Goals for Chapter 13

• To outline periodic motion
• To quantify simple harmonic motion
• To explore the energy in simple harmonic motion
• To consider angular simple harmonic motion
• To study the simple pendulum
• To examine the physical pendulum
• To explore damped oscillations
• To consider driven oscillations and resonance
Introduction

- If you look to the right, you’ll see a time-lapse photograph of a simple pendulum. It’s far from simple, but it is a great example of the regular oscillatory motion we’re about to study.

**WHY study oscillatory motion?**

It is EVERYWHERE around us. Nature likes to be in equilibrium (lowest energy), any time you deviate from this equilibrium it wants to get back to it. This leads many times to oscillatory motion of the pendulum type which we call simple harmonic motion.
Describing oscillations

- The spring drives the glider back and forth on the air-track and you can observe the changes in the free-body diagram as the motion proceeds from –A to A and back.

The pendulum is another example

Elements of harmonic motion:

- Frequency: \( f = \frac{1}{T} \)
- Angular frequency: \( \omega = 2\pi f \)
- Amplitude: maximum deflection/stretch/compression
Simple harmonic motion

- An ideal spring responds to stretch and compression linearly, obeying Hooke’s Law.

\[ F_x = -kx \]

The restoring force towards the equilibrium point is linear from the displacement from equilibrium.

Let's look at Newton’s 2\textsuperscript{nd} law:

\[ F_x = ma_x \Rightarrow -kx = ma_x = m \frac{d^2x}{dt^2} \]

\[ \Rightarrow \frac{d^2x}{dt^2} = -\frac{k}{m} x \]

\[ \Rightarrow \frac{d^2x}{dt^2} = -\omega^2 x \]
Simple harmonic motion viewed as a projection

- If you illuminate uniform circular motion (say by shining a flashlight on a candle placed on a rotating lazy-Susan spice rack), the shadow projection that will be cast will be undergoing simple harmonic motion.

The solution to this equation is \( A \cos(\omega t + \phi) \)

\[
\frac{d^2 x}{dt^2} = -\omega^2 x \\
x(t) = A \cos(\omega t + \phi) \\
\frac{dx}{dt} = -A \omega \sin(\omega t + \phi) \\
\frac{d^2 x}{dt^2} = -A \omega^2 \cos(\omega t + \phi) = -\omega^2 x
\]
Characteristics of SHM

• Frequency, period, amplitude

\[
\frac{d^2 x}{dt^2} = -\frac{k}{m} x \quad \Rightarrow \quad \frac{d^2 x}{dt^2} = -\omega^2 x
\]

\[x(t) = A \cos(\omega t + \phi)\]

For a spring

\[
\omega = \sqrt{\frac{k}{m}} \quad \Rightarrow \quad f = \frac{1}{2\pi} \sqrt{\frac{k}{m}} \quad \text{or} \quad T = 2\pi \sqrt{\frac{m}{k}}
\]

A and \(\Phi\) are obtained by the \(t=0\) (or other time) conditions (initial position AND velocity)

Example

(a) You first use a force-meter to measure the force when you pull on the spring 3 cm.

(b) Then you put a 0.5 Kg weight on one end, pull it 0.020 m and let go.

Find the angular frequency, frequency, and period of oscillation of the above situation.
X versus $t$ for SHO then simple variations on a theme

Very important: frequency and period of oscillations DO NOT depend on the amplitude!!
SHM phase, position, velocity, and acceleration

- SHM can occur with various phase angles.
- For a given phase we can examine position, velocity, and acceleration.

These three curves show SHM with the same period $T$ and amplitude $A$ but with different phase angles $\phi$.

$$x(t) = A \cos(\omega t + \phi)$$
$$v(t) = -\omega A \sin(\omega t + \phi)$$
$$a(t) = -A \omega^2 \cos(\omega t + \phi)$$

(a) Displacement $x$ as a function of time $t$

(b) Velocity $v_x$ as a function of time $t$

(c) Acceleration $a_x$ as a function of time $t$
Watch variables change for a glider example

- As the glider undergoes SHM, you can track changes in velocity and acceleration as the position changes between the turning points.

\[ x(t) = A \cos(\omega t + \phi) \]
\[ v(t) = -\omega A \sin(\omega t + \phi) \]
\[ a(t) = -A \omega^2 \cos(\omega t + \phi) \]
Energy in SHM

- Energy is conserved during SHM and the forms (potential and kinetic) interconvert as the position of the object in motion changes.

\[ E = \frac{1}{2} m v_x^2 + \frac{1}{2} k x^2 = \frac{1}{2} k A^2 = \frac{1}{2} m v_{\text{max}}^2 \]
Energy in SHM II

(a) The potential energy $U$ and total mechanical energy $E$ for a body in SHM as a function of displacement $x$

The total mechanical energy $E$ is constant.

