Chapter 13

Gravitation

PowerPoint® Lectures for
University Physics, 14th Edition
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Lectures by Jason Harlow

Next one week

- Today: Ch 13
- Wed: Review of Ch 8-11, focusing on additional problems, clarifications on frequent mistakes (sin/cos, static friction), etc.
- Mon (4/17): No lectures, since it has not been very helpful to have lectures just before the exam. You focus on the exam and come fully prepared.
- No Fri 1-3 office hour, since it is a half day. Come find me on Thu anytime between 8 am - 3 pm.
- Make sure to go through lecture slides for quick recap, clicker quizzes and supplemental problems (work them out, not just look at answers to see if it makes sense)

Common Errors

- Sin vs Cos, Vector product
- Static friction
- Using energy consv for (not elastic) collision problems
- Which component of mom. is conserved? Look at net force in that direction. Must have net force = 0 to conserve momentum.
- Being able to set up Before and After for collision problems
- Set up energy conservation for follow up problem after collision
- Right-hand rule and direction of w, τ, L
- Setting linear speed = tangential speed for rolling w/o slipping
- Two forms of angular momentum = Iw, r x p

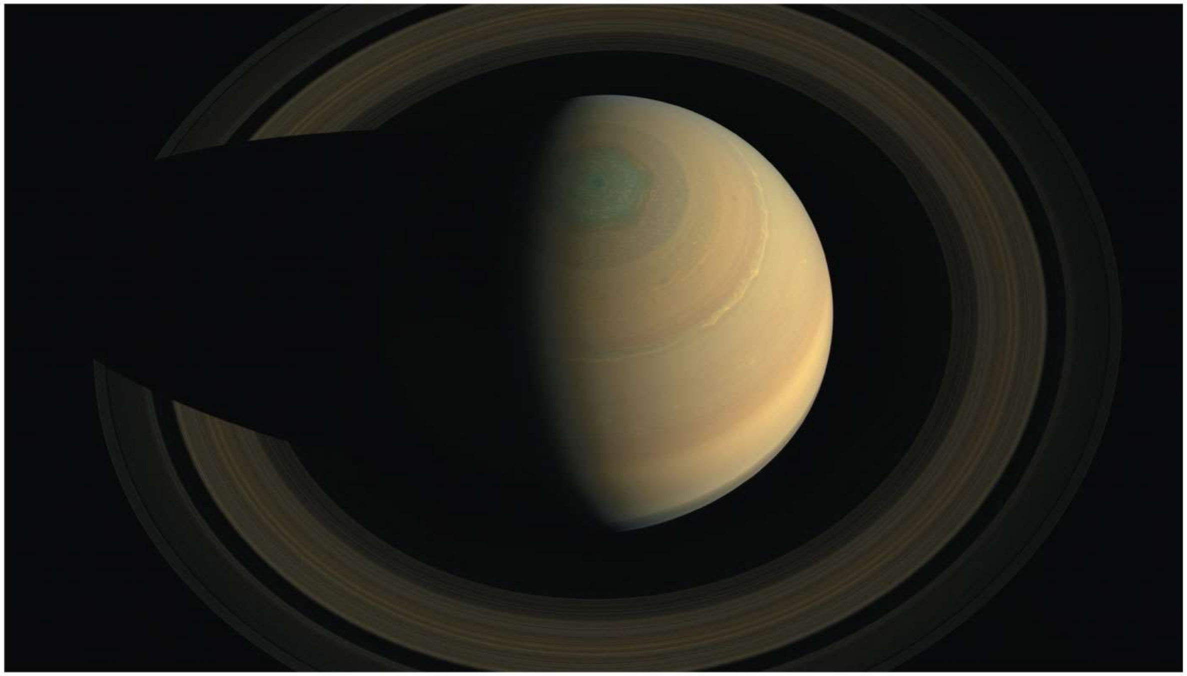
Learning Goals for Chapter 13

Looking forward at …

- how to calculate the gravitational forces that any two bodies exert on each other.
- how to relate the weight of an object to the general expression for gravitational force.
- how to calculate the speed, orbital period, and mechanical energy of a satellite in a circular orbit.
- how to apply and interpret Kepler's three laws that describe the motion of planets.

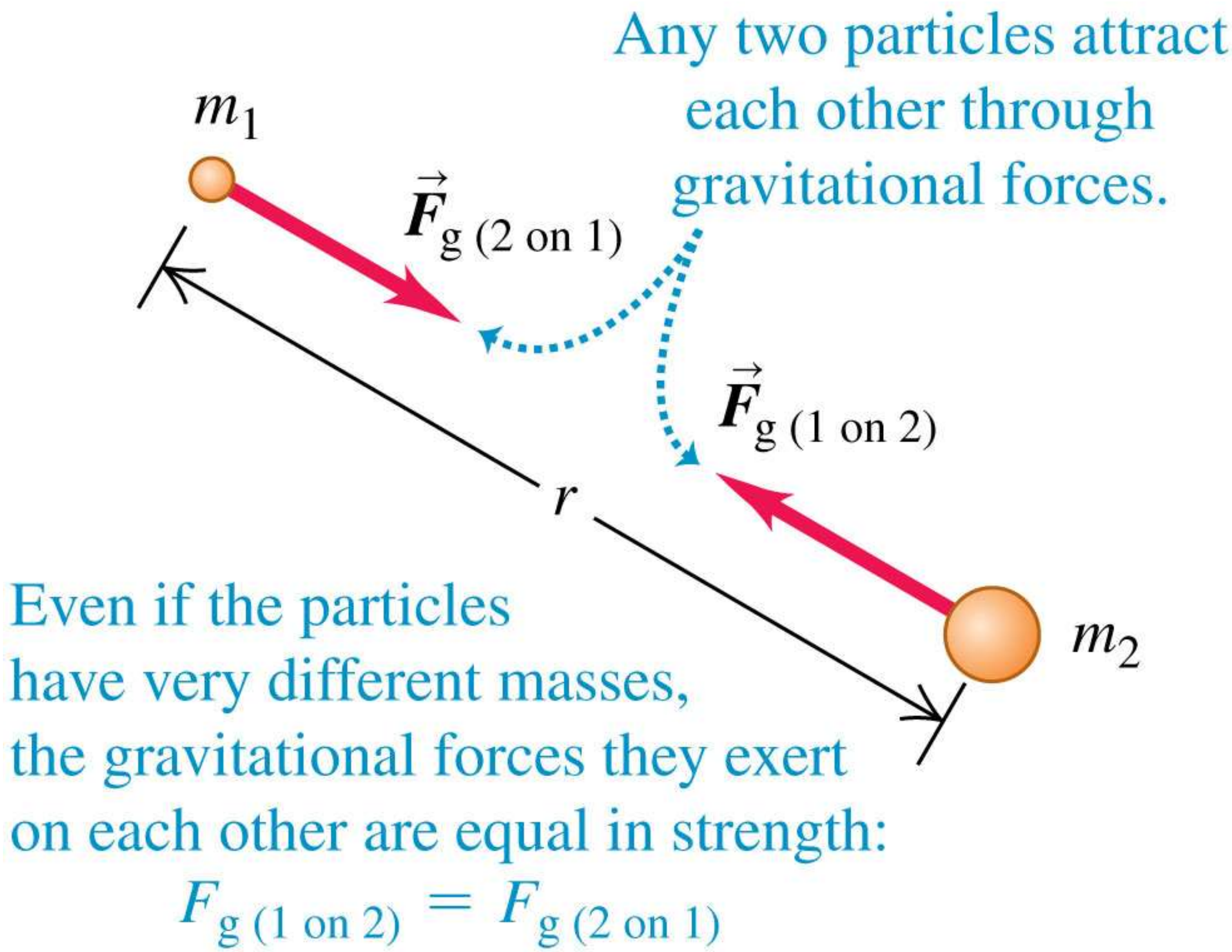
Introduction

- What can we say about the motion of the particles that make up Saturn's rings?
- Why doesn't the moon fall to earth, or the earth into the sun?



