Chapter 2

Motion Along a Straight Line

Figure 2.5

Describing Motion

Interested in two key ideas:

How objects move as a function of time

- Kinematics
- Chapters 2 and 3

Why objects move the way they do

- Dynamics
- Do this in Chapter 4 and later

Table 2.2

Table 2.2 Typical Velocity Magnitudes

A snail's pace	10^{-3} m/s
A brisk walk	2 m/s
Fastest human	11 m/s
Freeway speeds	30 m/s
Fastest car	341 m/s
Random motion of air molecules	500 m/s
Fastest airplane	1000 m/s
Orbiting communications satellite	3000 m/s
Electron orbiting in a hydrogen atom	2×10^{6} m/s
Light traveling in a vacuum	3×10^{8} m/s

Chapter 2: Motion in 1-Dimension

Velocity & Acceleration

- Equations of Motion
- Definitions
- Some calculus (derivatives)
- Understanding displacement, velocity curves
- Strategies to solve kinematics problems
- Motion with constant acceleration
  - Free falling objects

Equations of Motion

We want Equations that describe:

Where am I as a function of time?

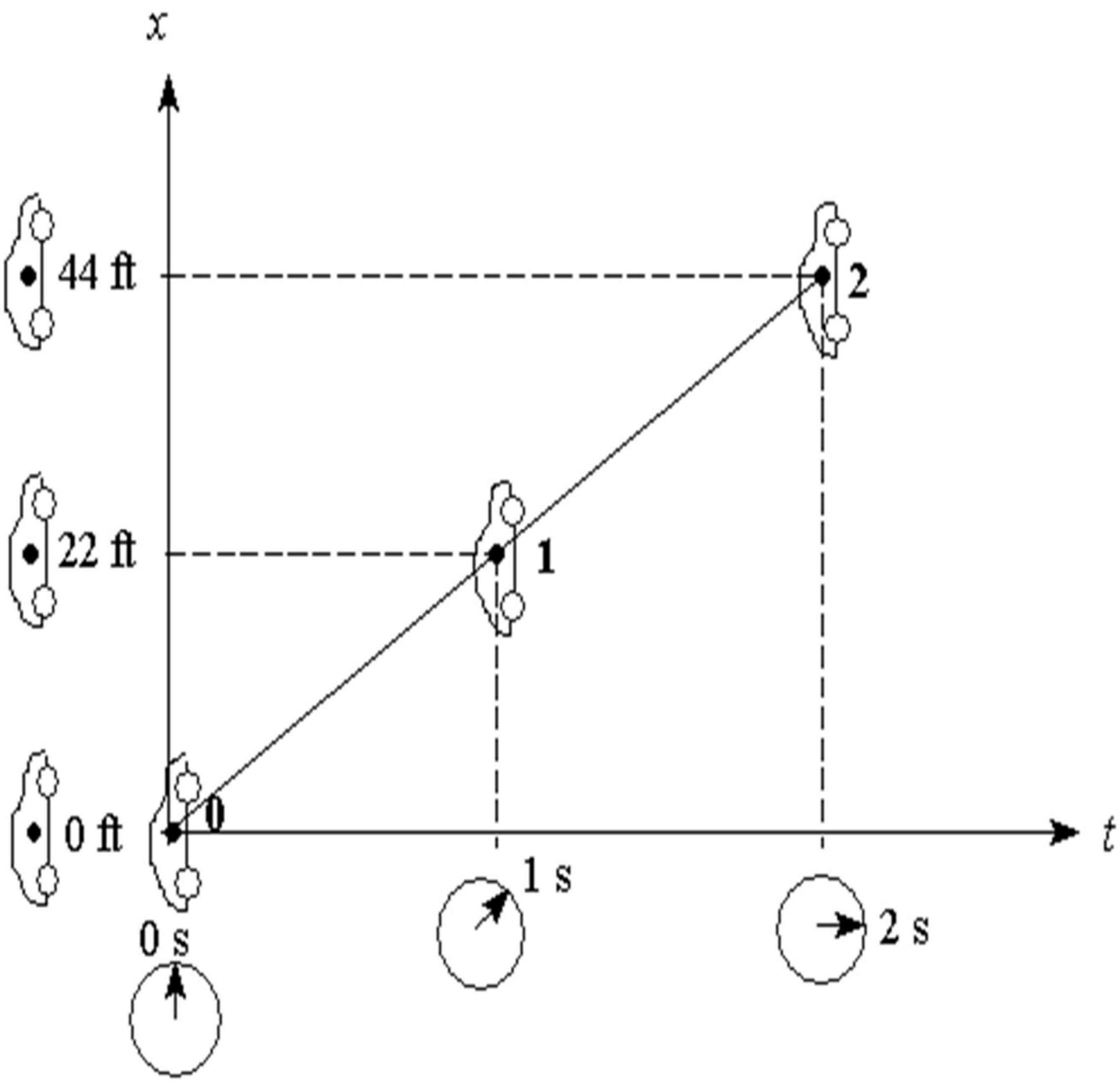
How fast am I moving as a function of time?

What direction am I moving as a function of time?

Is my velocity changing? Etc.

Where will I land?

Motion in One Dimension



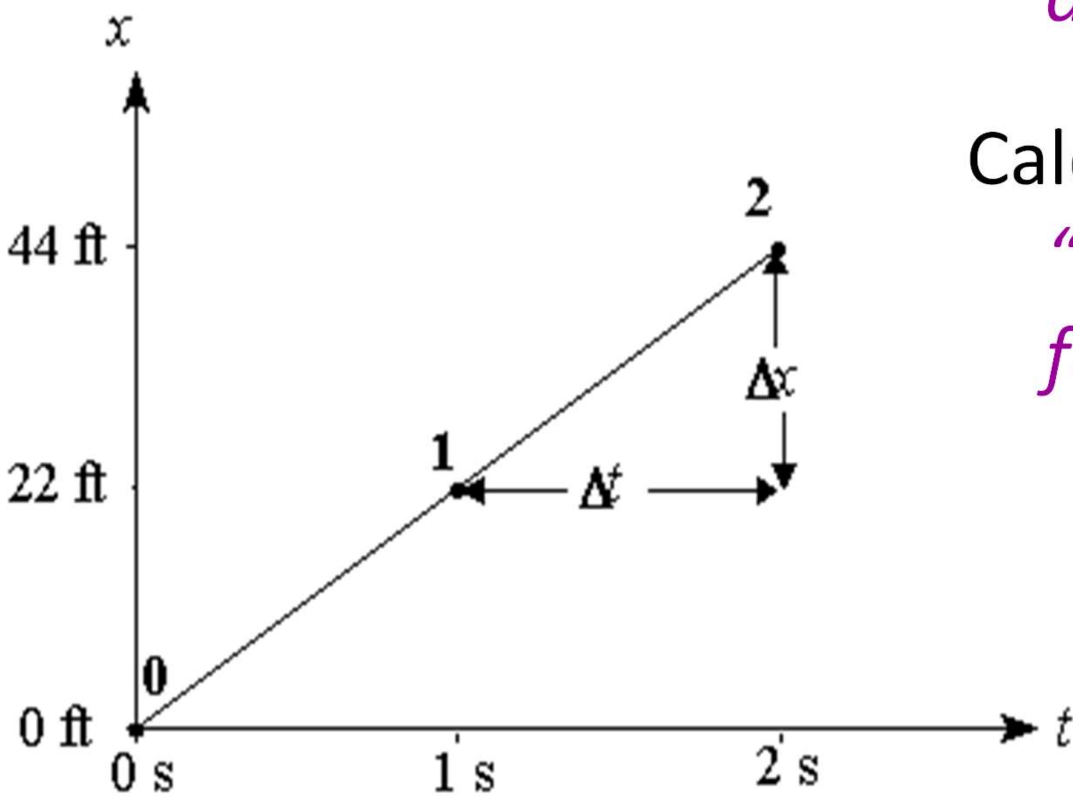
Where is the car?

- $X=0$ *feet* at $t_0=0$ *sec*
- $X=22$ *feet* at $t_1=1$ *sec*
- $X=44$ *feet* at $t_2=2$ *sec*

We say this car has
“velocity” or “Speed”

Plot position vs. time.
How do we get the velocity from graph?

