# Chapter 7

# Potential Energy and Energy Conservation

PowerPoint® Lectures for
***University Physics, Twelfth Edition***
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**Lectures by James Pazun**

# Goals for Chapter 7

- To study gravitational and elastic potential energy (conservative forces)
- To determine when total mechanical energy is conserved
- To examine situations when total mechanical energy is not conserved
- To examine conservative forces, nonconservative forces, and the law of energy conservation
- To determine force from potential energy

# Potential Energy

- Things with potential:
  - Could do potentially do work
- Here we mean the same thing
- *E.g.* Gravitation potential energy:
  - If you lift up a brick it has the *potential* to do damage
  - Compressed spring

# Example: Gravity & Potential Energy

You (very slowly) lift up a brick (at rest) from the ground and then hold it at a height $Z$.

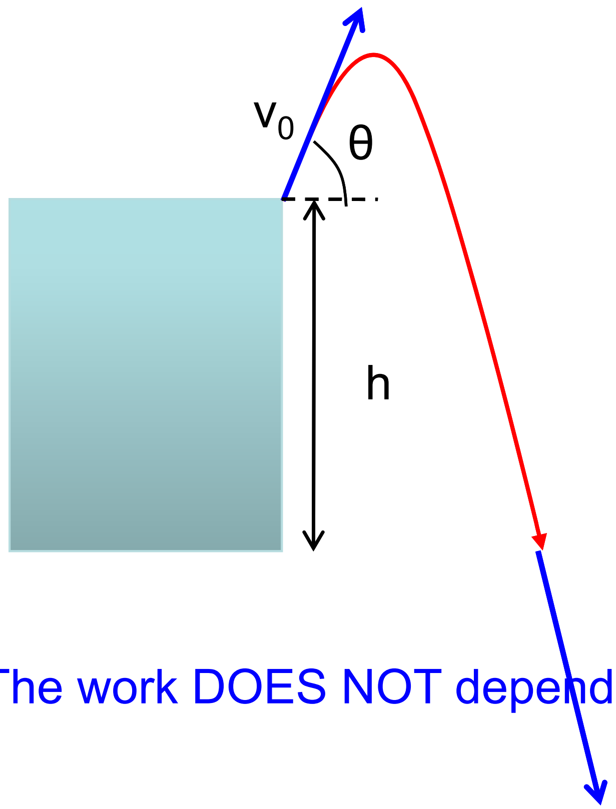
- How much work has been done on the brick?
- How much work did you do?
- If you let it go, how much work will be done by gravity by the time it hits the ground?

We say it has potential energy:

$$U = mgZ$$

- Gravitational potential energy

# What is the work done by the force of gravity?

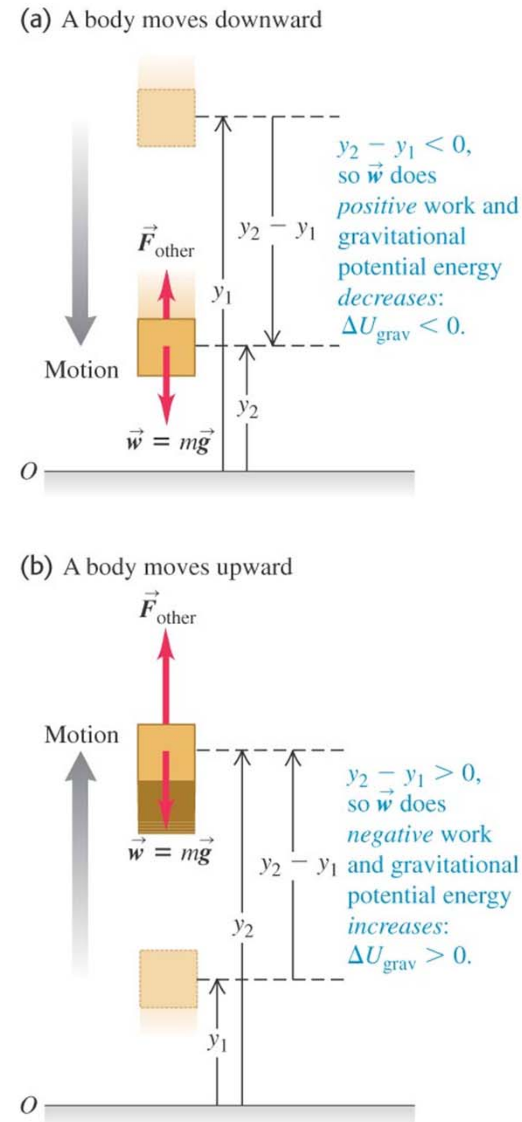



The work DOES NOT depend on the trajectory!!!

$$W_{mg} = \int \vec{F}_{mg} \cdot d\vec{s} = \int (-mg\,\hat{j}) \cdot (x\hat{i} + y\hat{j})$$

$$= 0 + (-mg)(y_f - y_i) = -UE_f + UE_i$$

UE=mgy=Gravitational potential energy



# Mechanical Energy

- We <u>define</u> the *total mechanical energy* in a system to be the *kinetic energy* plus the *potential energy*
- Define $E \equiv K + U$

# Conservation of Mechanical Energy

- For *some types* of problems, Mechanical Energy is *conserved* (more on this next week)
- *E.g.* Mechanical energy *before* you drop a brick is equal to the mechanical energy *after* you drop the brick

$$K_2+U_2 = K_1+U_1$$

Conservation of Mechanical Energy

$$E_2=E_1$$

# Problem Solving

- What are the types of examples we'll encounter?
  - Gravity
  - Things falling
  - Springs
- Converting their potential energy into kinetic energy and back again

$$\text{Gravity: } E = K + U = \tfrac{1}{2}mv^2 + mgy$$

$$\text{Spring: } E = K + U = \tfrac{1}{2}mv^2 + \tfrac{1}{2}kx^2$$

# Athletes (projectile motion) and the conservation of energy

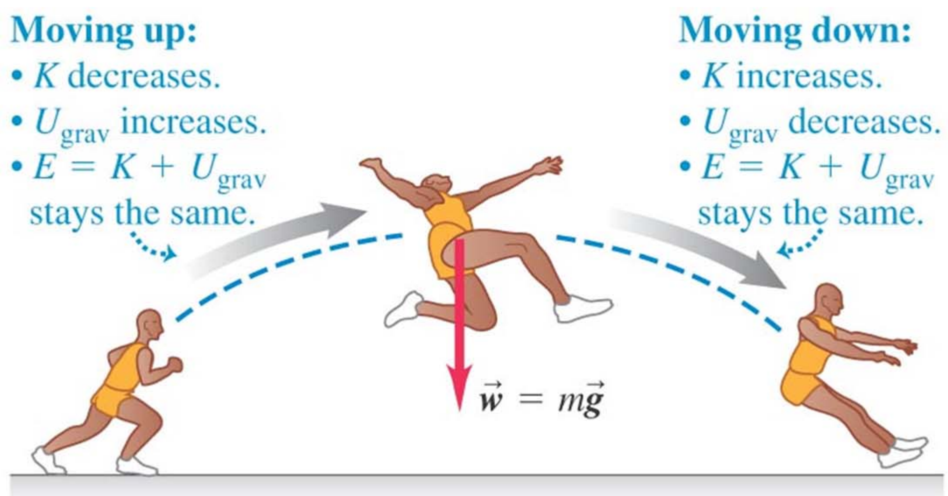
If we have a projectile motion the work energy theorem says