- By studying gravitation and celestial mechanics, we will be able to answer these and other questions.

Newton's law of gravitation



Newton's law of gravitation

- *Law of gravitation*: Every particle of matter attracts every other particle with a force that is directly proportional to the product of their masses and inversely proportional to the square of the distance between them:

Newton's **law of gravitation:**
Magnitude of attractive gravitational force between any two particles

$$F_g = \frac{Gm_1m_2}{r^2}$$

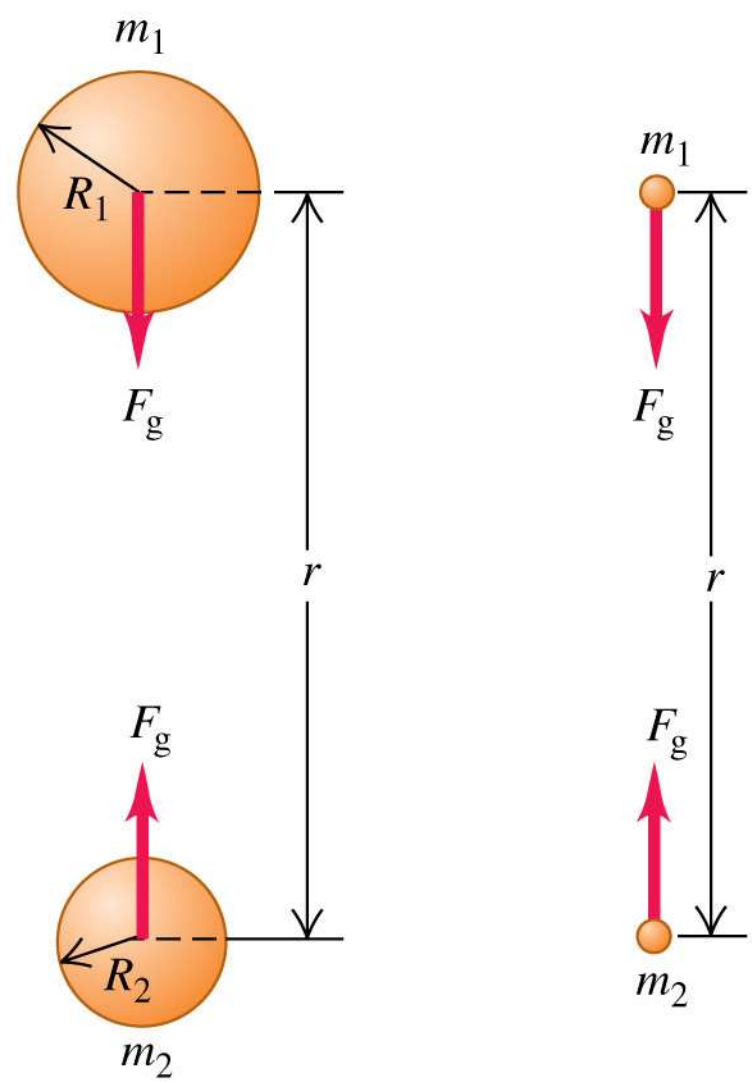
Gravitational constant (same for any two particles)
Masses of particles
Distance between particles

- The gravitational constant G is a fundamental physical constant that has the same value for any two particles.

- $G = 6.67 \times 10^{-11}\ \text{N} \cdot \text{m}^2/\text{kg}^2$.

Gravitation and spherically symmetric bodies

- The gravitational effect *outside* any spherically symmetric mass distribution is the same as though all of the mass were concentrated at its center.
- The force F_g attracting m_1 and m_2 on the left is equal to the force F_g attracting the two point particles m_1 and m_2 on the right, which have the same masses and whose centers are separated by the same distance.



Gravitational attraction

- Our solar system is part of a spiral galaxy like this one, which contains roughly 10^{11} stars as well as gas, dust, and other matter (mostly dark matter).

- The entire assemblage is held together by the mutual gravitational attraction of all the matter in the galaxy.

Weight

- The **weight** of a body is the total gravitational force exerted on it by all other bodies in the universe.

- At the surface of the earth, we can neglect all other gravitational forces, so a body's weight is:

$$w = F_g = \frac{Gm_E m}{R_E^2}$$

Weight of a body at the earth's surface ... (w)
... equals gravitational force the earth exerts on body. (F_g)
Gravitational constant (G)
Mass of the earth (m_E)
Mass of body (m)
Radius of the earth (R_E)

- The acceleration due to gravity at the earth's surface is:

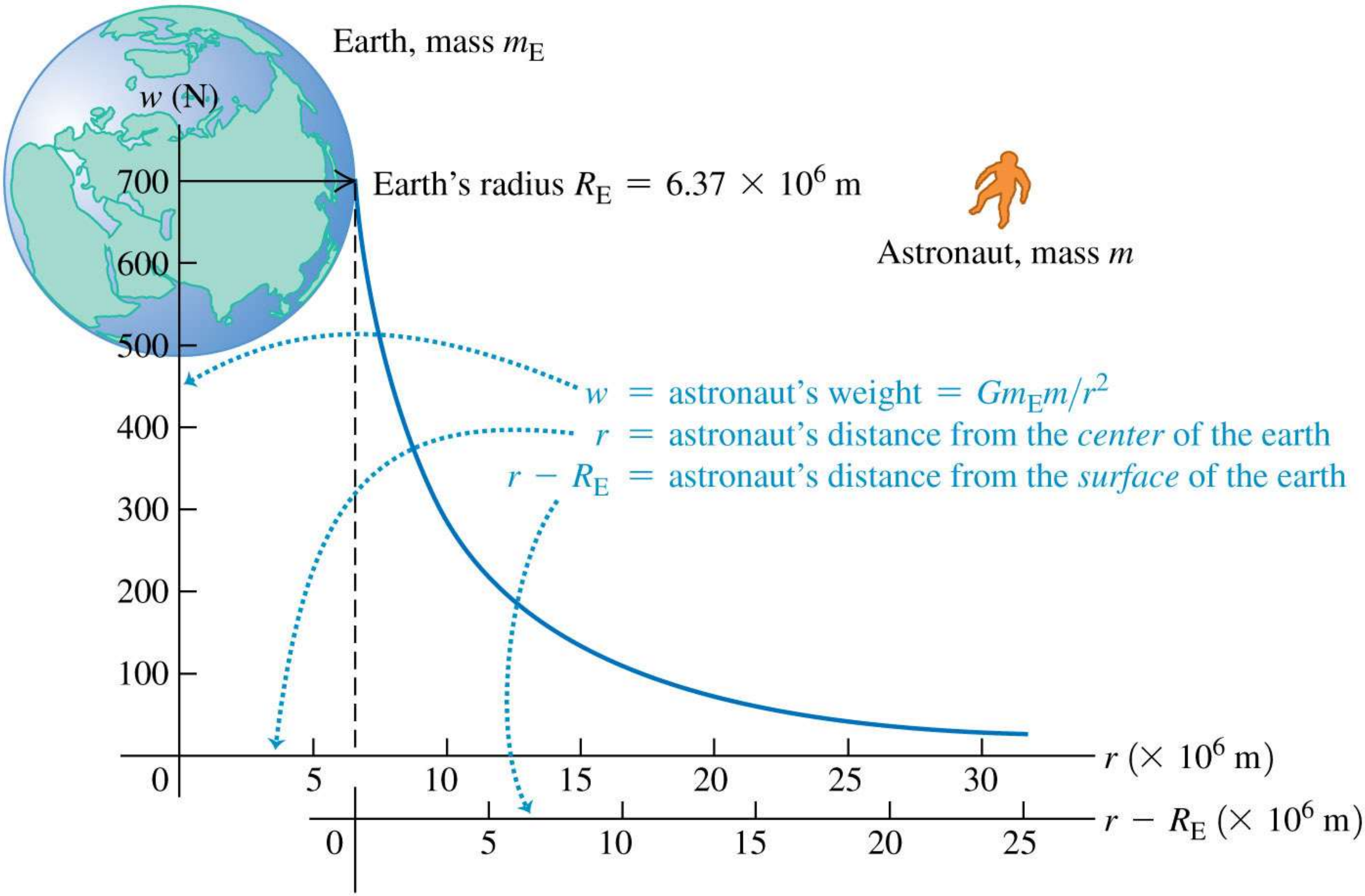
$$g = \frac{Gm_E}{R_E^2}$$

Acceleration due to gravity at the earth's surface (g)
Gravitational constant (G)
Mass of the earth (m_E)
Radius of the earth (R_E)

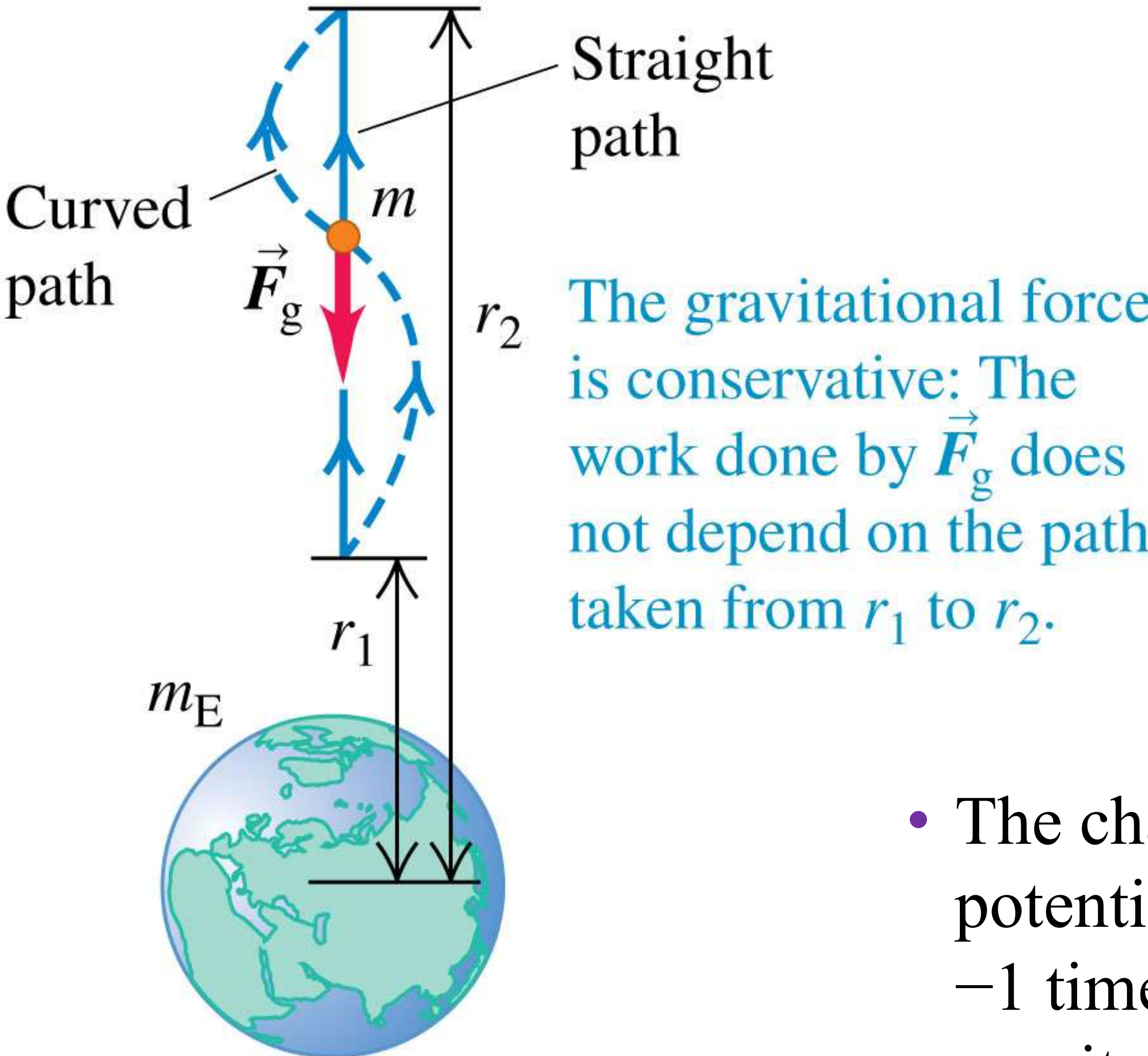
Walking and running on the moon

- You automatically transition from a walk to a run when the vertical force the ground exerts on you exceeds your weight.
- This transition from walking to running happens at much lower speeds on the moon, where objects weigh only 17% as much as on earth.
- Hence, the Apollo astronauts found themselves running even when moving relatively slowly during their moon "walks."

Weight decreases with altitude



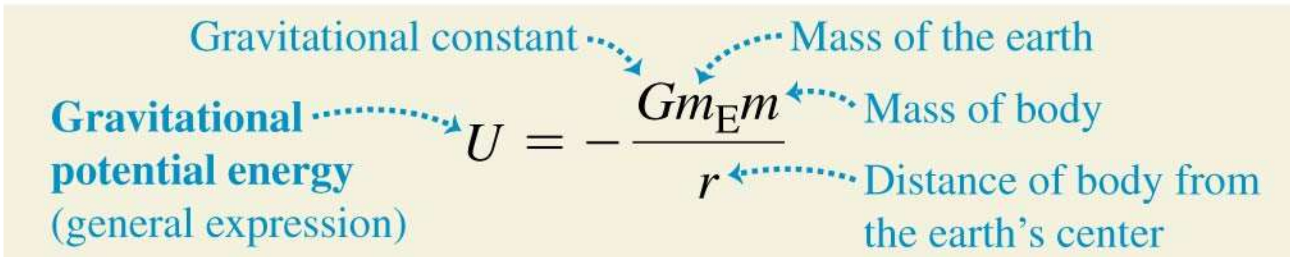
Gravitational potential energy



- The change in gravitational potential energy is defined as −1 times the work done by the gravitational force as the body moves from r_1 to r_2.