Motion in One Dimension Cont…



Velocity: *"Change in position during a certain time period"*

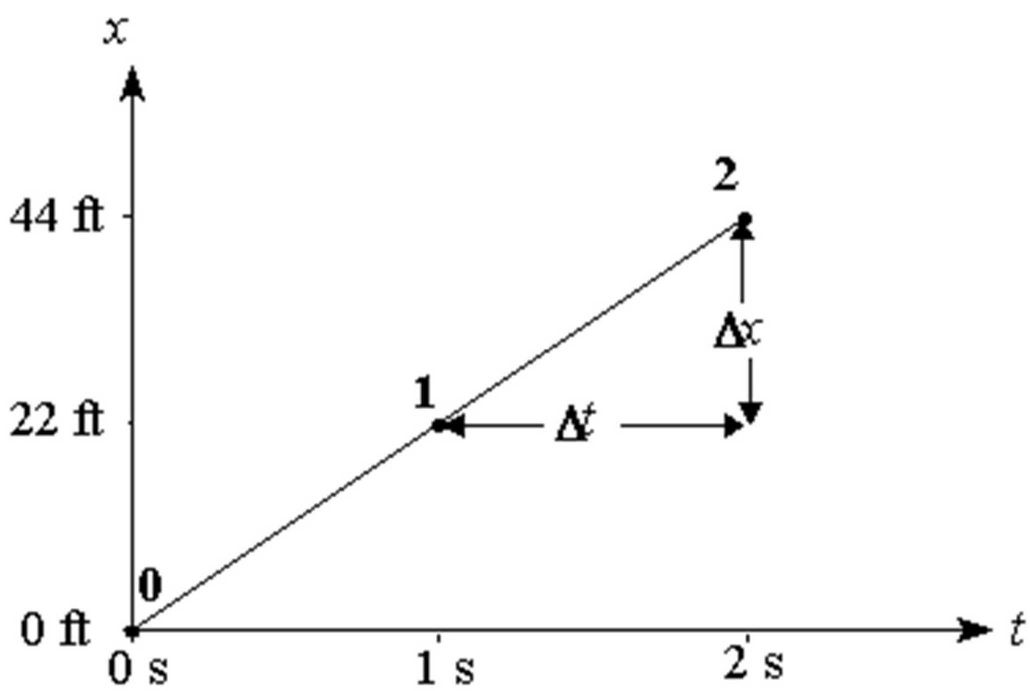
Calculate from the *Slope*: The *"Change in position as a function of time"*

- Change in Vertical
- Change in Horizontal

Change: *D*

Velocity ≡ *DX/Dt*

Constant Velocity

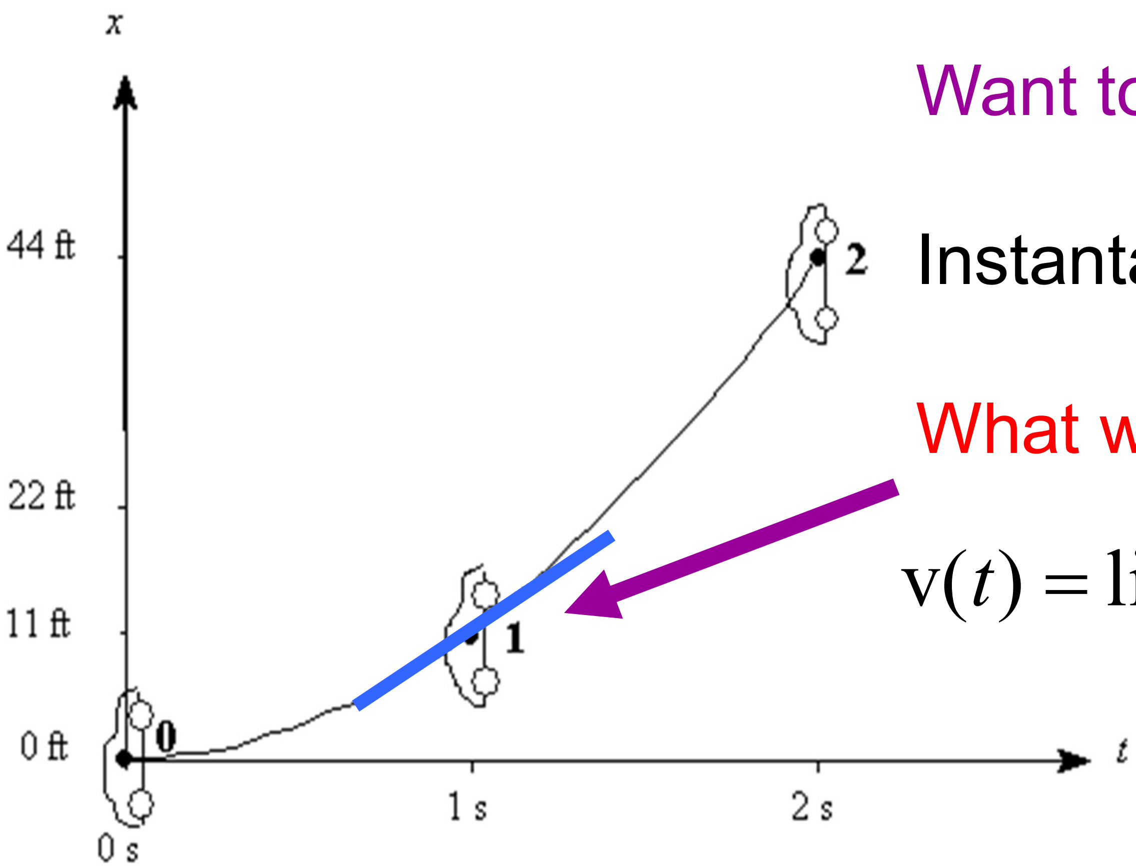


Equation of Motion for this example:

$$X = bt + x_0 \text{ with } x_0 = 0$$

- Slope is constant
- Velocity is constant
  - Easy to calculate
  - Same everywhere

Moving Car



A harder example:

$$X = ct^2$$

What's the velocity at *t=1 sec*?

Want to calculate the *"Slope"* here.

Instantaneous Velocity

What would my speedometer read?

$$\mathrm{v}(t) = \lim_{\Delta t \to 0} \frac{x(t+\Delta t) - x(t)}{\Delta t} = \frac{dx}{dt}$$

Math: Derivatives

To find the slope at time *t*, just take the "derivative"

For $X=ct^2$, *Slope* = $V = dx/dt = 2ct$

Derivative for most common case:

- If $x = at^n \rightarrow V = dx/dt = nat^{n-1}$
- "Derivative of *X* with respect to *t*"

More examples

- $X = qt^2 \rightarrow V = dx/dt = 2qt$
- $X = ht^3 \rightarrow V = dx/dt = 3ht^2$

Derivatives continued

Basic rules:

- Sum: $x(t)= f(t) + g(t) \Rightarrow dx/dt = df/dt+dg/dt$
  - $x(t)= 12+6t+3t^2+4t^3 \Rightarrow dx/dt =0+6+6t+12t^2$
- Constant: $x(t)= C \Rightarrow dx/dt = 0$;follows from n=0
- Multiplication:
  $x(t)=f(t)*g(t) \Rightarrow dx/dt = df/dt * g + f * dg/dt$
  - $x(t)= (2t)*(t^2) \Rightarrow dx/dt =2*(t^2)+(2t)*(2t)=6t^2$
  - Compare to $x(t)= (2t)*(t^2)=2t^3 \Rightarrow$
    $dx/dt=2*3*t^2=6t^2$

Common Mistakes

The trick is to remember what you are taking the derivative "with respect to"

More Examples (with a=*constant*):

What if $X = 2a^3t^n$?