$$-mg(y_2 - y_1) = KE_2 - KE_1$$

With some rearranging

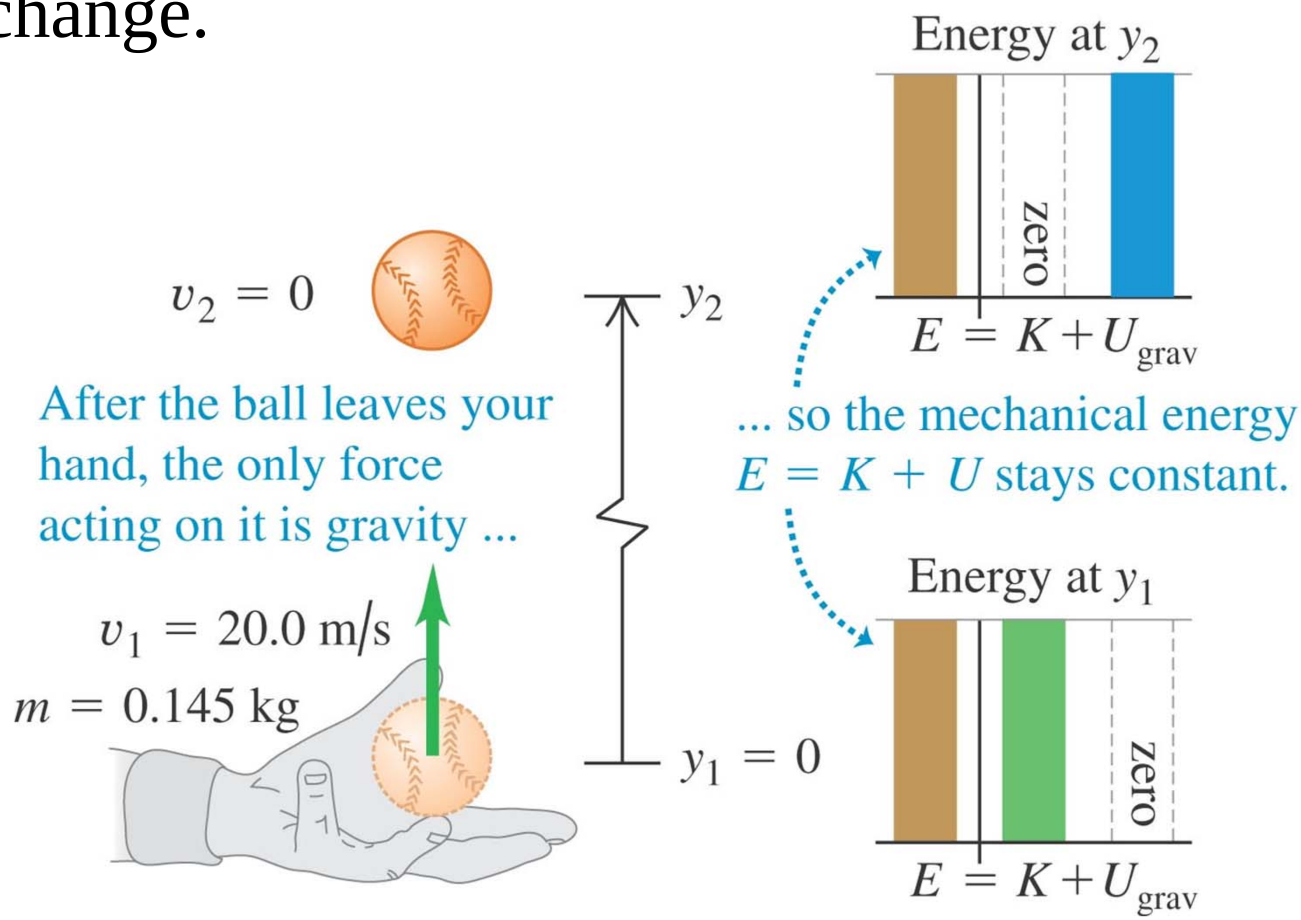
$$KE_1 + UE_1 = KE_2 + UE_2$$

Energy is conserved!!

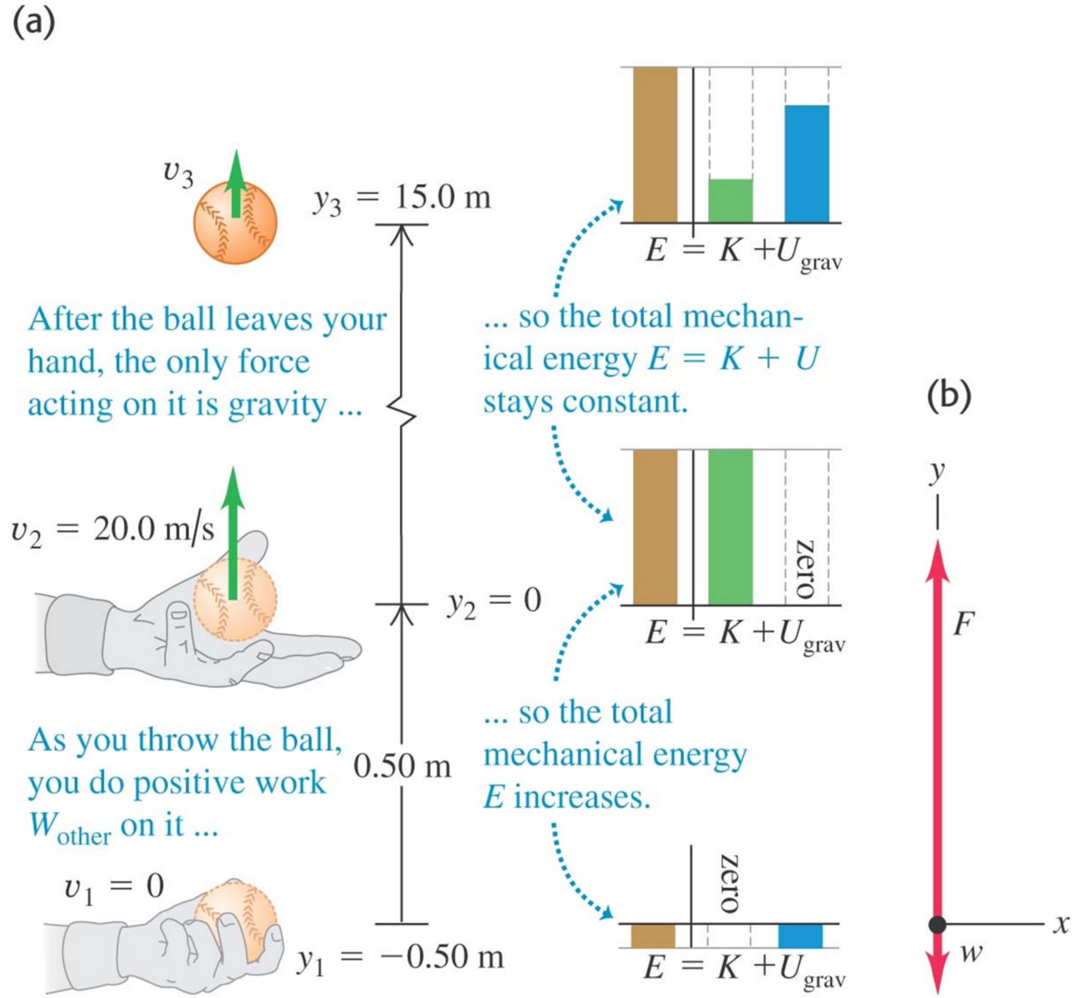


# Athletes and energy II—Example 7.1

- Refer to Figure 7.4 as you follow Example 7.1.
- Notice how velocity changes as forms of energy interchange.