Gravitational potential energy

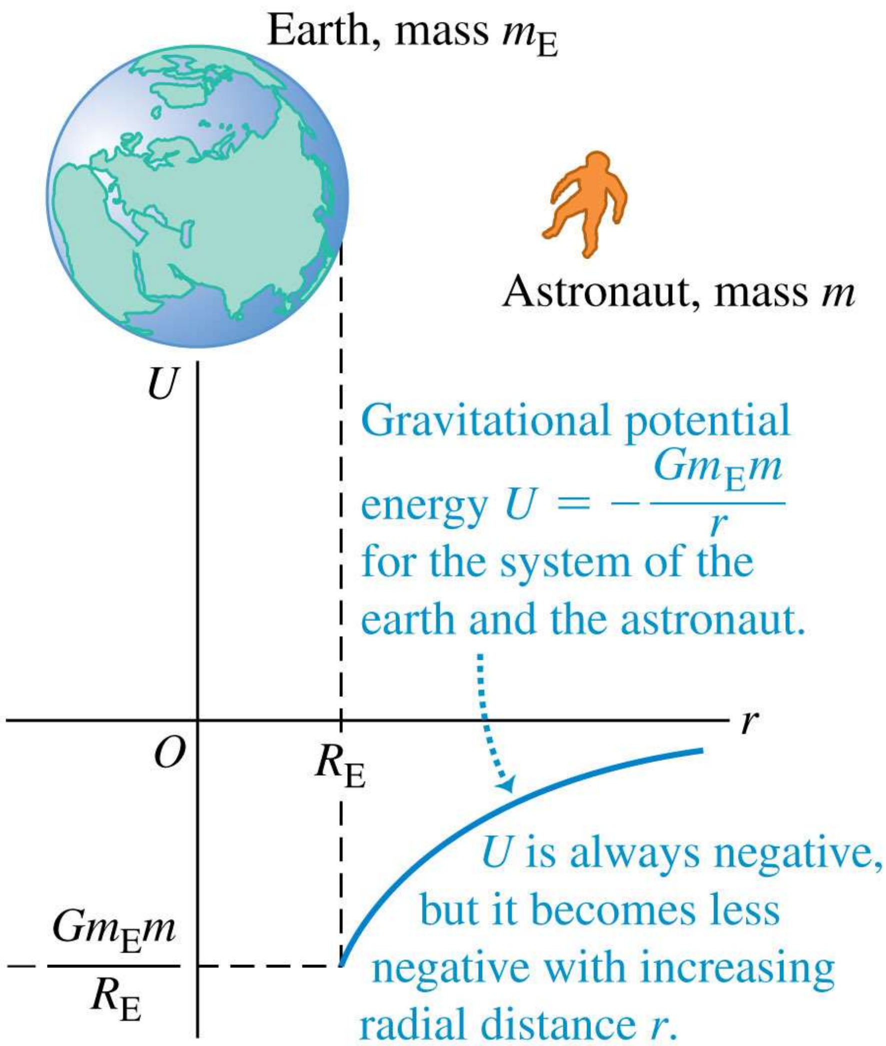
- We define the gravitational potential energy U so that $W_{grav} = U_1 - U_2$:



- If the earth's gravitational force on a body is the only force that does work, then the total mechanical energy of the system of the earth and body is constant, or **conserved**.

Gravitational potential energy depends on distance

- The gravitational potential energy of the earth–astronaut system *increases* (becomes less negative) as the astronaut moves away from the earth.



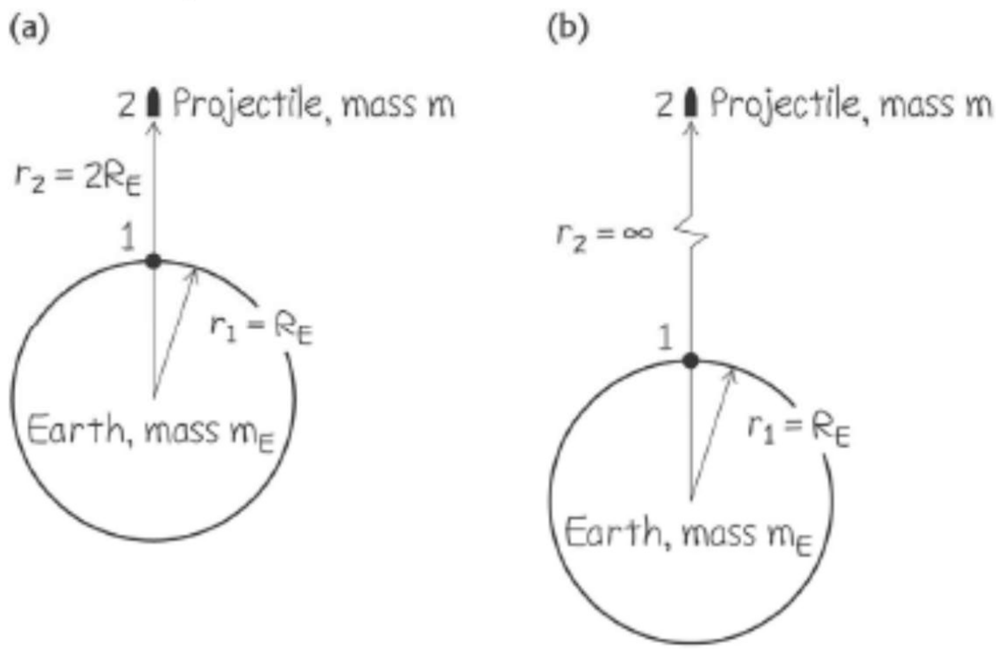
Escaping from the Earth

What is the velocity you need to shoot straight up from the surface of the earth an not come back (conservation of energy)?