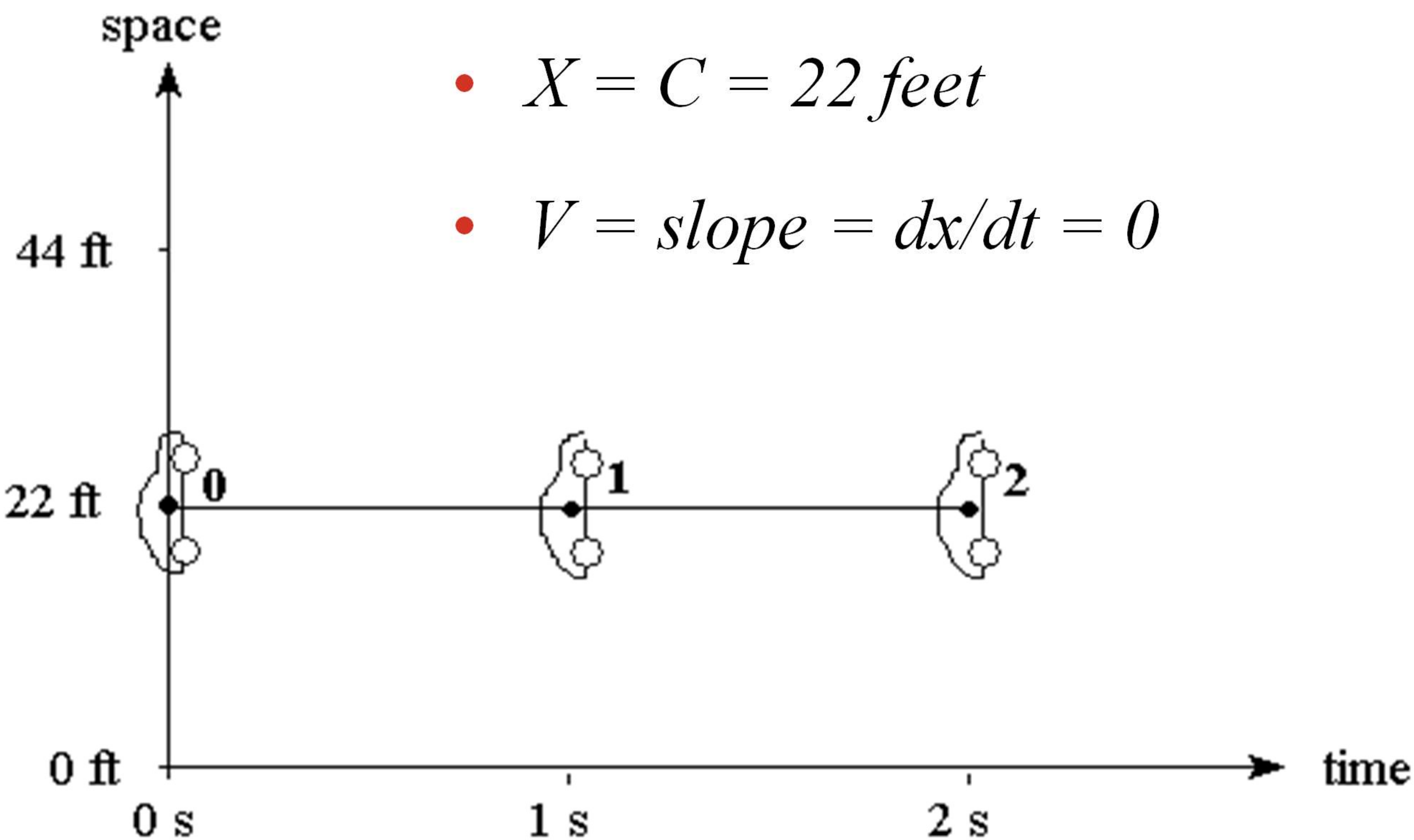
- Why not $dx/dt = 3(2a^2t^n)$?
- Why not $dx/dt = 3n(2a^2t^{n-1})$?

What if $X = 2a^3$?

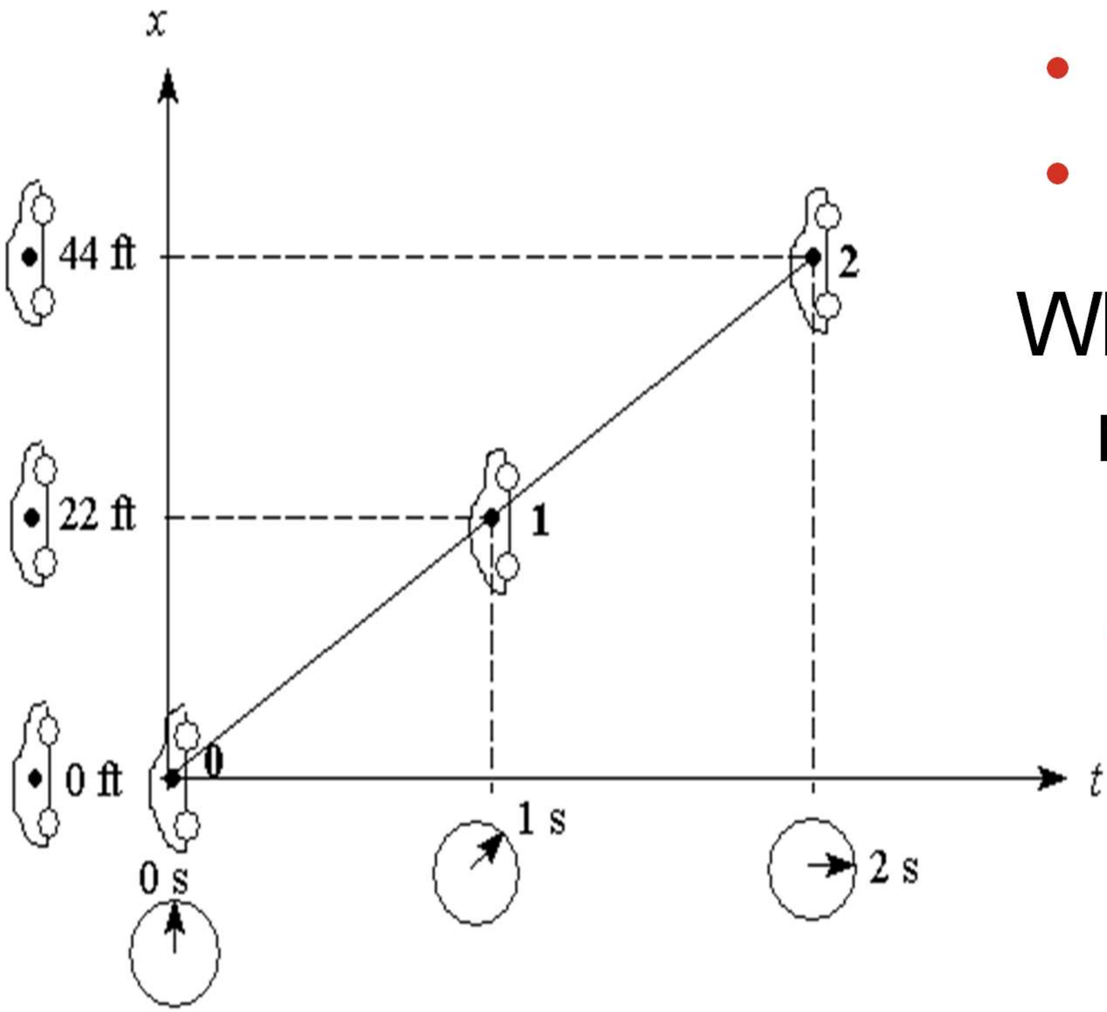
- What is dx/dt?
- There are no t's!!! $dx/dt = 0$!!!
- If $X=22$ feet for all t, what is the velocity? =0!!!