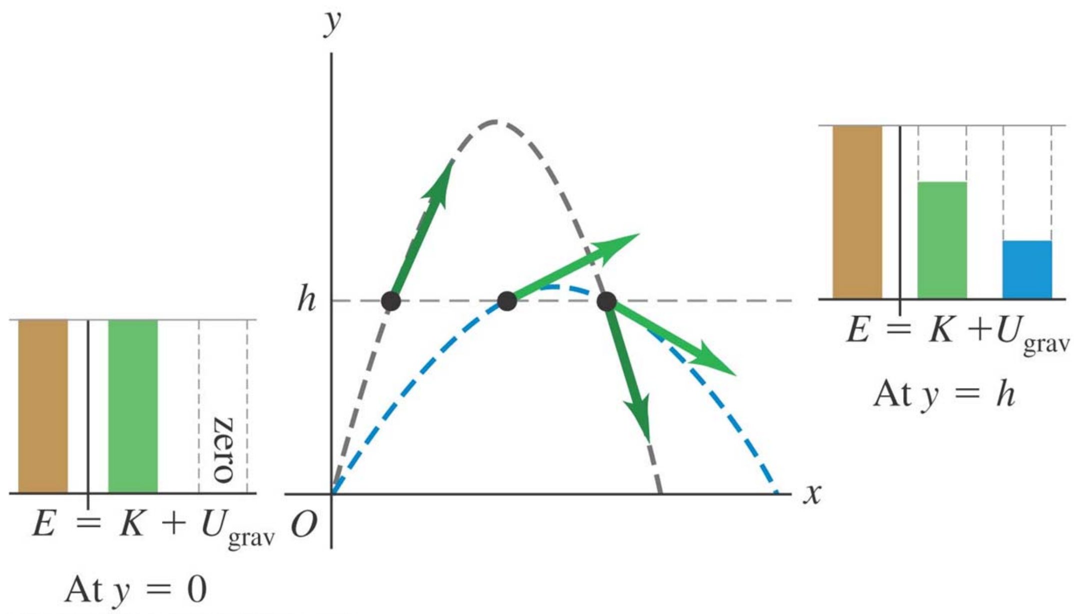


# Forces other than gravity doing



# Consider projectile motion using energetics

- Consider the speed of a projectile as it traverses its parabola in the absence of air resistance.
- Refer to Conceptual Example 7.3 and Figure 7.8.



# Revisiting the work energy theorem

The work energy theorem says the total work is equal to the change in KE

$$W_{net} = KE_2 - KE_1$$

On the other hand, we have seen that the work due to gravity ONLY DEPENDS ON THE INITIAL AND FINAL POINT OF THEIR PATH, NOT ON THE ACTUAL PATH. These type of forces (of which gravity is one) are called conservative forces. Let's brake the total work done into two parts, the one done by the conservative forces and the ones done by non-conservative forces (e.g. friction)

$$W_{conserv} + W_{non-conserv} = KE_2 - KE_1$$

$$-UE_2 + UE_1 + W_{non-conserv} = KE_2 - KE_1$$

$$W_{non-conserv} = E_2 - E_1 \quad \text{where} \quad E = KE + UE$$

If the work done by the non-conservative forces is zero then the total energy is conserved. This is a very powerful tool!!

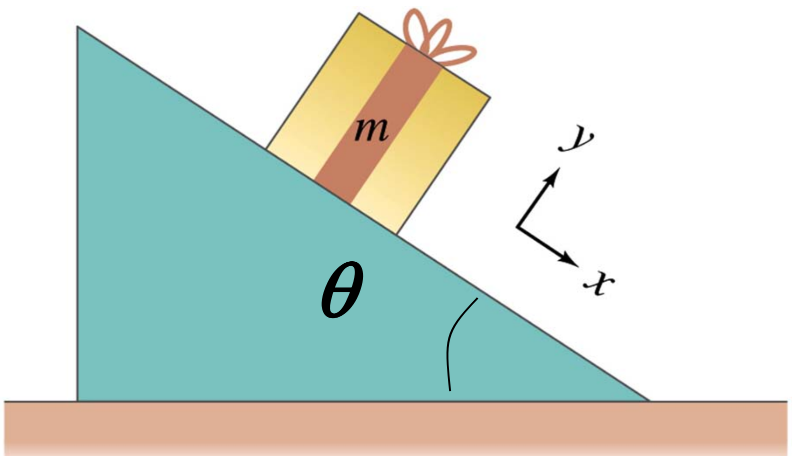
# Box on an inclined plane

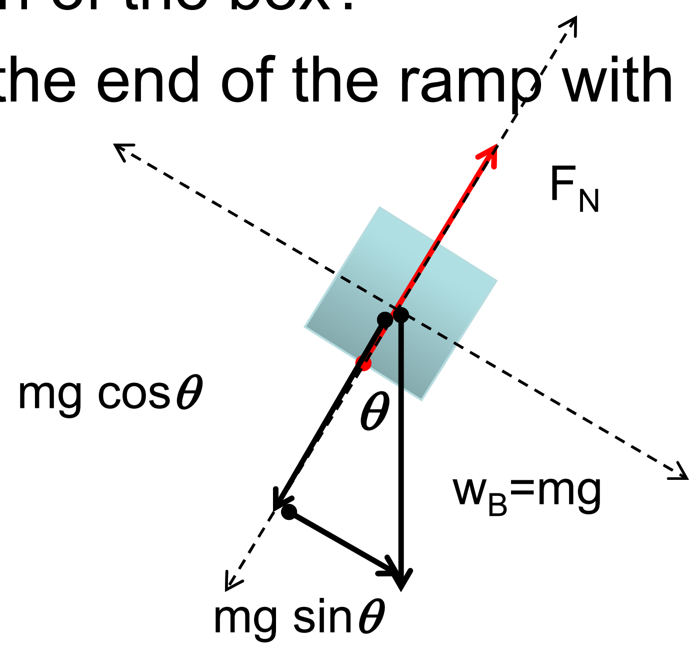
A box with mass *m* is placed on a frictionless incline with angle $\theta$ and is allowed to slide down.

a) What is the normal force?

b) What is the acceleration of the box?

c) What is the velocity at the end of the ramp with length L?