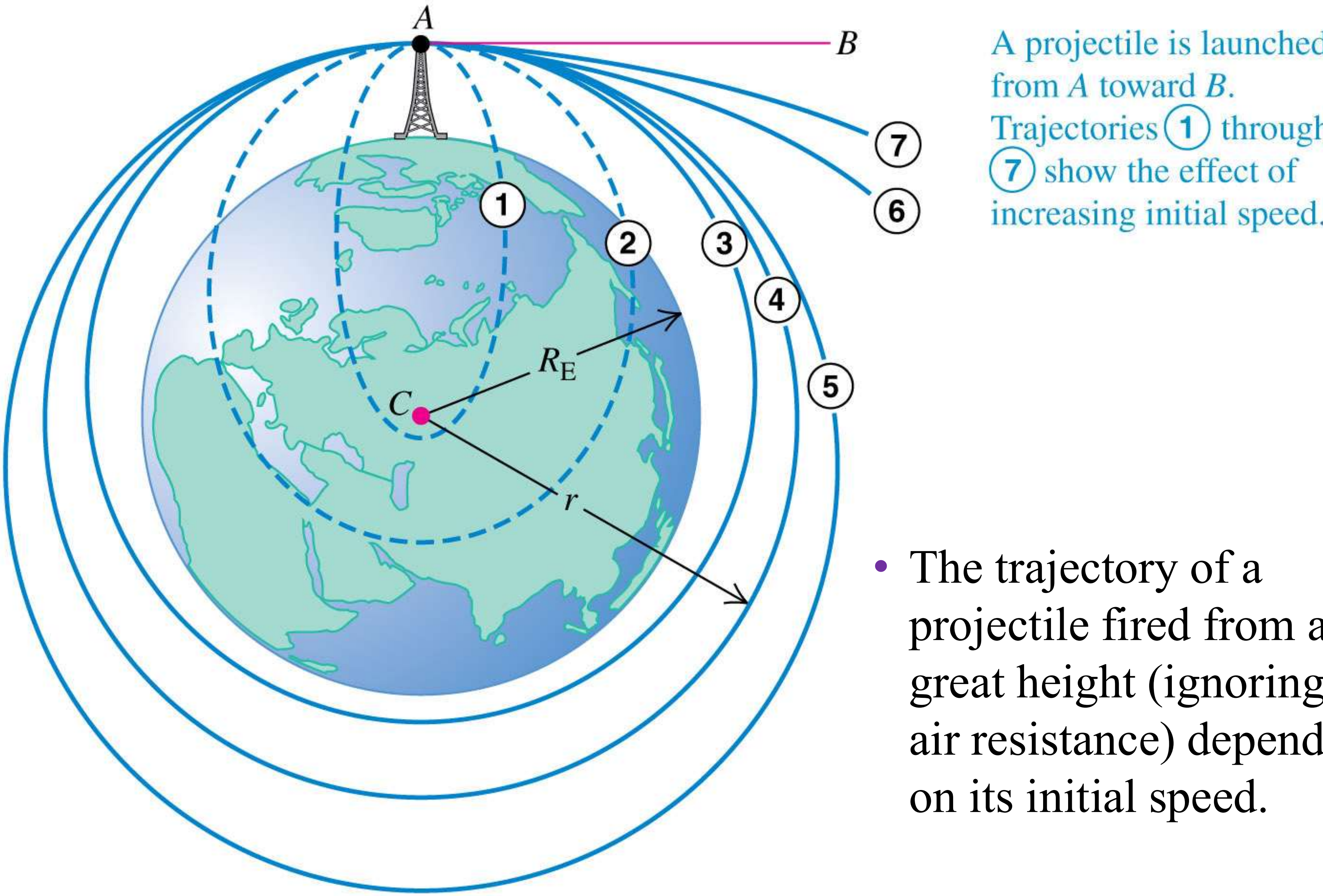
$$KE_1 + PE_1 = KE_\infty + PE_\infty$$

$$\frac{1}{2} m \mathrm{v}^2 - G\frac{mM_E}{R_E} = 0 + 0$$

$$\Rightarrow \mathrm{v} = \sqrt{\frac{2GM_E}{R_E}} = 1.12 \times 10^4 \,\mathrm{m/s} = 25{,}000 \text{ mph}$$



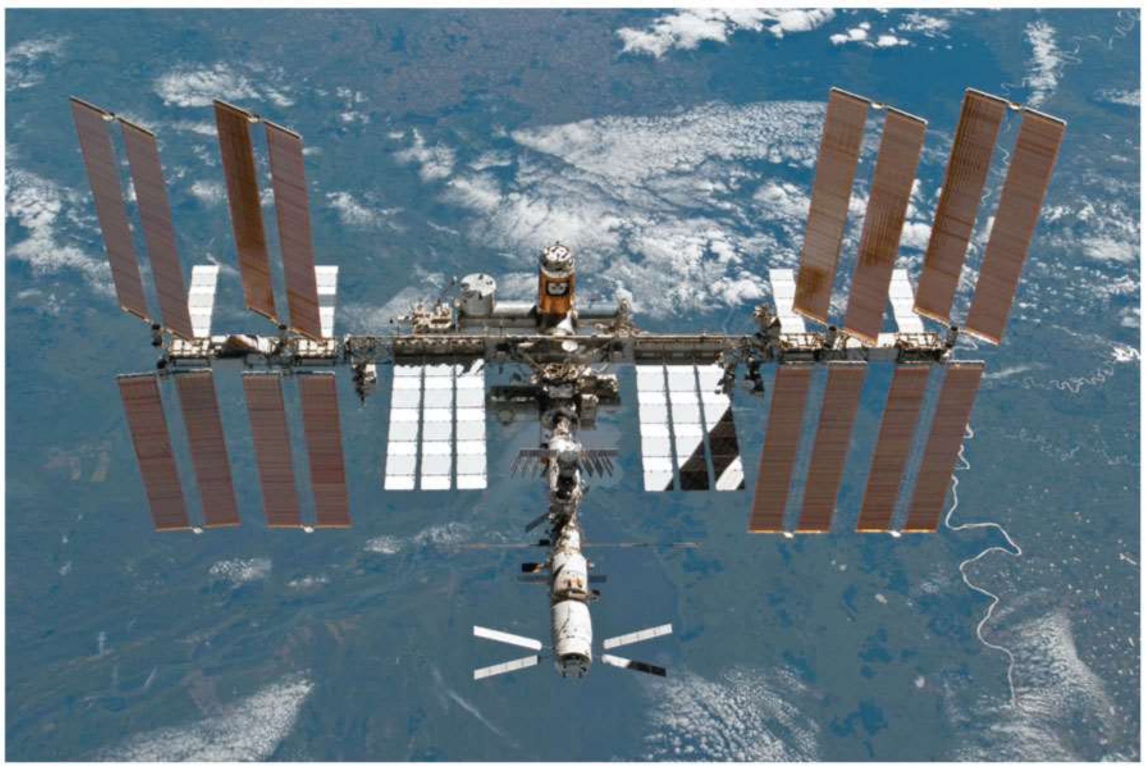
The motion of satellites



- The trajectory of a projectile fired from a great height (ignoring air resistance) depends on its initial speed.

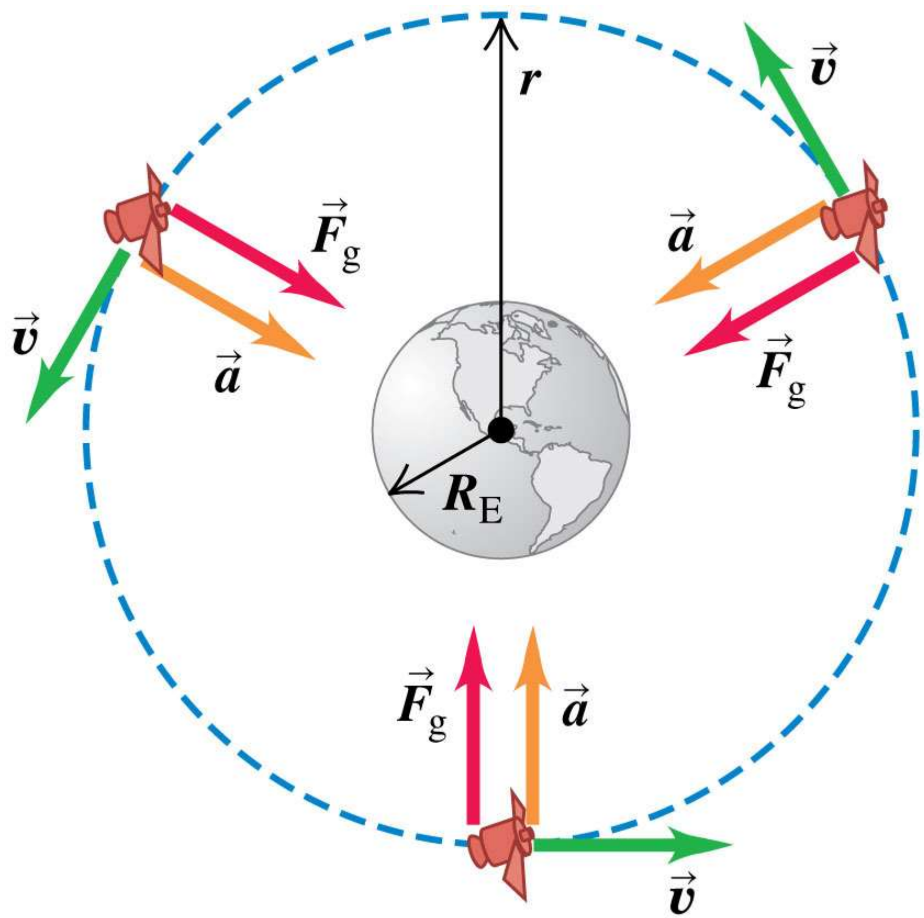
Circular satellite orbits

- With a mass of approximately 4.5×10^5 kg and a width of over 108 m, the International Space Station is the largest satellite ever placed in orbit. It spins around earth once every 90 mins. It rotates at an orbit 250 miles from the earth. Earth rad ~4000 miles).