Check: Constant Position

- $X = C = 22\ feet$
- $V = slope = dx/dt = 0$



Check: Constant Velocity



Car is moving

- $X=0\ feet$ at $t_0=0\ sec$
- $X=22\ feet$ at $t_1=1\ sec$
- $X=44\ feet$ at $t_2=2\ sec$

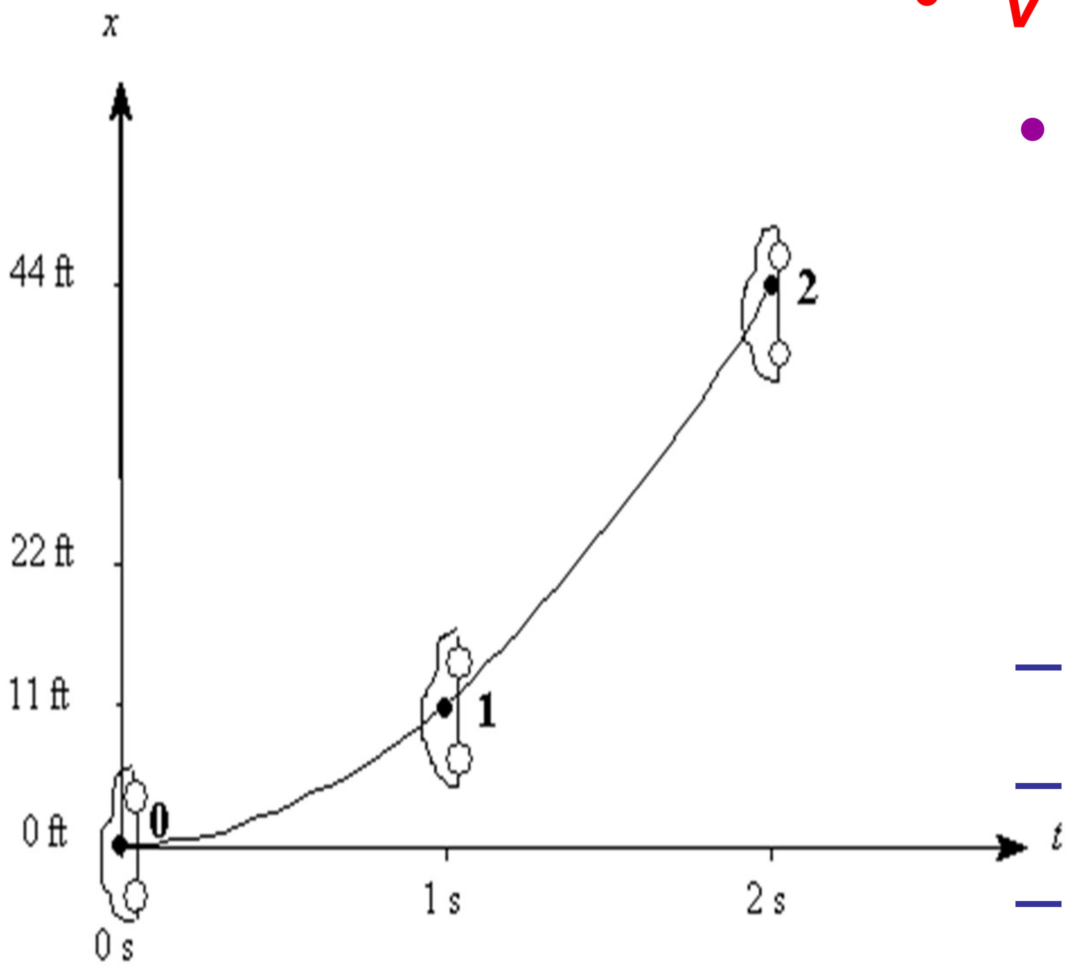
What is the equation of motion?

$X = bt$ with $b=22\ ft/sec$

$V = dX/dt$

$\rightarrow V = b = 22\ ft/sec$

Check: Non-Constant Velocity

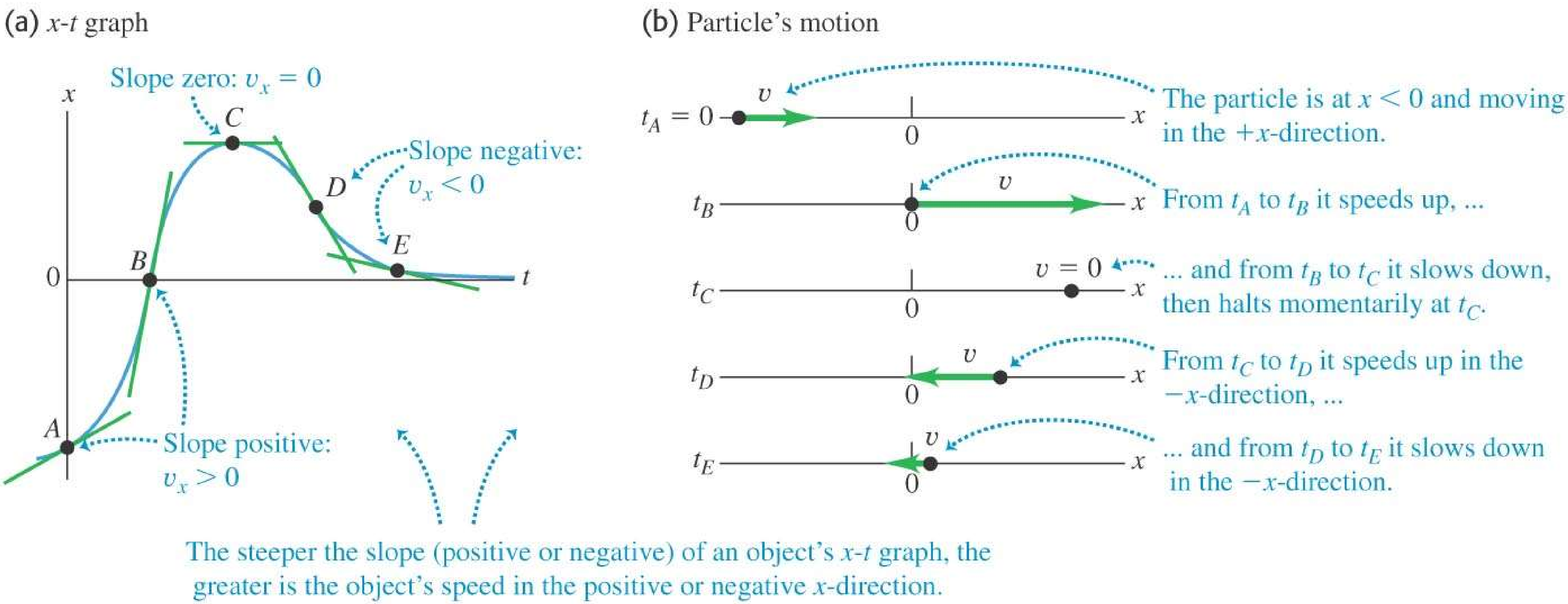


- $X = ct^2$ with $c=11\ ft/sec^2$
- $V = dX/dt = 2ct$
  - The velocity is:
    - "non-Constant"
    - a "function of time"
    - "Changes with time"

– $V=0$ *ft/s* at $t_0=0$ *sec*
– $V=22$ *ft/s* at $t_1=1$ *sec*
– $V=44$ *ft/s* at $t_2=2$ *sec*

Follow the motion of a particle

- A graph of position versus time may be constructed.
- The motion of the particle may be described at selected moments in time.



Acceleration

If the velocity is changing, we are *"accelerating"*

Acceleration is the "Rate of change of velocity"

- Just like velocity is the rate of change in distance (or position)

You hit the *accelerator* in your car to speed up

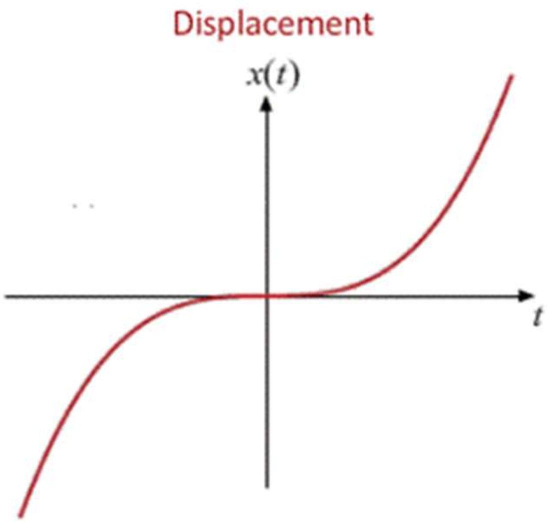
- (It's true you also hit it to stay at constant velocity, but that's because friction is slowing you down…we'll get to that later…)