$$x: \quad mg\sin\theta = ma_x$$

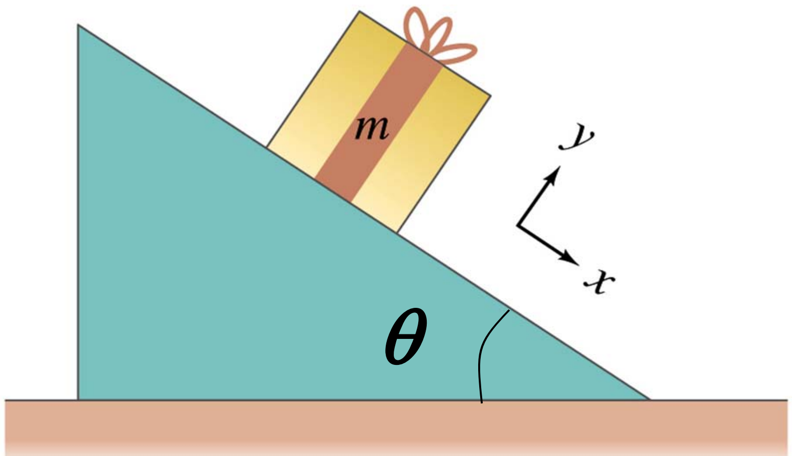
$$y: \quad F_N - mg\cos\theta = 0$$

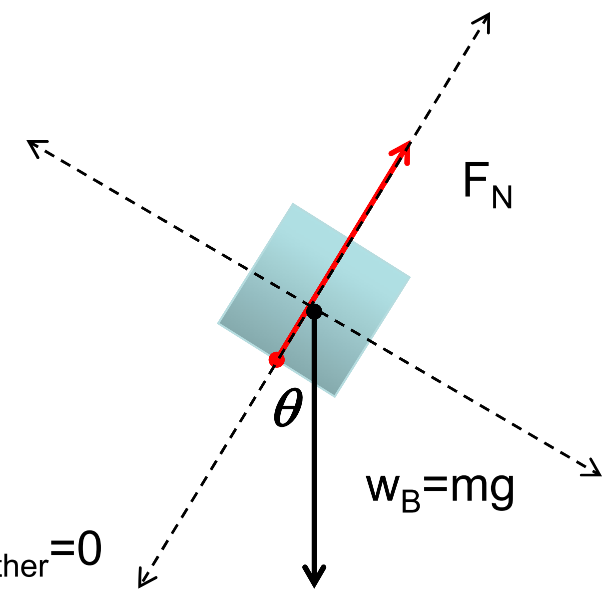
$$a_x = g\sin\theta$$

# Box on an inclined plane REVISITED

A box with mass $m$ is placed on a frictionless incline with angle $\theta$ and is allowed to slide down.

a) What is the velocity at the end of the ramp with length L?




Normal force DOES NO WORK so $W_{other}=0$

Then $E_{top}=E_{bottom}$

$$KE_{top}+UE_{top}=KE_{bott}+UE_{bott}$$

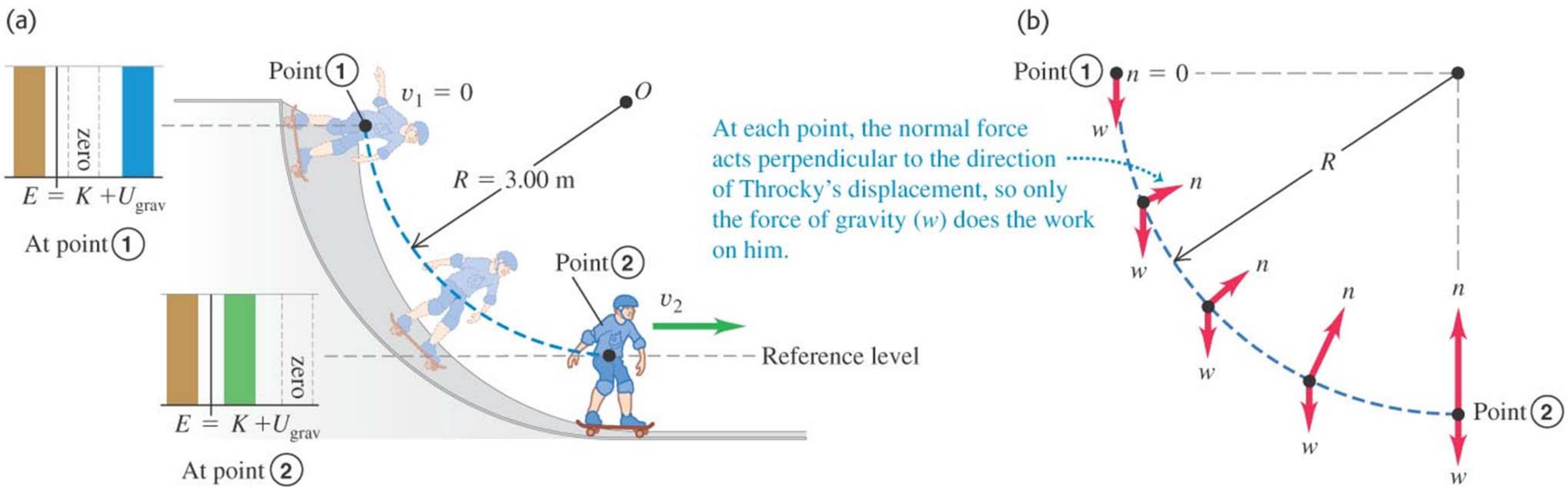
$$0+mgh=\frac{1}{2}m\mathrm{v}^2+0$$

$$mgL\sin\theta=\frac{1}{2}m\mathrm{v}^2$$

$$\Rightarrow \quad \mathrm{v}=\sqrt{2gL\sin\theta}$$

# What's the speed in a vertical circle?

Refer to Example 7.4 and Figure 7.9.




$$KE_{top} + UE_{top} = KE_{bott} + UE_{bott}$$

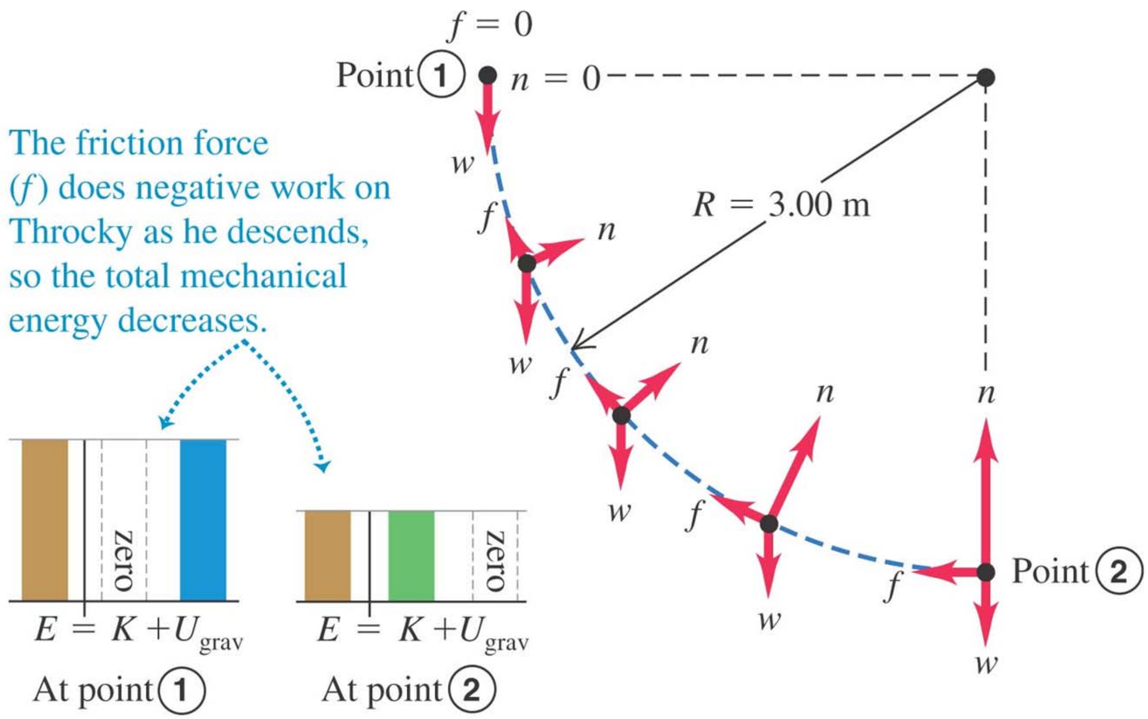
$$0 + mgh = \frac{1}{2}m\,\mathrm{v}^2 + 0$$

$$mgR = \frac{1}{2}m\,\mathrm{v}^2 \quad \Rightarrow \quad \mathrm{v} = \sqrt{2gR}$$

# Speed in a vertical circle with friction

– Consider how things change when friction is introduced.

