



# Chapter 11

# Equilibrium and Elasticity

PowerPoint® Lectures for  
*University Physics, 14th Edition*  
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# Learning Goals for Chapter 11

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## *Looking forward at ...*

- the conditions that must be satisfied for a body or structure to be in equilibrium.
- what the center of gravity of a body is and how it relates to the body's stability.
- how to solve problems that involve rigid bodies in equilibrium.

# Introduction: Static Equilibrium

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- This Roman aqueduct uses the principle of the arch to sustain the weight of the structure and the water it carries.
- In construction we're interested in making sure that bodies *don't* accelerate.

# Conditions for equilibrium

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- For an extended body to be in static equilibrium, two conditions must be satisfied.
- The first condition is that the vector sum of all external forces acting on the body must be zero:

**First condition for equilibrium:**

For the center of mass of a body at rest to remain at rest ...

$$\sum \vec{F} = \mathbf{0}$$

*... net external force on the body must be zero.*

- The second condition is that the sum of external torques must be zero about any point:

**Second condition for equilibrium:**

For a nonrotating body to remain nonrotating ...

$$\sum \vec{\tau} = \mathbf{0}$$

*... net external torque around any point on the body must be zero.*

# Conditions for equilibrium: Example 1

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(a) This body is in static equilibrium.

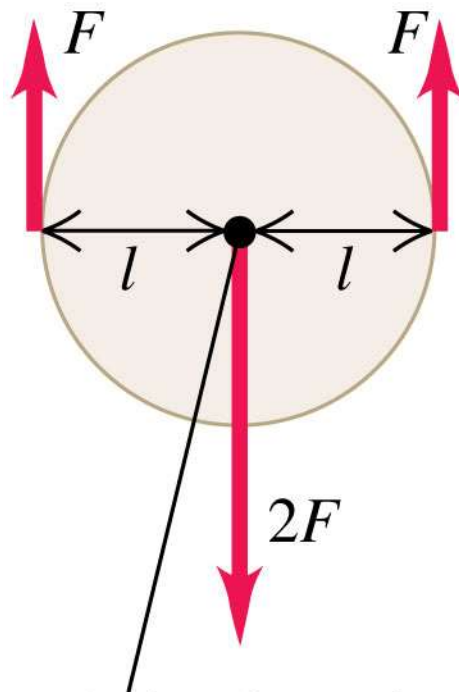
**Equilibrium conditions:**

**First condition satisfied:**

Net force = 0, so body at rest has no tendency to start moving as a whole.

**Second condition satisfied:**

Net torque about the axis = 0, so body at rest has no tendency to start rotating.



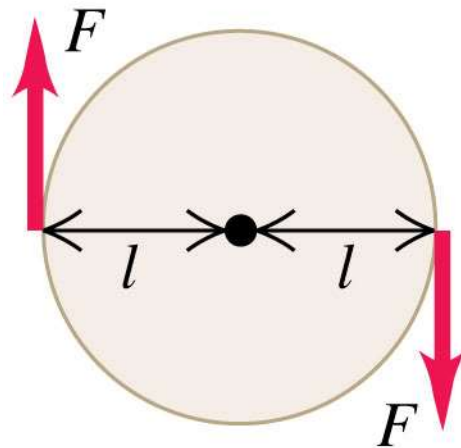
Axis of rotation (perpendicular to figure)



## Conditions for equilibrium: Example 2

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(b) This body has no tendency to accelerate as a whole, but it has a tendency to start rotating.



### First condition satisfied:

Net force = 0, so body at rest has no tendency to start moving as a whole.