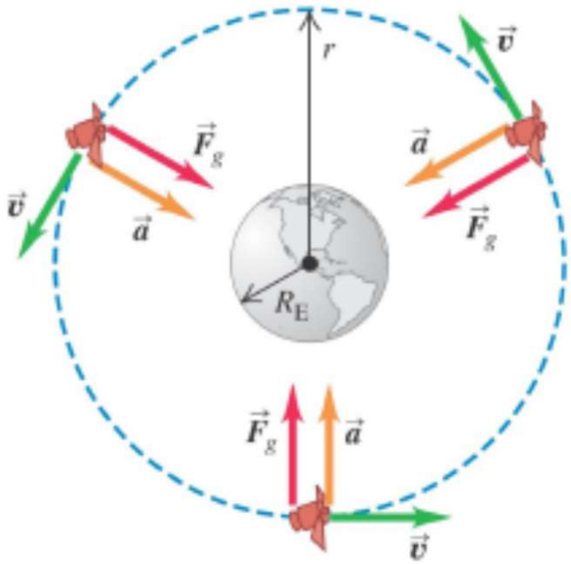
Circular satellite orbits

- For a circular orbit, the speed of a satellite is just right to keep its distance from the center of the earth constant.
- The force $\vec{F}_g$ due to the earth's gravitational attraction provides the centripetal acceleration that keeps a satellite in orbit.



The satellite is in a circular orbit: Its acceleration $\vec{a}$ is always perpendicular to its velocity $\vec{v}$, so its speed v is constant.

Satellite circular motion



The satellite is in a circular orbit: Its acceleration $\vec{a}$ is always perpendicular to its velocity $\vec{v}$, so its speed v is constant.

The force is radial so Newton's 2nd law reads:

$$r:\quad G\frac{M_E m_s}{R^2} = m_s\frac{v^2}{R} = m_s\frac{4\pi^2 R}{T^2}$$

$$\Rightarrow v = \sqrt{G\frac{M_E}{R}}$$

$$\Rightarrow T = \sqrt{\frac{4\pi^2 R^3}{GM_E}}$$

$$T \propto R^{3/2}$$

Circular satellite orbits

- A satellite is constantly falling *around* the earth.
- Astronauts inside the satellite in orbit are in a state of *apparent weightlessness* because they are falling with the satellite.

How big is a black hole?

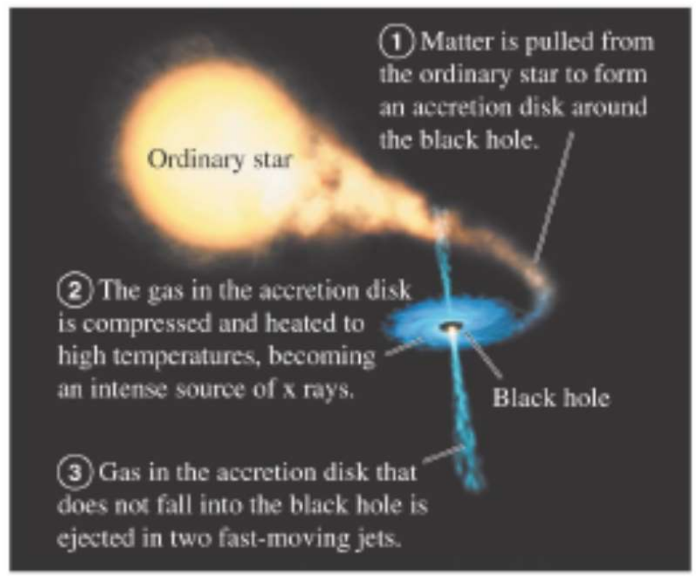
Remember the escape velocity? $v = \sqrt{\frac{2GM_E}{R_E}}$

But the maximum speed in the universe is the speed of light, c. What does this mean?

$$c = \sqrt{\frac{2GM}{R}} \Rightarrow R = \frac{2GM}{c^2}$$

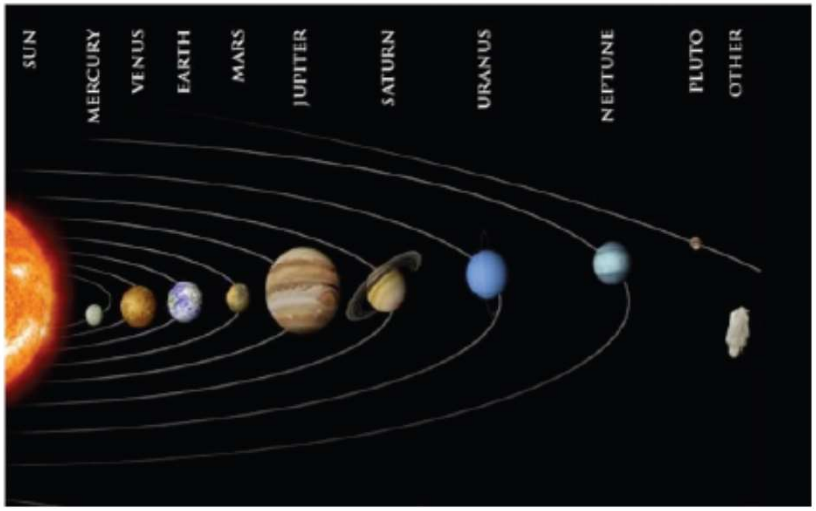
The Earth has a mass of 5.97 x10^{24} Kg

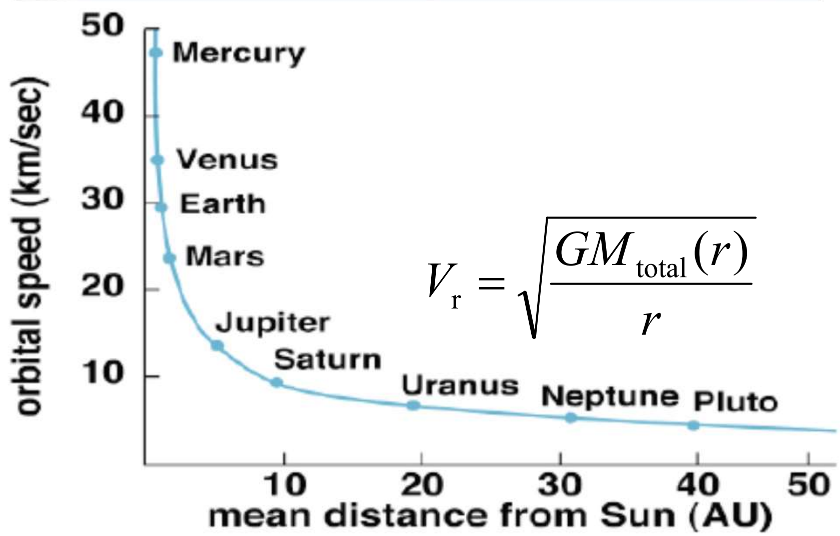
$$R = \frac{2(6.6742 \times 10^{-11})(5.97 \times 10^{24})}{(3 \times 10^{8})^2} \text{ m} = 8.9 \text{ mm}$$



