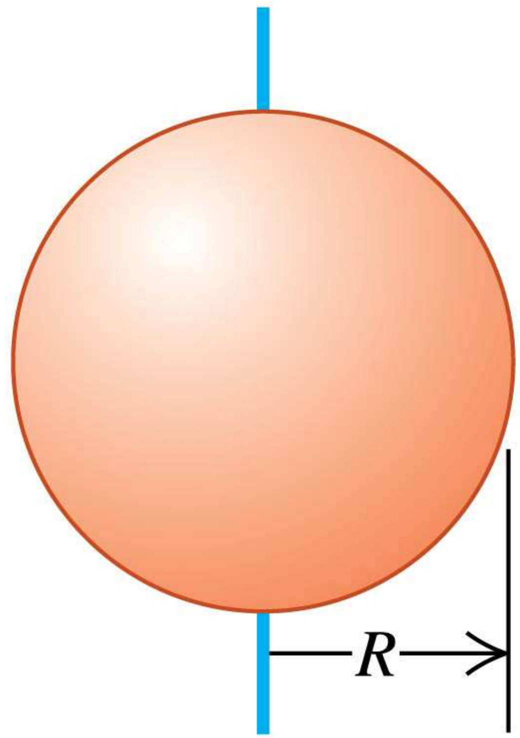
(h) Solid sphere

$$I = \frac{2}{5}MR^2$$



(i) Thin-walled hollow sphere

$$I = \frac{2}{3}MR^2$$



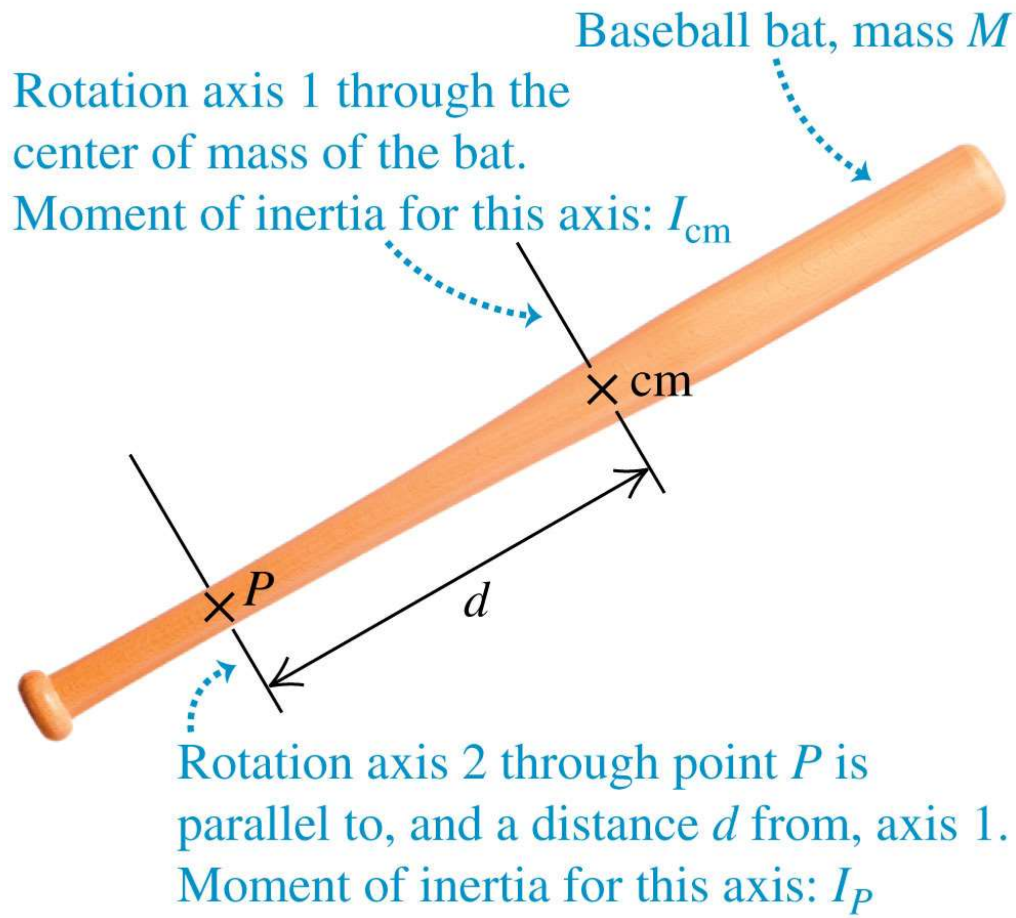
Gravitational potential energy of an extended body

- The gravitational potential energy of an extended body is the same as if all the mass were concentrated at its center of mass: $U_{grav} = Mgy_{cm}$.

- This athlete arches her body so that her center of mass actually passes *under* the bar.
- This technique requires a smaller increase in gravitational potential energy than straddling the bar.

The parallel-axis theorem

- There is a simple relationship, called the **parallel-axis theorem**, between the moment of inertia of a body about an axis through its center of mass and the moment of inertia about any other axis parallel to the original axis.



Parallel-axis theorem: $I_P = I_{cm} + Md^2$

Moment of inertia calculations

- The moment of inertia of any distribution of mass can be found by integrating over its volume:

$$I = \int r^2 \rho \, dV$$

- By measuring small variations in the orbits of satellites, geophysicists can measure the earth's moment of inertia.
- This tells us how our planet's mass is distributed within its interior.
- The data show that the earth is far denser at the core than in its outer layers.