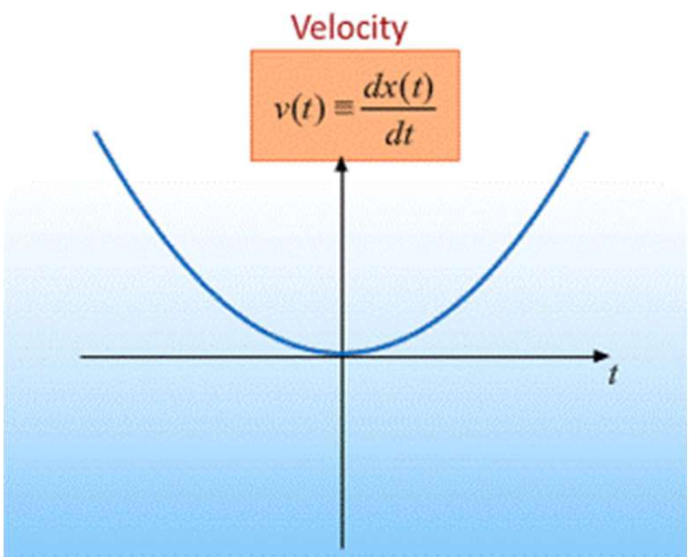
Acceleration

Acceleration is the "Rate of change of velocity"

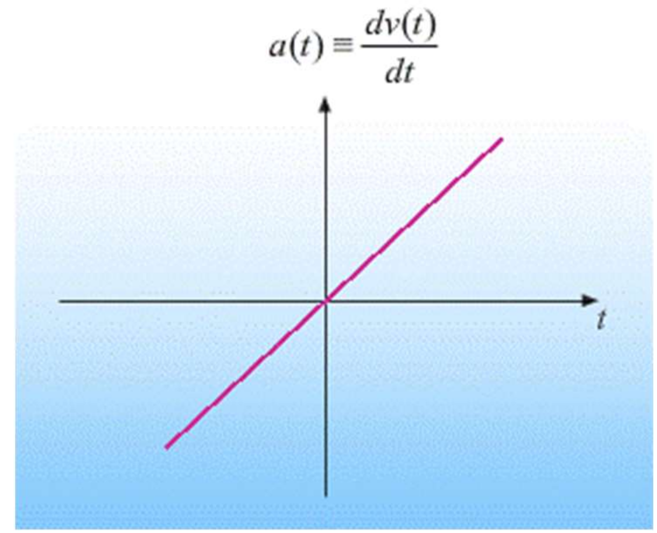
Said differently: "How fast is the Velocity changing?" "What is the change in velocity as a function of time?"

$$a = \frac{\Delta \mathrm{v}}{\Delta t} = \frac{V_2 - V_1}{t_2 - t_1} = \frac{d\,\mathrm{v}}{dt}$$





A



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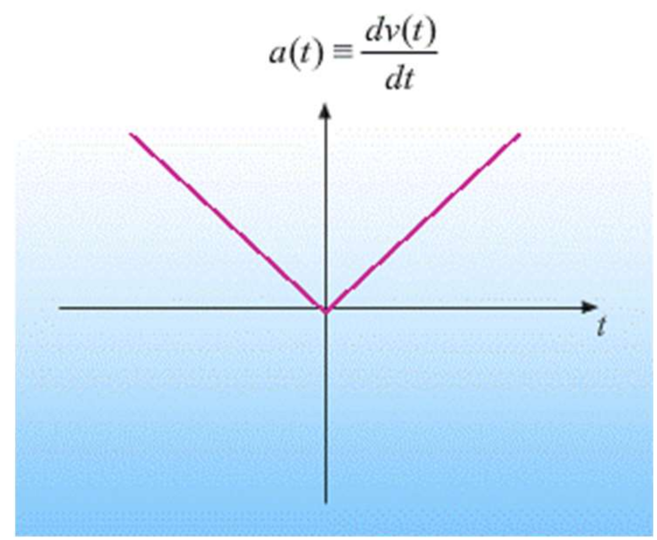


Which plot best represents the acceleration curve associated with the displacement and velocity curves shown above?

Important Equations of Motion

If the acceleration is constant

$$\vec{v} = \vec{v}_0 + \vec{a}t$$

$$\vec{x} = \vec{x}_0 + \vec{v}_0 t + \frac{1}{2}\vec{a}t^2$$

Position, velocity and Acceleration are vectors. More on this in Chap 3

Example

You have an equation of motion of:

$$X = X_0 + V_0 t + \tfrac{1}{2} a t^2$$

where X_0, V_0, and a are constants.

What is the velocity and the acceleration?

$\rightarrow v = dx/dt = 0 + V_0 + at$ (recall that $d(ct^n)/dt = nct^{n-1}$)

- Remember that derivative of a constant is 0!!

$\rightarrow a = dv/dt = d^2x/dt^2 = 0 + 0 + a$

The acceleration of this example is constant

Position, Velocity and Acceleration

All three are related

- Velocity is the *derivative* of position with respect to time
- Acceleration is the *derivative* of velocity with respect to time
- Acceleration is the *second derivative* of position with respect to time

Calculus is REALLY important

Derivatives are something we'll come back to over and over again

Equations of motion for constant acceleration

$$x(t) = x_0 + v_0 t + \tfrac{1}{2} a t^2$$

$$v(t) = v_0 + at$$

$$a(t) = a$$

It follows from the one above if you take the derivative wrt time

Just says that acceleration is constant in time; it is not useful in terms of algebra.

From now on writing x or v implies x(t) or v(t) so as not to write (t) all the time.

Algebra reminder: if you have a set of equations and a set of unknown variables their number have to be the same to be able to solve it.

We have TWO independent equations and 6 variables $x,\ x_0, v, v_0, t, a$

Truly we really have only 5 because x and x_0 always come together as $x\text{-}x_0$

Problem

Show that for constant acceleration:

$$2a(\Delta x) = V_f^2 - V_0^2$$

The equations of motion under constant acceleration

$$x - x_0 = +v_0 t + \tfrac{1}{2} a t^2$$

$$v = v_0 + at$$

To make life easier lets get a third equation (BUT we only still have TWO INDEPENDENT ones since we are getting this third by combining the two)

First solve for t on the second and substitute it in the first

$$t = \frac{v - v_0}{a}$$

$$x - x_0 = v_0\left(\frac{v - v_0}{a}\right) + \tfrac{1}{2} a \left(\frac{v - v_0}{a}\right)^2 =$$

$$x - x_0 = \frac{v_0 v - v_0^2}{a} + \tfrac{1}{2} a \frac{v^2 - 2v_0 v + v_0^2}{a^2}$$

$$x - x_0 = \frac{v_0 v - v_0^2}{a} + \tfrac{1}{2} \frac{v^2 - 2v_0 v + v_0^2}{a}$$

$$2a(x - x_0) = 2v_0 v - 2v_0^2 + v^2 - 2v_0 v + v_0^2$$

$$2a(x - x_0) = v^2 - v_0^2$$

$$x - x_0 = +v_0 t + \tfrac{1}{2} a t^2$$

$$v = v_0 + at$$

$$v^2 = v_0^2 + 2a(x - x_0)$$

Problem with Derivatives

A car is stopped at a traffic light. It then travels along a straight road so that its distance from the light is given by

- $x(t) = bt^2 - ct^3$, $b = 2.40\ \mathrm{m/s^2}$, $c = 0.12\ \mathrm{m/s^3}$

Calculate:

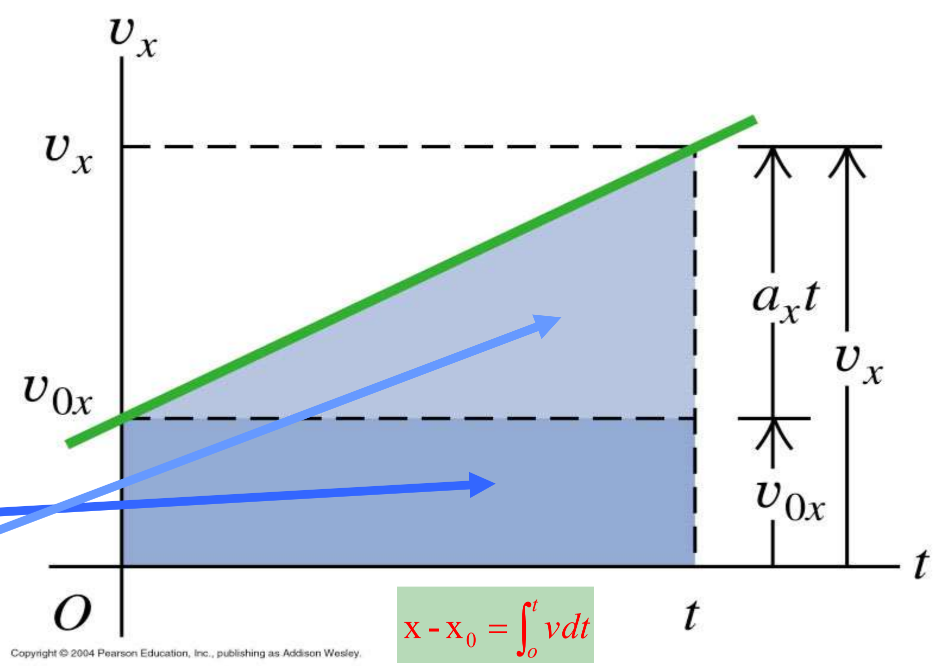
- instantaneous velocity of the car at $t = 0, 5, 10$ s
- How long after starting from rest is the car again at rest?