(b) The same graph as in (a), showing kinetic energy $K$ as well

At $x = \pm A$ the energy is all potential; the kinetic energy is zero.

At $x = 0$ the energy is all kinetic; the potential energy is zero.

At these points the energy is half kinetic and half potential.
Velocity, acceleration, and energy in SHM

In the oscillator described earlier $k=200 \text{ N/m}$, $m=0.50 \text{ Kg}$, and the oscillating mass is released from rest at $x=0.020 \text{ m}$. (a) Find the maximum and minimum speeds. (b) Compute the maximum acceleration. (c) Determine the velocity and acceleration when the body has moved halfway to the center from its original position. (d) Find the total energy, potential energy, and kinetic energy at this position.
Conservation of energy and momentum

At the instant the block $M$ passes through equilibrium a lump of putty collides with the block and sticks to it. (a) Find the new amplitude and period. (b) Do the same if the putty falls on the block when it is at one end of its path.
Vertical SHO

(a) A hanging spring that obeys Hooke’s law

(b) A body is suspended from the spring. It is in equilibrium when the upward force exerted by the stretched spring equals the body’s weight.

(c) If the body is displaced from equilibrium, the net force on the body is proportional to its displacement. The oscillations are SHM.
The simple pendulum

(b) An idealized simple pendulum

String is assumed to be massless and unstretchable.
Bob is modeled as a point mass.

The restoring force on the bob is proportional to \( \sin \theta \), not to \( \theta \). However, for small \( \theta \), \( \sin \theta \approx \theta \), so the motion is approximately simple harmonic.

\[ F_{\text{tan}} = -mg \sin \theta \approx -mg \theta \]

\[ F_{\text{tan}} = ma_{\text{tan}} = mL \frac{d^2 \theta}{dt^2} \]

\[ \Rightarrow mL \frac{d^2 \theta}{dt^2} = -mg \theta \]

\[ \frac{d^2 \theta}{dt^2} = -\frac{g}{L} \theta \]

\[ \Rightarrow \omega = \sqrt{\frac{g}{L}} \quad f = \frac{1}{2\pi} \sqrt{\frac{g}{L}} \quad T = 2\pi \sqrt{\frac{L}{g}} \]
The physical pendulum

- A physical pendulum is any real pendulum that uses an extended body in motion. This illustrates a physical pendulum.

\[ T = 2\pi \sqrt{\frac{I}{mgd}} \]

The body is free to rotate around the z-axis.

The gravitational force acts on the body at its center of gravity (cg).

The restoring torque on the body is proportional to \( \sin \theta \), not \( \theta \). However, for small \( \theta \), \( \sin \theta \approx \theta \), so the motion is approximately simple harmonic.
Dinosaurs, long tails, and the physical pendulum

All walking animals have a natural walking pace. This is the number of steps per minute that is more comfortable than a faster or slower pace. One can take this pace from considering the leg as a physical pendulum. (I for a rod pivoted at one end is $ML^{2/3}$)

(a) Estimate the natural pace of a human.
(b) Tyrannosaurus rex had a leg of length $L=3.1$ m and a stride of $S=4.0$ m. Estimate the walking speed of Tyrannosaurus rex.
Forced (driven) oscillations and resonance

- A force applied “in synch” with a motion already in progress will resonate and add energy to the oscillation.
- A singer can shatter a glass with a pure tone in tune with the natural “ring” of a thin wine glass.
Forced (driven) oscillations and resonance II

- The Tacoma Narrows Bridge suffered spectacular structural failure after absorbing too much resonant
**SUMMARY**

Periodic motion: motion that repeats itself in a defined cycle. 

\[
f = \frac{1}{T} \quad T = \frac{1}{f} \quad \omega = 2\pi f = \frac{2\pi}{T}
\]

Simple harmonic motion: if the restoring force is proportional to the distance from equilibrium, the motion will be of the SHM type. The angular frequency and period do not depend on the amplitude of oscillation.

\[
F_x = -kx \quad a_x = \frac{F_x}{m} = -\frac{k}{m}x
\]

\[
\omega = \sqrt{\frac{k}{m}} \quad f = \frac{\omega}{2\pi} = \frac{1}{2\pi} \sqrt{\frac{k}{m}} \quad T = 2\pi \sqrt{\frac{m}{k}}
\]

\[
x = A \cos(\omega t + \phi)
\]

Energy in SHM:

\[
E = \frac{1}{2} mv_x^2 + \frac{1}{2} kx^2 = \frac{1}{2} kA^2 = \frac{1}{2} mv_{\text{max}}^2
\]

Simple pendulum:

\[
\omega = \sqrt{\frac{g}{L}} \quad f = \frac{1}{2\pi} \sqrt{\frac{g}{L}} \quad T = 2\pi \sqrt{\frac{L}{g}}
\]