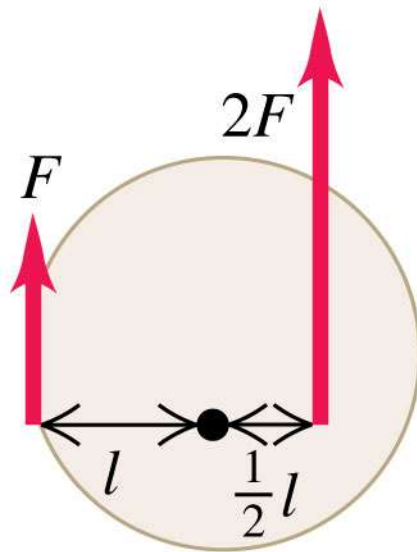
### Second condition NOT satisfied:

There is a net clockwise torque about the axis, so body at rest will start rotating clockwise.

## Conditions for equilibrium: Example 3

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(c) This body has a tendency to accelerate as a whole but no tendency to start rotating.

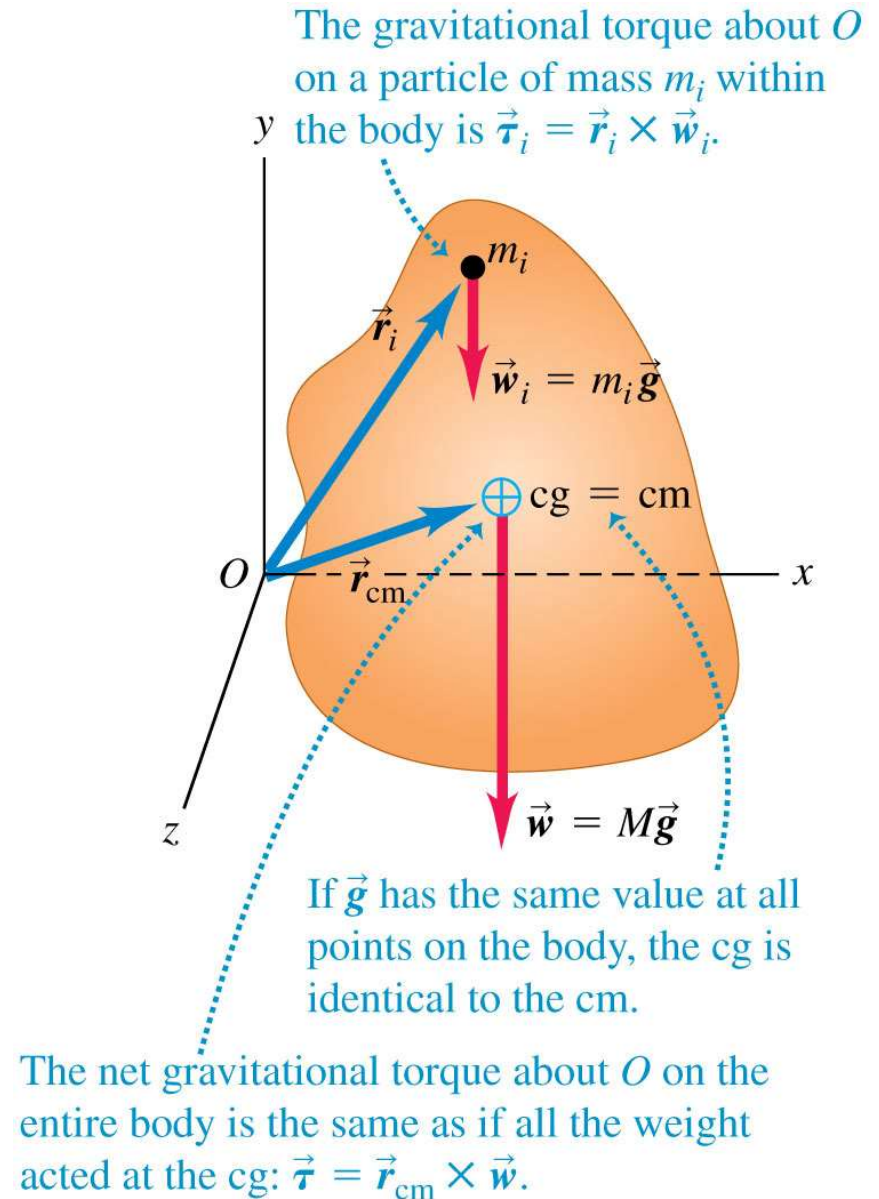


**First condition NOT satisfied:** There is a net upward force, so body at rest will start moving upward.

**Second condition satisfied:** Net torque about the axis  $= 0$ , so body at rest has no tendency to start rotating.

# Center of gravity

- We can treat a body's weight as though it all acts at a single point: the **center of gravity**.
- If we can ignore the variation of gravity with altitude, the center of gravity is the same as the center of mass.