Getting Displacement from Velocity

For const acceleration the <u>Equation of motion:</u> $X=X_0+V_0t + \frac{1}{2}at^2$

If you are given the <u>velocity</u> vs. time graph you can find the total distance traveled from the area under the curve:

- $X-X_0=V_0t + \frac{1}{2}at^2$



Can also find this from *integrating*…

Definite and Indefinite Integrals

How to you calculate the *Value* of an integral?

In many ways an integral is an *anti-derivitive*

$$\Rightarrow \int (c)\, dt = ct + b$$

where b is an arbitrary constant and is added to the right side of the equation $\Rightarrow \dfrac{d(ct+b)}{dt} = c$

If I know where my region of integratio n begins and ends :

(assuming $a,\ b,$ and c are constants)

$$\int_a^b c\,dt = ct\Big|_{t=a}^{t=b} = cb - ca$$

Some Integrals

Make this more general :

$$\int a\, dt = at + c$$

$$\int at^m\, dt = a\left(\frac{t^{m+1}}{m+1}\right) + c$$

Check "anti - derivitive "

$$\frac{d}{dt}\left(\int at^m\, dt\right) = \frac{d}{dt}\left(a\left(\frac{t^{m+1}}{m+1}\right) + c\right)$$

$$= (m+1)a\left(\frac{t^{(m+1)-1}}{m+1}\right) + 0$$

$$= at^m$$

Our Example with Const. Acceleration

$$\mathrm{x} - \mathrm{x}_0 = \int_o^t v\,dt$$

$$= \int_o^t (v_0 + at)\,dt$$

$$= \left(v_0 t + \tfrac{1}{2} a t^2\right)\Big|_0^t$$

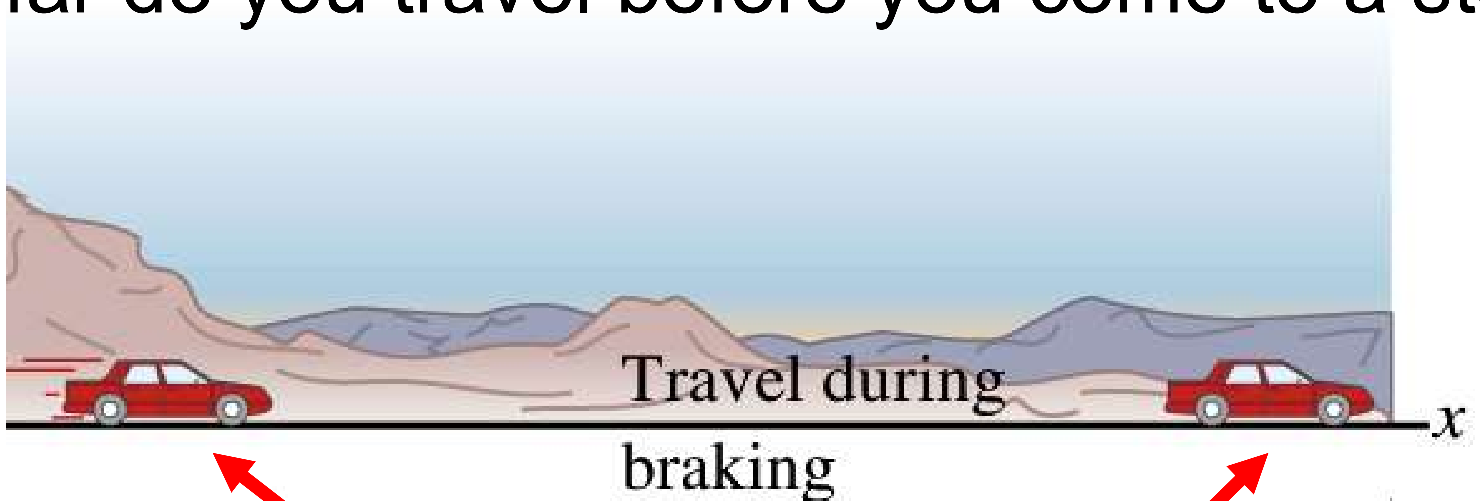
$$\boxed{= v_0 t + \tfrac{1}{2} a t^2}$$

How quickly can you stop a car?

You're driving along a road at some constant speed, V_0, and slam on the breaks and slow down with constant deceleration *a*.

1. How much time does it take to stop?

2. How far do you travel before you come to a stop?



When you hit the brakes

Where you stop

Speeder

A speeder passes you (a police officer) sitting by the side of the road and maintains their constant velocity V. You immediately start to move after the speeder from rest with constant acceleration a.

How much time does it take to ram the speeder?

How far do you have to travel to catch the speeder?

What is your final speed?

Equations of motion for constant acceleration

$$x(t) = x_0 + v_0 t + \tfrac{1}{2} a t^2$$

$$v(t) = v_0 + at$$

$$a(t) = a$$

It follows from the one above if you take the derivative wrt time

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We have TWO independent equations and 6 variables $x, \; x_0, v, v_0, t, a$

Truly we really have only 5 because x and x_0 always come together as $x\text{-}x_0$

Strategy to solve constant acceleration problems

1. Read the problem. (how many objects are moving? Write the 5/6 possible variable for each)

$$x - x_0 =$$
$$v =$$
$$v_0 =$$
$$a =$$
$$t =$$

Write one column set of these for each object.

2. THEN read the problem again slowly and identify every unknown that is given (careful, sometimes words ARE numbers, i.e. "start at rest" means v_0=0 which is important). Also, now convert EVERYTHING TO MKS units

3. (a) IF only one object then count number of unknowns, IF you have three then move to next step, IF not then you are missing some info, read again

(b) IF more than one object, identify what is common between the two objects (which variable)

4. Look at the equations and use the one for which the two unknown variables are given. If more than one object you will need to combine them. Do the algebra.

$$x - x_0 = +v_0 t + \tfrac{1}{2} a t^2$$
$$v = v_0 + at$$
$$v^2 = v_0^2 + 2a(x - x_0)$$

NEXT, LOTS OF EXAMPLES

One object

You are going along Texas Av. at 35 mph (texting on the phone). Suddenly you see an old lady crossing the street about 100 meters away and hit the breaks. Your car decelerates uniformly at 1.25 m/s^2. Did you run the poor lady over?

1st step

$x - x_0 =$

$v =$

$v_0 =$

$a =$

$t =$

2nd step

$x - x_0 = ?$

$v = 0$

Because it is assume you are stopped at the end

$v_0 = 35mph = 15.64\,\frac{m}{s}$

Unit conversion

$a = -1.25\,\frac{m}{s^2}$

$t =$

We have THREE unknowns so we can now find the other two.