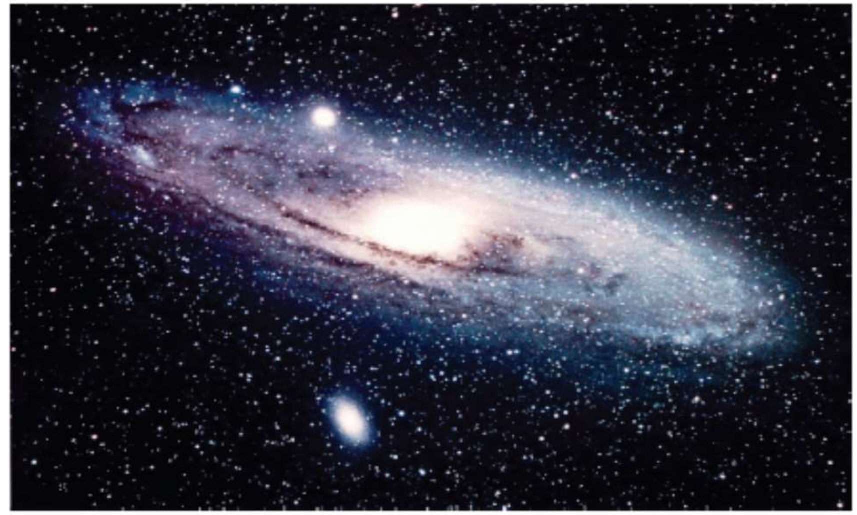
Dark Matter: Something Invisible?