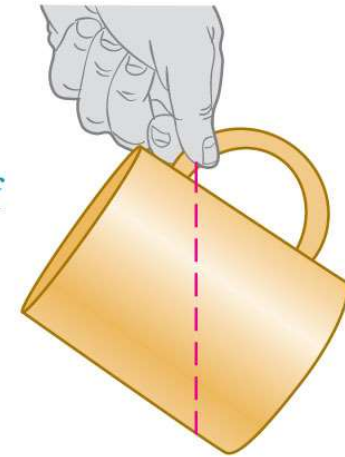
# Center of gravity

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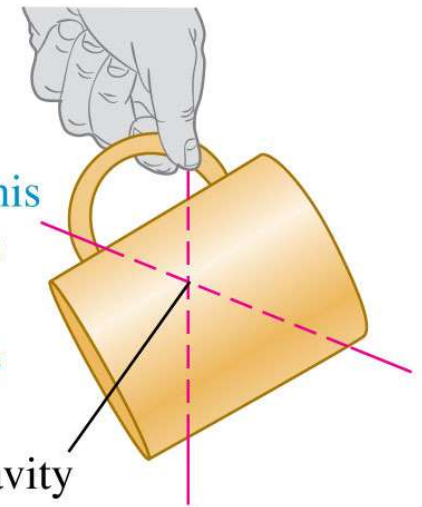
- When a body in rotational equilibrium and acted on by gravity is supported or suspended at a single point, the center of gravity is always at or directly above or below the point of suspension.

Where is the center of gravity of this mug?

① Suspend the mug from any point. A vertical line extending down from the point of suspension passes through the center of gravity.



② Now suspend the mug from a different point. A vertical line extending down from this point intersects the first line at the center of gravity (which is inside the mug).