3rd step

Which equation contains the three unknowns and the one we want?

$$x - x_0 = +v_0 t + \tfrac{1}{2} a t^2$$

$$v = v_0 + at$$

$$v^2 = v_0^2 + 2a(x - x_0)$$

$$(x - x_0) = \frac{v^2 - v_0^2}{2a} = \frac{0^2 - (15.64\,m/s)^2}{2(-1.25\,m/s^2)} = 97.8\,m$$

So the old lady survives the car

Free Fall

Free fall is a good example for one dimensional problems

Gravity:

- Things accelerate towards earth with a constant acceleration
- $a = g = 9.8 \text{m/s}^2$ towards the earth
- We'll come back to Gravity a lot!

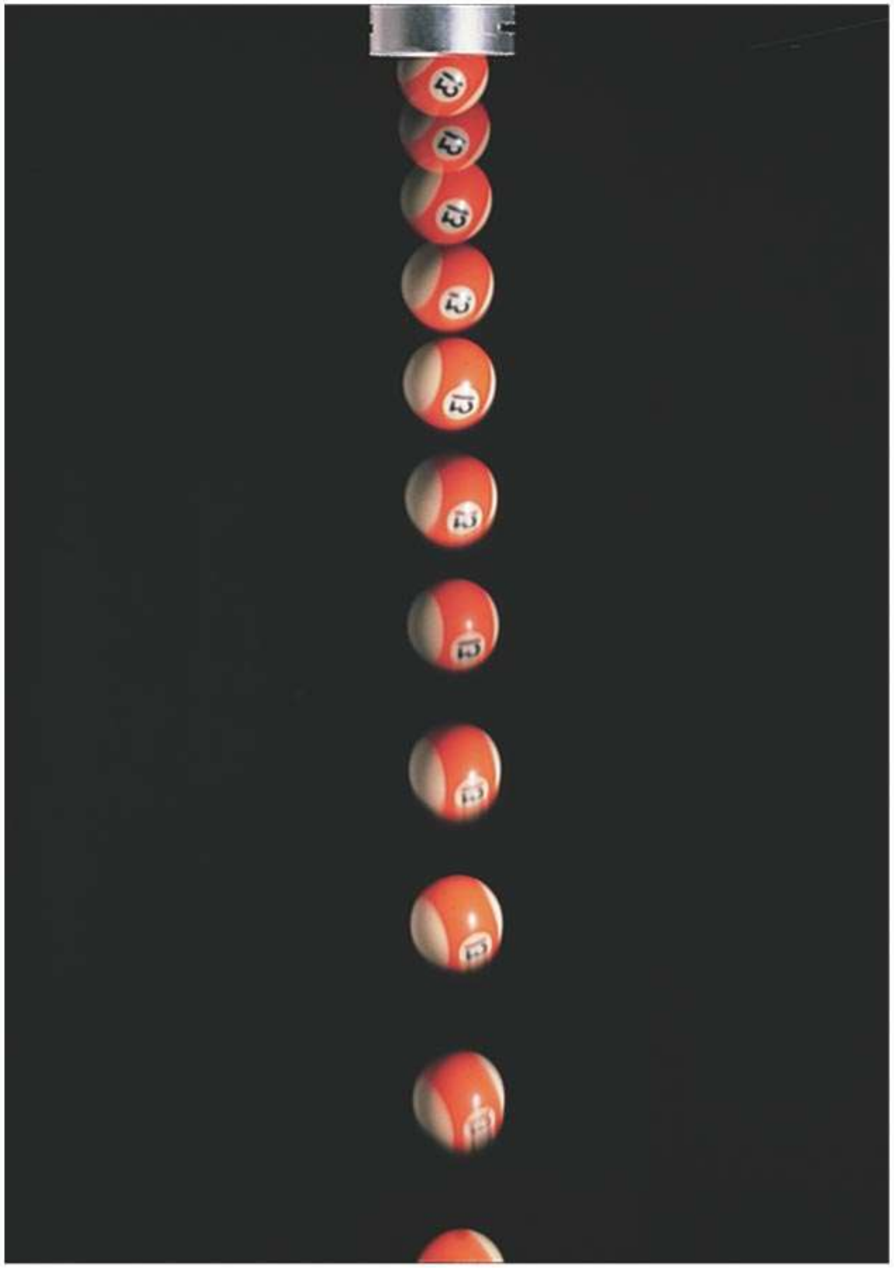
Free fall

Whenever an object is released in the air the acceleration is constant and pointing downwards.

The magnitude is $g=9.8\ m/s^2$. This is also known as the acceleration due to gravity, sometimes called simply gravity.

$$a_y=-9.8\ m/s^2=-g$$

NOTE: it does not matter which way the initial velocity is pointing (up or down), the acceleration is always downwards!!



Throw a Ball up

You throw a ball upward into the air with initial velocity V_0. Calculate:

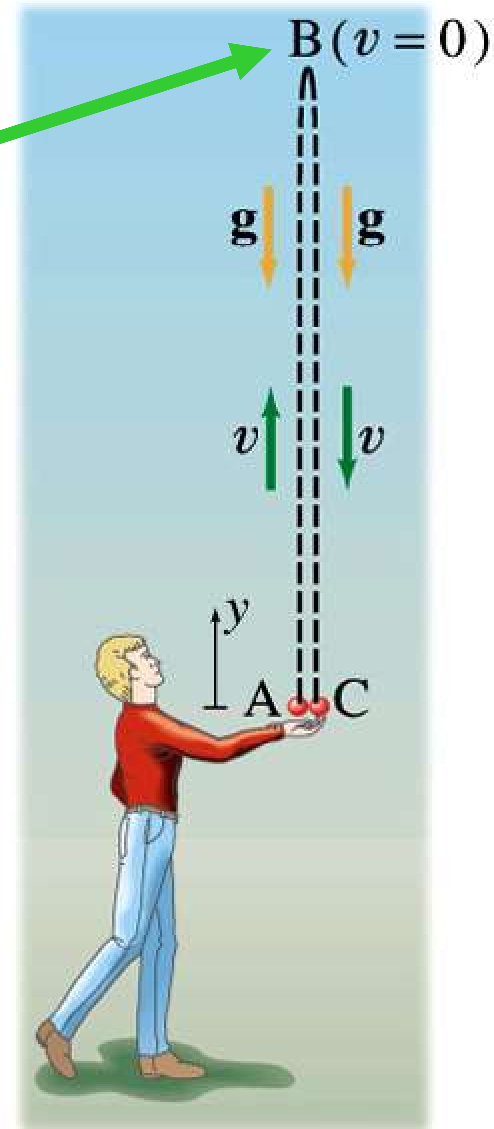
a) The time it takes to reach its highest point (the top).

b) Distance from your hand to the top

c) Time to go from your hand and come back to your hand

d) Velocity when it reaches your hand

e) Time from leaving your hand to reach some random height *h*.



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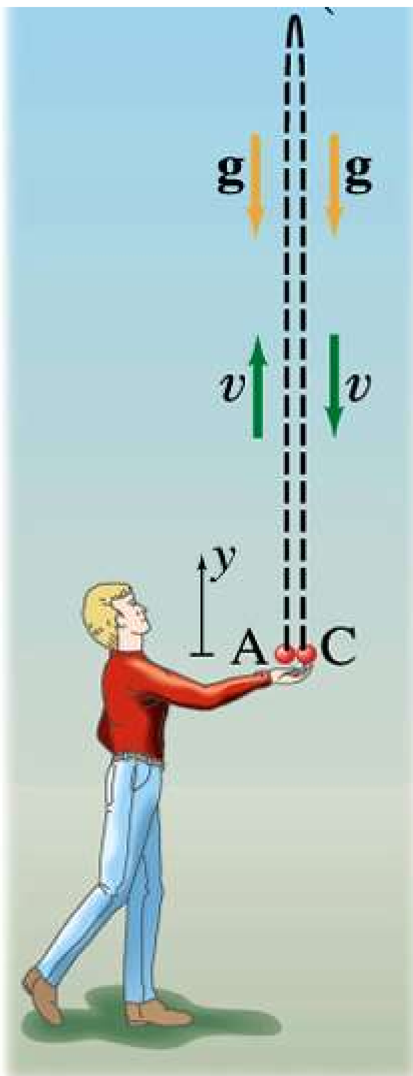


1st and 2nd step

$$y - y_0 =$$
$$v = 0$$
$$v_0 = v_0$$
$$a = -g$$
$$t = ?$$

Key is to realize that v=0 at the top

3rd step

Which equation contains the three unknowns and the one we want?