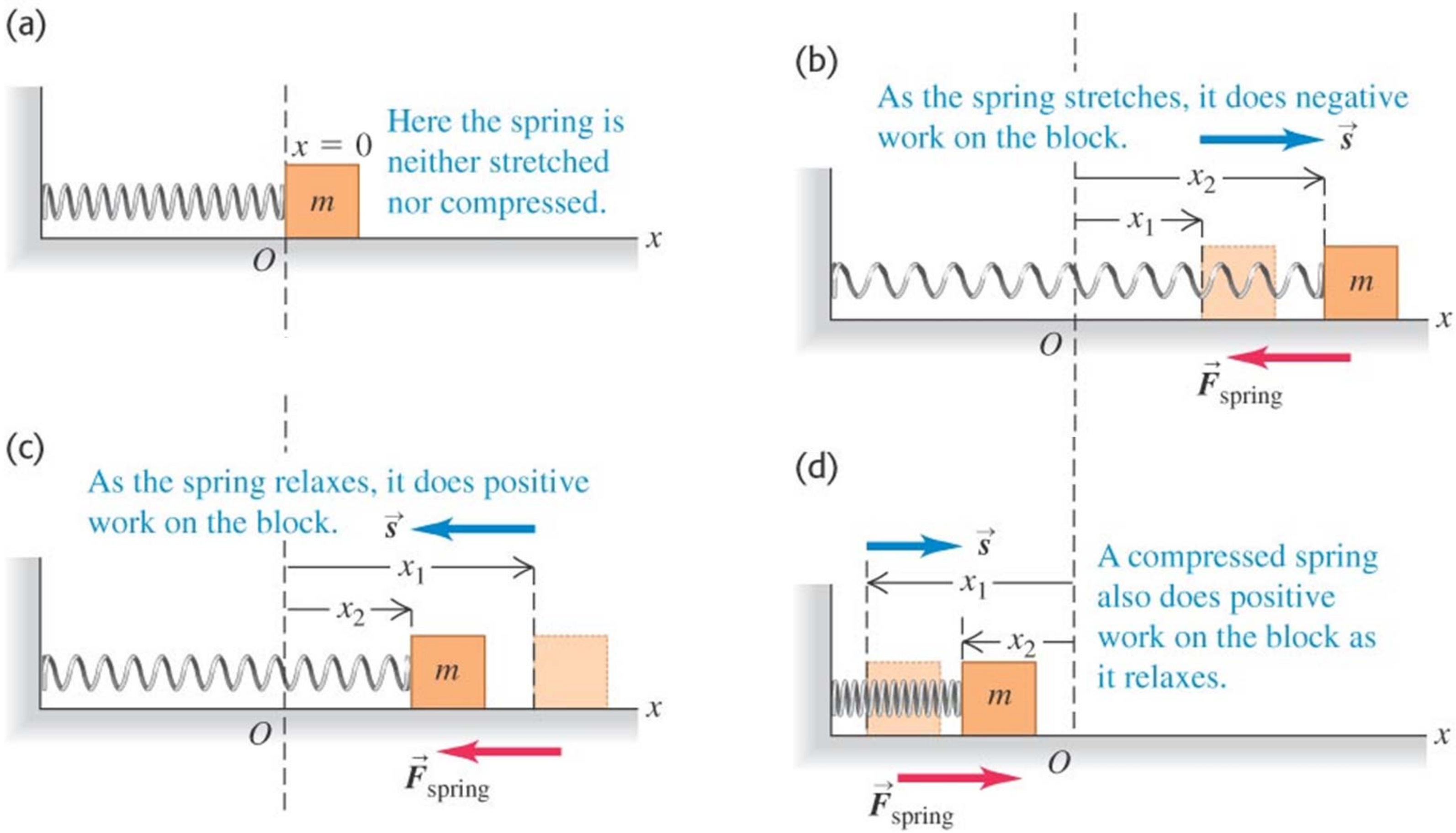
– Refer to Example 7.5 and Figure 7.10.



# Work and energy in the motion of a mass on a spring

$$W_{spring} = \int_{x1}^{x2} (-kx\ )dx = -\frac{1}{2}kx_2^{\ 2} + \frac{1}{2}kx_1^{\ 2}$$

$$= -UE_{spring\,2} + UE_{spring1}$$



# Work energy theorem: situations with both gravitational and elastic potential energy

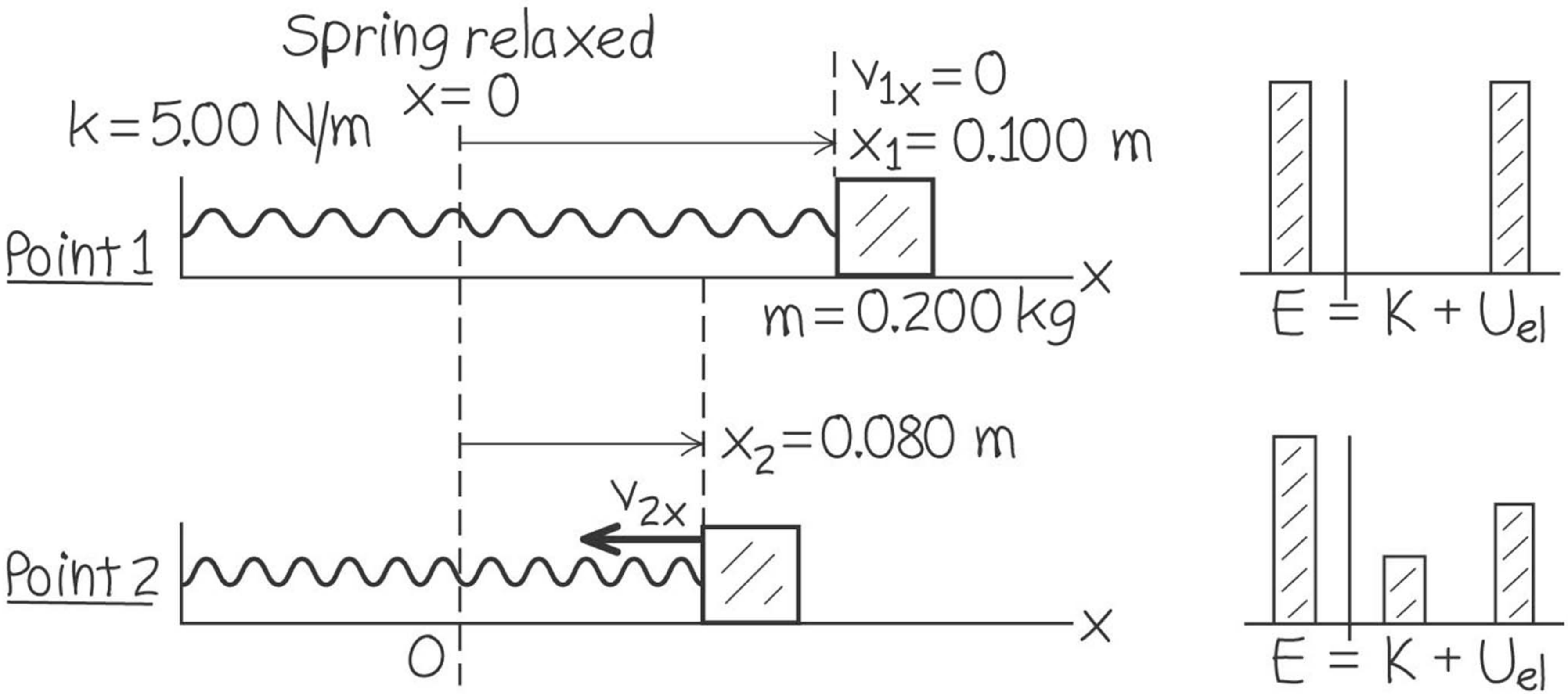
$$W_{conserv} + W_{non-conserv} = KE_2 - KE_1$$

$$-UE_{grav\,2} + UE_{grav\,1} - UE_{spring\,2} + UE_{spring\,1} + W_{non-conserv} = KE_2 - KE_1$$

$$W_{non-conserv} = E_2 - E_1$$ where

$$E = KE + UE_{grav} + UE_{spring} = \frac{1}{2}m\mathrm{v}^2 + mgy + \frac{1}{2}kx^2$$