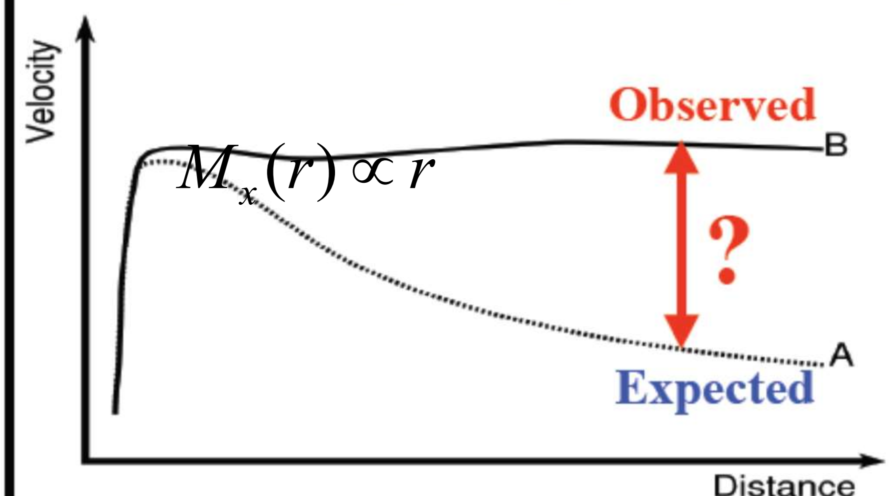
Solar System





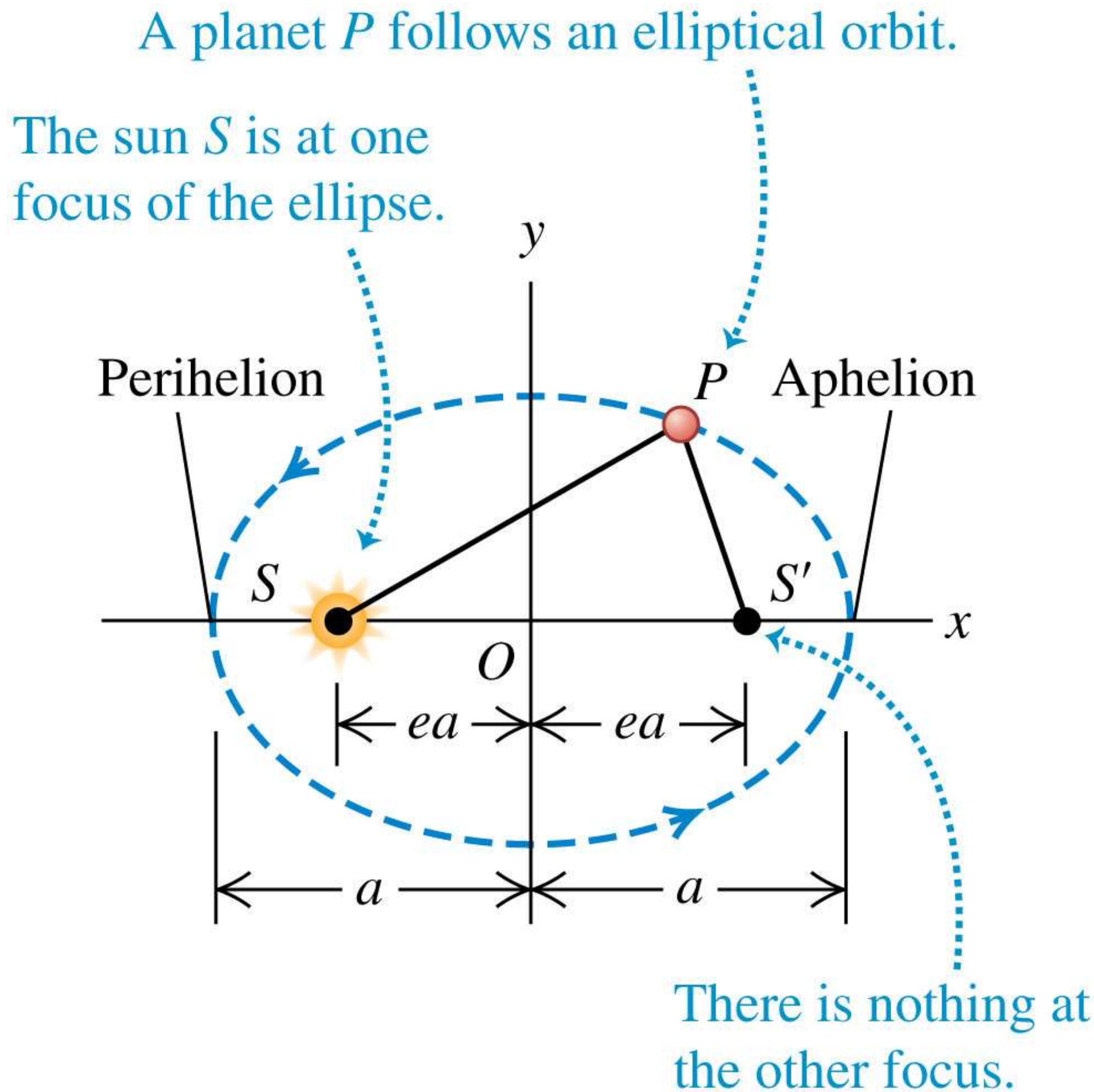
Spiral Galaxy





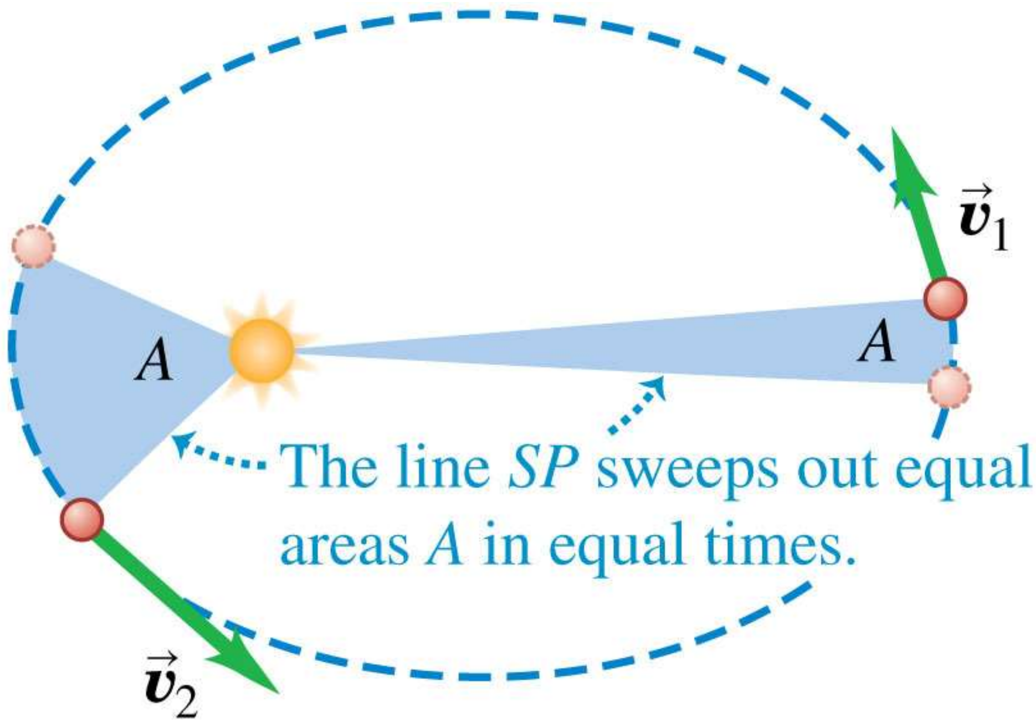
Kepler's first law

- Each planet moves in an elliptical orbit with the sun at one focus of the ellipse.



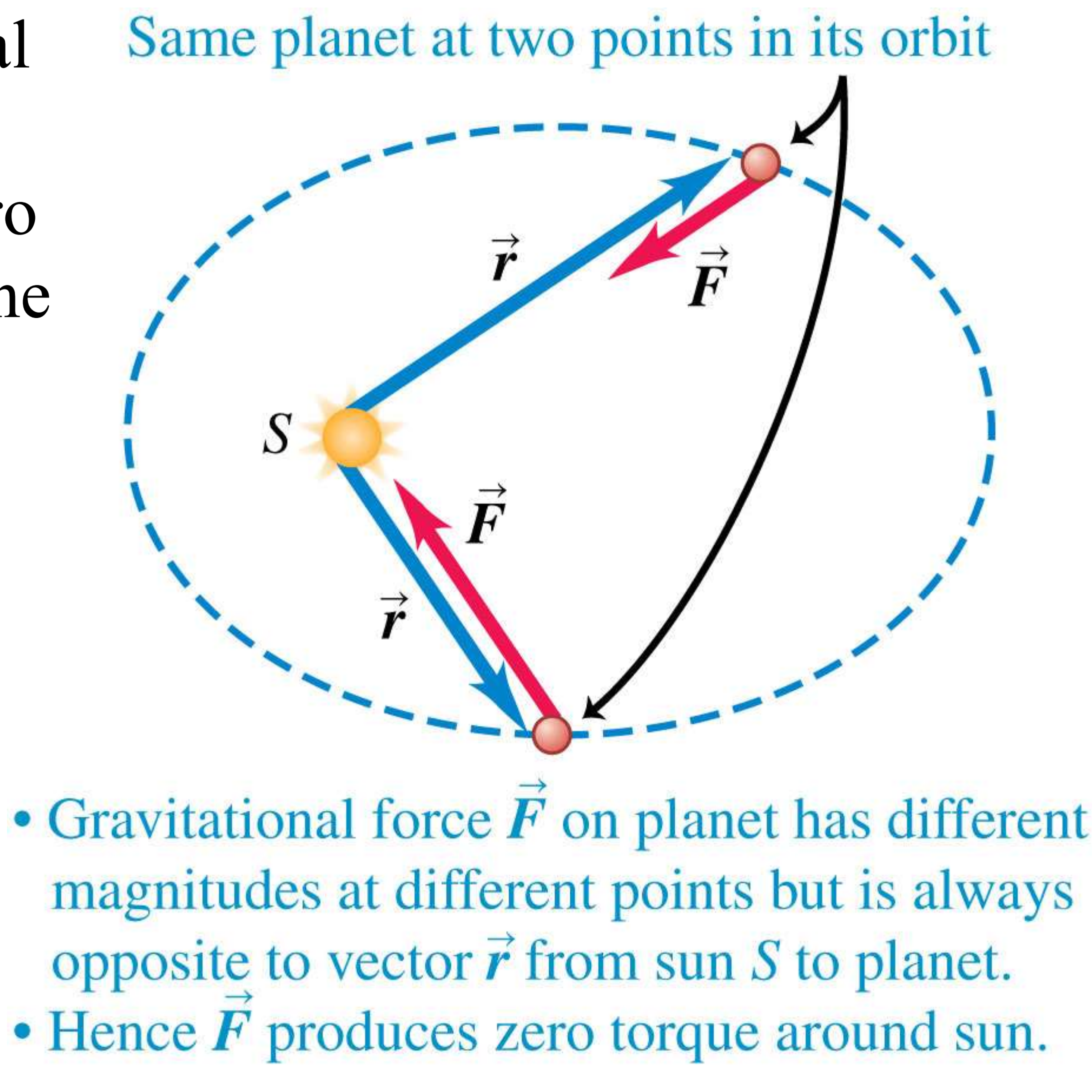
Kepler's second law

- A line from the sun to a given planet sweeps out equal areas in equal times (due to conservation of angular momentum)



Kepler's second law

- Because the gravitational force that the sun exerts on a planet produces zero torque around the sun, the planet's angular momentum around the sun remains constant.



Kepler's third law

- The periods of the planets are proportional to the three-halves powers of the major axis lengths of their orbits.

$$T = \frac{2\pi a^{3/2}}{\sqrt{Gm_S}} \qquad \text{(elliptical orbit around the sun)}$$

- Note that the period does not depend on the eccentricity e.
- An asteroid in an elongated elliptical orbit with semi-major axis a will have the same orbital period as a planet in a circular orbit of radius a.

Comet Halley

- At the heart of Comet Halley is an icy body, called the **nucleus**, that is about 10 km across.
- When the comet's orbit carries it close to the sun, the heat of sunlight causes the nucleus to partially evaporate.
- The evaporated material forms the tail, which can be tens of millions of kilometers long.