$$y - y_0 = +v_0 t + \tfrac{1}{2} a t^2$$
$$\boxed{v = v_0 + at}$$
$$v^2 = v_0^2 + 2a(y - y_0)$$

$$0 = v_0 + (-g)t \rightarrow t = \frac{v_0}{g}$$

Throw a Ball up

You throw a ball upward into the air with initial velocity V_0. Calculate:

b) Distance from your hand to the top

1st and 2nd step

$$y - y_0 = h = ?$$
$$v = 0$$

Key is to realize that v=0 at the top

$$v_0 = v_0$$
$$a = -g$$
$$t =$$

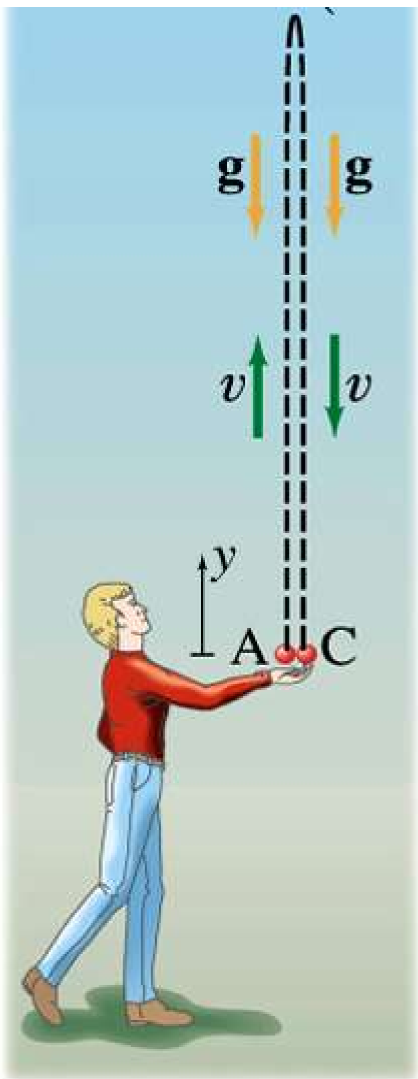
3rd step

Which equation contains the three unknowns and the one we want?

$$y - y_0 = +v_0 t + \tfrac{1}{2} a t^2$$
$$v = v_0 + at$$
$$\boxed{v^2 = v_0^2 + 2a(y - y_0)}$$

$$0^2 = v_0^2 + 2(-g)(h)$$

$$h = \frac{v_0^2}{2g}$$



Throw a Ball up

You throw a ball upward into the air with initial velocity V_0. Calculate:

c) Time it takes for the ball to come back to your hand

1st and 2nd step

$y - y_0 = 0 \longrightarrow$ Key part

$v =$

$v_0 = v_0$

$a = -g$

$t = ?$

3rd step

Which equation contains the three unknowns and the one we want?

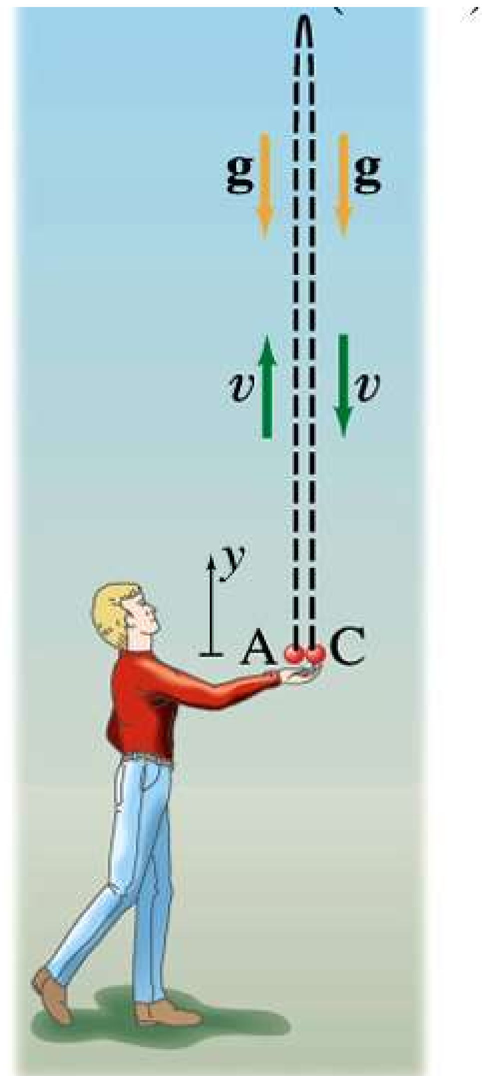
$$\boxed{y - y_0 = +v_0 t + \tfrac{1}{2} a t^2}$$

$$v = v_0 + at$$

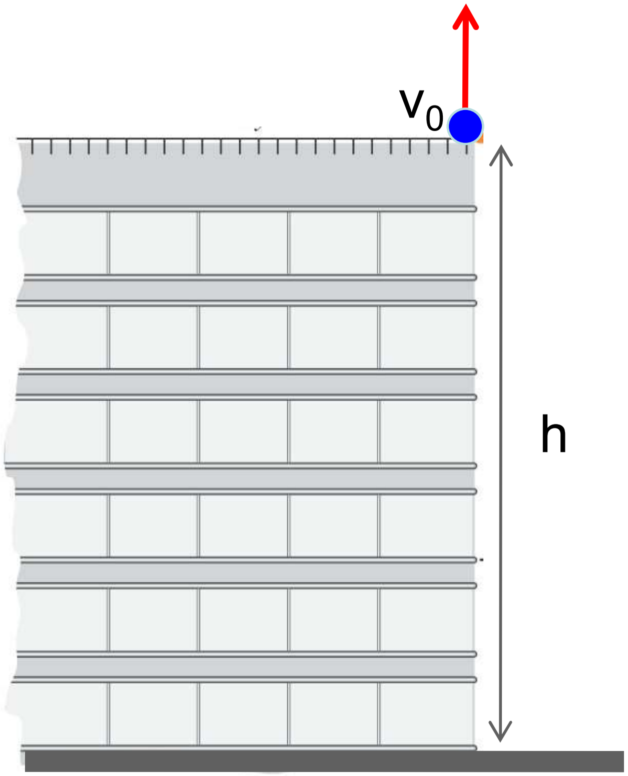
$$v^2 = v_0^2 + 2a(y - y_0)$$

$$0 = +v_0 t + \tfrac{1}{2}(-g)t^2 = t(v_0 - \tfrac{1}{2} g t)$$

$$\Rightarrow t = 0 \text{ and } t = \frac{2v_0}{g}$$



Free fall example



Q: A ball is thrown upwards with an initial velocity v_0.
What is the speed of the ball when it hits the ground?

1st and 2nd step

$$y - y_0 = -h$$
$$v = ?$$
$$v_0 = v_0$$
$$a = -g$$
$$t =$$

3rd step

Which equation contains the three unknowns and the one we want?

$$y - y_0 = +v_0 t + \tfrac{1}{2} a t^2$$
$$v = v_0 + at$$
$$\boxed{v^2 = v_0^2 + 2a(y - y_0)}$$

$$v = \sqrt{v_0^2 + 2(\text{-}g)(-h)} = \boxed{\sqrt{v_0^2 + 2hg}}$$

Notice that solution does NOT depend on the sign of v_0

Q: A ball is thrown upwards with an initial velocity v_0.
What is maximum height it gets from the top of the building?

This is a new problem. Start from the beginning b/c the end point is different!

1st and 2nd step

$$y - y_0 = ?$$
$$v = 0$$
$$v_0 = v_0$$
$$a = -g$$
$$t =$$

Key is to realize that v=0 at the top

3rd step

Which equation contains the three unknowns and the one we want?

$$y - y_0 = +v_0 t + \tfrac{1}{2} a t^2$$
$$v = v_0 + at$$
$$\boxed{v^2 = v_0^2 + 2a(y - y_0)}$$

$$(y - y_0) = \frac{0^2 - v_0^2}{2(-g)} = \frac{v_0^2}{2g}$$

Graphs

Describe motion in each point:

- Direction
- Velocity
- Acceleration