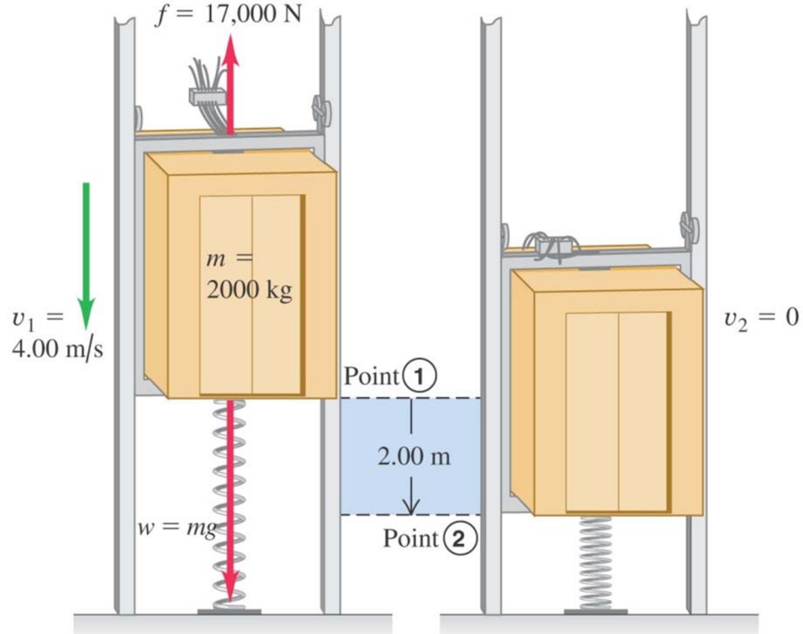
# Motion with elastic potential energy—Example 7.7



# Bring together two potential energies and friction

Example 7.9 What is the spring constant needed?

$$W_{non-conserv} = E_2 - E_1$$




$$W_{non-conserv} = -F_f s = -(17{,}000N)(2m)$$

$$E_1 = \frac{1}{2} m \mathrm{v}_1^{\ 2} + mgy_1 + \frac{1}{2} kx_1^{\ 2} =$$

$$= \frac{1}{2}(2000\,\mathrm{kg})(4\ \mathrm{m/s})^{\ 2} + (2000\,\mathrm{kg})g(2\,\mathrm{m}) + 0$$

$$E_2 = \frac{1}{2} m \mathrm{v}_2^{\ 2} + mgy_2 + \frac{1}{2} kx_2^{\ 2} =$$

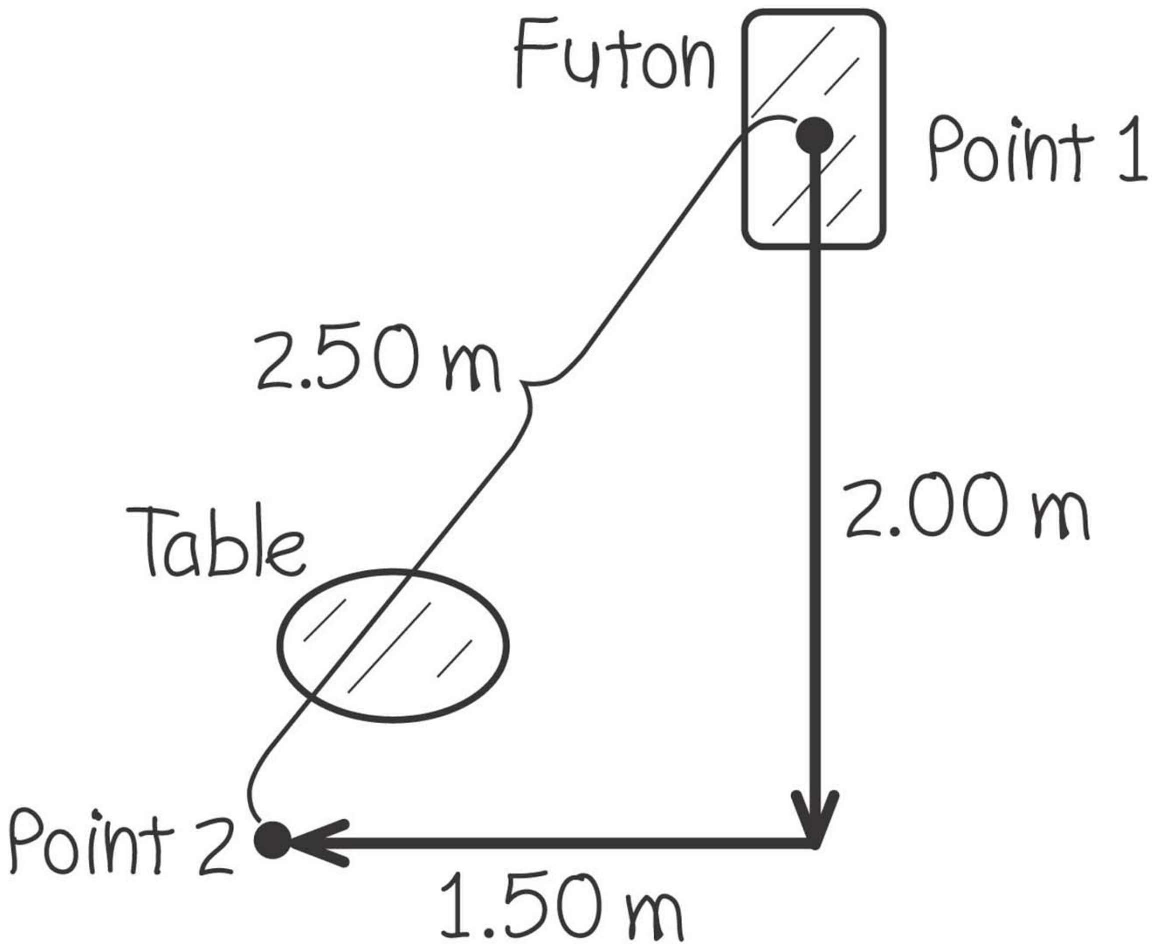
$$= 0 + 0 + \frac{1}{2} k(2m)^2$$

$$-34000 = 2k - 55200$$

$$k = 1.06 \times 10^4\ \mathrm{N/m}$$

## Friction does depend on the path taken

- Consider Example 7.10 where the nonconservative frictional force changes with path.



# Relation between potential energy and force

$$W_{cons} = \int \vec{F}_{cons} \cdot d\vec{s} = -\Delta UE$$

$$dW_{cons} = \vec{F}_{cons} \cdot d\vec{s} = -dUE$$

$$\vec{F}_{cons} = \frac{-dUE}{d\vec{s}}$$

$$F_{cons-x} = \frac{-dUE}{dx} \qquad F_{cons-y} = \frac{-dUE}{dy} \qquad F_{cons-z} = \frac{-dUE}{dz}$$

For example. For gravity

$$F_{grav-y} = \frac{-d(mgy)}{dy} = -mg$$

For example. For gravity

$$F_{spring-x} = \frac{-d(\frac{1}{2}kx^2)}{dx} = -kx$$

This is the end of Ch 7. Let’s next review briefly the main concepts so far and then do more examples

# CH1-3: Kinematics: equations of motion

- **Time** of flight, rotation, etc.
- **IF** you know acceleration then all motion follows
- **General understanding of acceleration:** Acceleration component along/against velocity vector increases/decreases speed; perpendicular acceleration component changes direction (left or right). **IF** particle is going along a circle the radial component is equal to $v^2/r$ (due to geometry, otherwise it spirals in or out).

$$\vec{a} = \frac{d\vec{v}}{dt} \Leftrightarrow \vec{v} = \frac{d\vec{r}}{dt} \Leftrightarrow \vec{r} = x(t)\hat{i} + y(t)\hat{j} + z(t)\hat{k}$$

**Constant velocity**

$$\boxed{x - x_0 = v_x t}$$

**Constant acceleration**

$$x - x_0 = v_{0x} t + \frac{1}{2} a_x t^2$$

$$v_x = v_{0x} + a_x t$$

$$v_x^2 = v_{0x}^2 + 2a_x (x - x_0)$$

**Projectile motion:**
x-comp. is constant velocity
y-comp. is constant acceleration

# CH 4-5: Newton's laws of motion

For constant coordinate system

$$\sum F_x = ma_x \,; \sum F_y = ma_y \,; \sum F_z = ma_z$$

Newton's 3rd law

$$\vec{F}_{\text{on A by B}} = -\vec{F}_{\text{on B by A}}$$

- **They are the ones from which you find the acceleration of objects (connection to Ch. 1-3)**
- **Steps**: (1) draw sketch, (2) draw all forces and label 3rd law pairs, (3) draw free body diagram for each object, (4) choose coordinates for each object (if circular motion there is no choice, one has to be radial –positive towards center- and the other tangential), (5) decompose forces that are not along axis chosen, (6) write Newt. 2nd law for EACH object, (7) are there relations among objects (e.g. same velocity, or one twice the other, etc.), (8) how many equations and how many unknowns. NOW you are ready to solve for the question – this is a good time to look back at the question.
- **Force of friction:** know distinction between static (no acceleration) and kinetic (there is motion relative to the surface)
- **Circular motion:** if moving along a circle sum of forces along the radial direction MUST add to $mv^2/r$

Coordinate system is NOT constant, it rotates!!

$$\sum F_r = m\frac{v^2}{R} \,; \sum F_{\tan} = ma_{\tan} \,; \sum F_z = ma_z$$

# CH 6-7: Work and Energy

Work done by a force is

$$W_{\text{by F}} = \underbrace{\int_{\vec{r}_1}^{\vec{r}_2} \vec{F} \cdot d\vec{r}}_{\text{general}} = \underbrace{\vec{F} \cdot (\vec{r}_2 - \vec{r}_1) = F \Delta r \cos\theta_{F\Delta r}}_{\text{constant force}}$$

**Work energy theorem (also contains conservation of energy)**

$$W_{non-conserv} = E_2 - E_1$$

$$E = KE + UE_{grav} + UE_{spring} = \frac{1}{2} m v^2 + mgy + \frac{1}{2} kx^2$$

# Math that I should know VERY WELL by now

•Algebra (solve ANY complicated equation and pairs of equations)

•Derivatives (what they mean, know how to use them to find maximums/minim.)

•Scalar products of vectors (use them to calculate work, angles, etc.)

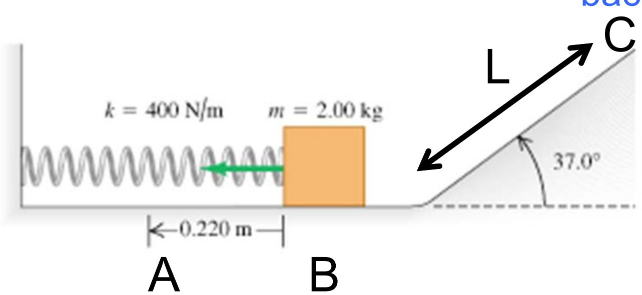
•Basic integrals (calculate work of changing forces, complicated equations of motion)

# Problem 7.42 Conservation of energy: gravity and spring

A 2.00 kg block is pushed against a spring with negligible mass and force constant k = 400 N/m, compressing it 0.220 m. When the block is released, it moves along a frictionless, horizontal surface and then up a frictionless incline with slope 37.0 degrees.

(a) What is the speed of the block as it slides along the horizontal surface after having left the spring?

(b) How far does the block travel up the incline before starting to slide back down?




Both of these are conservation of energy

(a)

$$E_A = E_B$$

$$\frac{1}{2}m{v_A}^2 + mgy_A + \frac{1}{2}kx_A^2 = \frac{1}{2}m{v_B}^2 + mgy_B + \frac{1}{2}kx_B^2$$

$$0 \quad + \quad 0 \quad + \frac{1}{2}kx_A^2 = \frac{1}{2}m{v_B}^2 + \quad 0 \quad + \quad 0$$

$$\sqrt{\frac{kx_A^2}{m}} = v_B$$

(b)

$$E_A = E_B$$

$$\frac{1}{2}m{v_A}^2 + mgy_A + \frac{1}{2}kx_A^2 = \frac{1}{2}m{v_C}^2 + mgy_C + \frac{1}{2}kx_C^2$$

$$0 \quad + \quad 0 \quad + \frac{1}{2}kx_A^2 = 0 + \quad mgy_C \quad + \quad 0$$

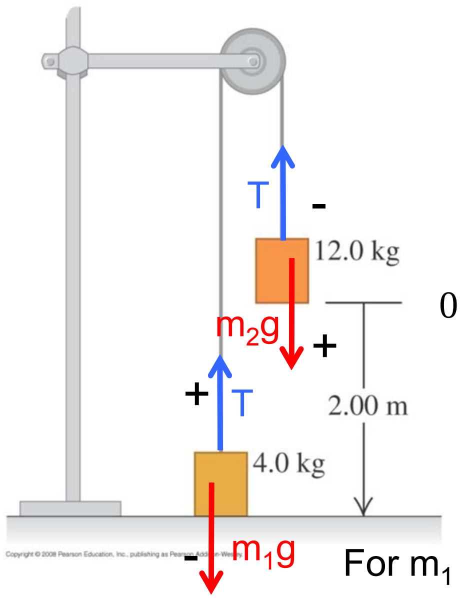
$$\frac{1}{2}\frac{kx_A^2}{mg} = y_C = L\sin\theta$$

$$\frac{1}{2}\frac{kx_A^2}{mg\sin\theta} = L$$

# NOW A PULLEY PROBLEM

# Problem 7.55 Pulley problem

What is the speed of the larger block before it strikes the ground?




TWO CHOICES: CH 4-5 style or CH 6-7 style

Because there is no friction then energy is conserved

$$E_A = E_B$$

$$\frac{1}{2}m_1 \mathrm{v}_{A1}{}^2 + m_1 g y_{A1} + \frac{1}{2}m_2 \mathrm{v}_{A2}{}^2 + m_2 g y_{A2} = \frac{1}{2}m_1 \mathrm{v}_{B1}{}^2 + m_1 g y_{B1} + \frac{1}{2}m_2 \mathrm{v}_{B2}{}^2 + m_2 g y_{B2}$$

$$0+0+0+(12\,\mathrm{Kg})(9.8\,\mathrm{m/s^2})(2.00\,\mathrm{m}) = \frac{1}{2}(4\,\mathrm{Kg})\mathrm{v}^2 + (4\,\mathrm{Kg})(9.8\,\mathrm{m/s^2})(2.00\,\mathrm{m}) + \frac{1}{2}(12\,\mathrm{Kg})\mathrm{v}^2 + 0$$

$$\mathrm{v} = 4.43\frac{\mathrm{m}}{\mathrm{s}}$$

Now let's do it the longish way (Ch 4-5)

For $m_1$ Newton's 2[nd] law reads $y: \quad -m_1 g + T = m_1 a \quad \Rightarrow T = m_1 a + m_1 g$

For $m_2$ Newton's 2[nd] law reads $y: \quad +m_2 g - T = m_2 a$

$$+m_2 g - (m_1 a + m_1 g) = m_2 a$$
$$+m_2 g - m_1 a - m_1 g = m_2 a$$
$$(m_2 - m_1) g = (m_2 + m_1) a$$
$$\frac{(m_2 - m_1)}{(m_2 + m_1)} g = a$$

Then we use kinematic to solve for v (Ch 1-3)

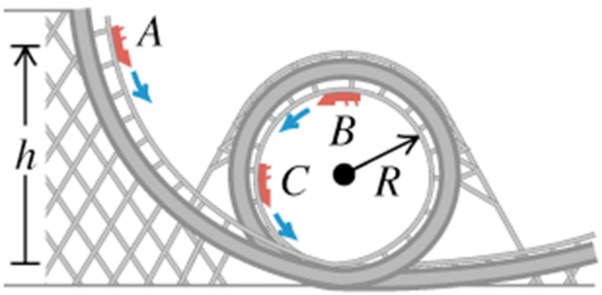
$$y - y_0 = -h$$
$$\mathrm{v}_y = ?$$
$$\mathrm{v}_{y0} = 0$$
$$a_y = -\left(\frac{m_2 - m_1}{m_1 + m_2}\right) g$$
$$t =$$

$$\mathrm{v}_y{}^2 = \mathrm{v}_{y0}{}^2 + 2a_y(y - y_0)$$
$$\mathrm{v}_y{}^2 = 0 + 2 - \left(\frac{m_2 - m_1}{m_1 + m_2}\right) g(-h)$$
$$\mathrm{v}_y = \sqrt{2\left(\frac{m_2 - m_1}{m_1 + m_2}\right) gh} = 4.43\frac{\mathrm{m}}{\mathrm{s}}$$

What if I had asked for the time? Would I have a choice?

# Problem 7.46: Energy + circular motion

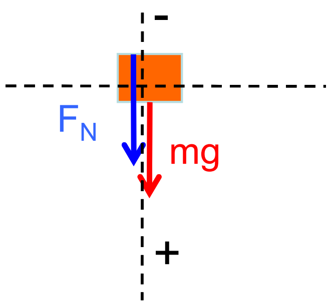
A car in an amusement park ride rolls without friction around the track shown in the figure . It starts from rest at point A at a height h above the bottom of the loop. Treat the car as a particle.




(a) What is the minimum value of h (in terms of R) such that the car moves around the loop without falling off at the top (point B)?

(b) If the car starts at height h= 4.00 R and the radius is R = 20.0 m, compute the radial acceleration of the passengers when the car is at point C, which is at the end of a horizontal diameter.

What is the minimum velocity so at B we are going around a CIRCLE? You will feel like you are flying and not touching the track?




$$r: \quad mg + F_N = m\frac{v^2}{R}$$

$$g = \frac{v_{min}^2}{R} \Rightarrow v_{min}^2 = gR$$

Now we know the velocity (or KE) we need at B so we can use conservation of energy (remember $F_N$ does no work so $W_{other}$=0) to get it

$$E_A = E_B$$

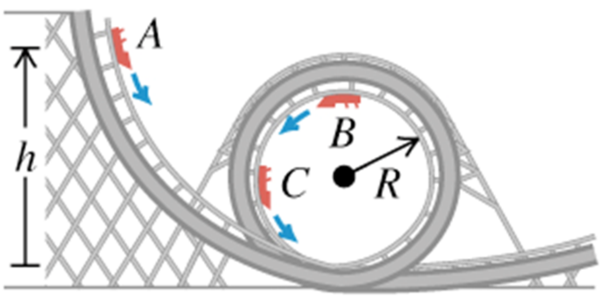
$$\frac{1}{2}m v_A^2 + mgy_A = \frac{1}{2}m v_B^2 + mgy_B$$

$$0 \quad + mgh = \frac{1}{2}mgR + mg2R$$

$$h = \frac{5}{2}R$$

# Problem 7.46: Energy + circular motion

A car in an amusement park ride rolls without friction around the track shown in the figure . It starts from rest at point A at a height h above the bottom of the loop. Treat the car as a particle.




(a) ~~What is the minimum value of h (in terms of R) such that the car moves around the loop without falling off at the top (point B)?~~

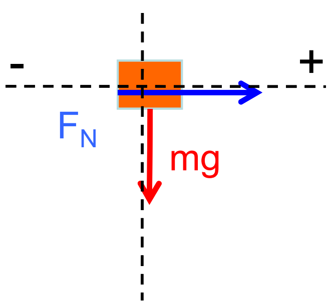
(b) If the car starts at height h= 4.00 R and the radius is R = 20.0 m, compute the radial acceleration of the passengers when the car is at point C, which is at the end of a horizontal diameter.

The radial acceleration is $v^2/R$ so we need v at C. We can use conservation of energy (remember $F_N$ does no work so $W_{other}=0$) to get it

$$E_A = E_C$$

$$\frac{1}{2}m\mathrm{v}_A{}^2 + mgy_A = \frac{1}{2}m\mathrm{v}_C{}^2 + mgy_C$$

$$0 \quad + mg4R = \frac{1}{2}m\mathrm{v}^2 + mgR$$

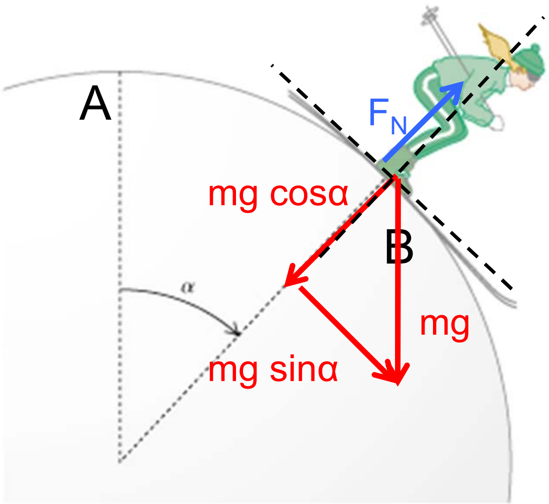



$$r: \quad F_N = m\frac{\mathrm{v}^2}{R} = 3mg$$

$$\mathrm{v}^2 = 3gR \quad \Rightarrow \quad a_r = \frac{\mathrm{v}^2}{R} = 3g$$

# Problem 7.63 Conservation of energy and circular motion

A skier starts at the top of a very large, frictionless snowball, with a very small initial speed, and skis straight down the side (the figure ). At what point does she lose contact with the snowball and fly off at a tangent?




Similar to the roller coaster from A to B use conservation and at B use $F_N$=0 when it flies off.

$$E_A = E_B$$

$$\frac{1}{2} m v_A{}^2 + mgy_A = \frac{1}{2} m v_B{}^2 + mgy_B$$

$$0 \quad + mgR = \frac{1}{2} m v^2 + mgR\cos\alpha$$

$$v^2 = 2Rg(1-\cos\alpha)$$

$$r: \quad mg\cos\alpha - F_N = m\frac{v^2}{R} \quad \Rightarrow \quad mg\cos\alpha - 0 = m2g(1-\cos\alpha)$$

$$\tan: \quad mg\sin\alpha = ma_{\tan} \qquad \cos\alpha = \frac{2}{3} \Rightarrow 48^o$$