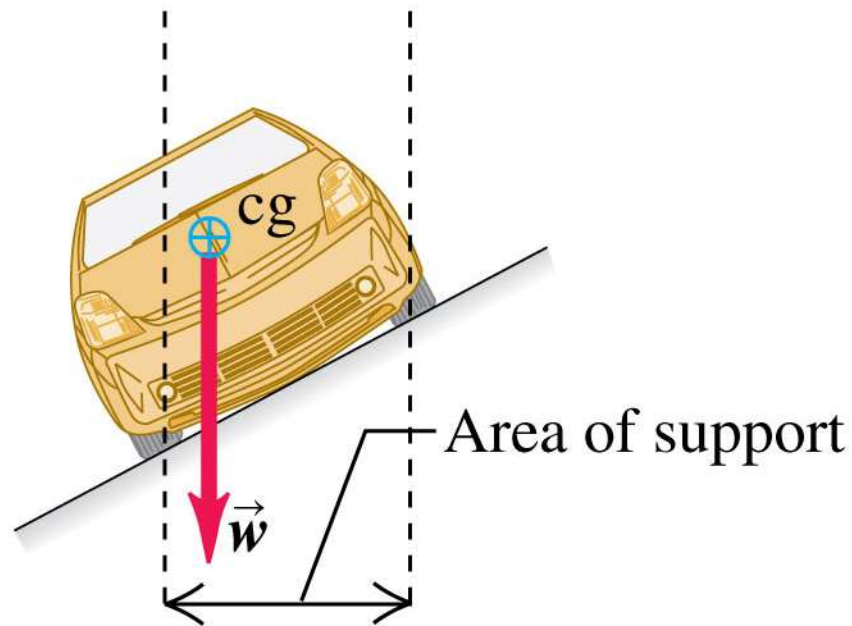


Center of gravity

# Center of gravity

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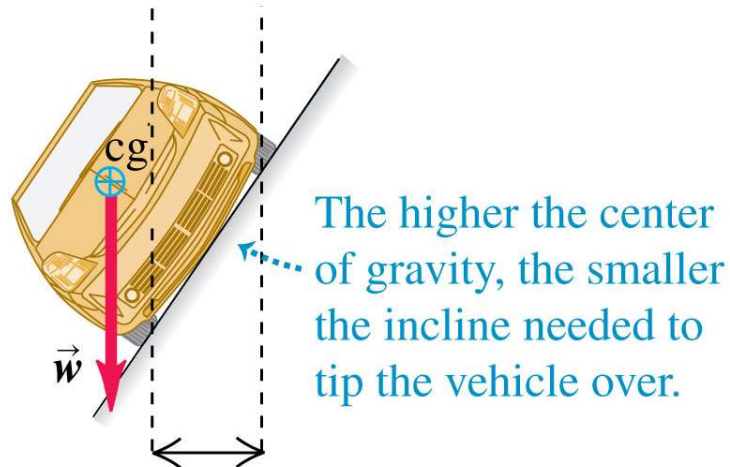
- To be in equilibrium, a body supported at several points must have its center of gravity somewhere within the area bounded by the supports.



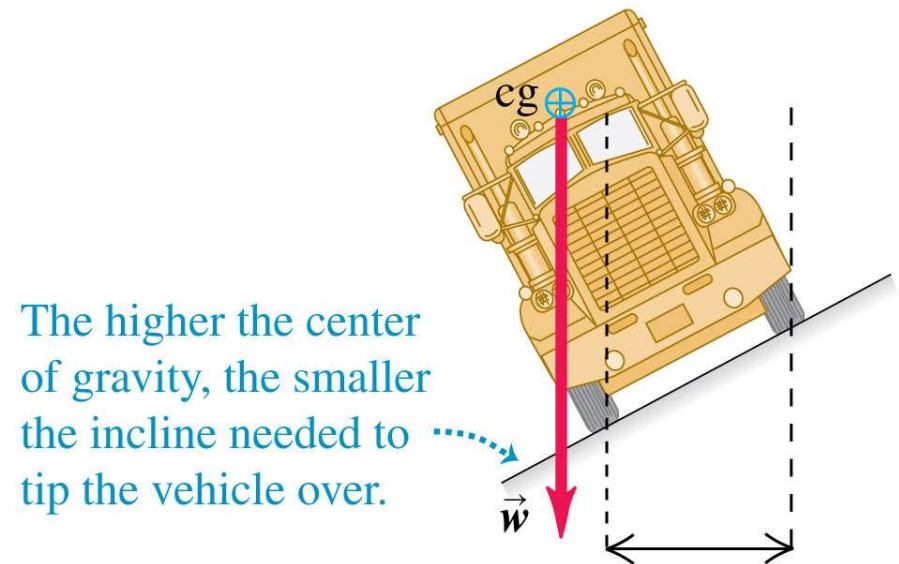
Center of gravity is over the area of support: car is in equilibrium.

# Center of gravity

- A body is *not* in equilibrium if its center of gravity lies outside the area of support.



Center of gravity is outside the area of support: vehicle tips over.



Center of gravity is outside the area of support: vehicle tips over.

# Problem-solving strategy for static equilibrium

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- *Identify* the relevant concepts: The first and second conditions for equilibrium are  $\sum F_x = 0$ ,  $\sum F_y = 0$ , and  $\sum \tau_z = 0$ .
- *Set up* the problem by using the following steps:
  1. Sketch the physical situation and identify the body in equilibrium to be analyzed.
  2. Draw a free-body diagram showing all forces acting on the body. Show the point on the body at which each force acts.
  3. Choose coordinate axes and specify their direction. Specify a positive direction of rotation for torques.
  4. Choose a reference point about which to compute torques.

# Problem-solving strategy for static equilibrium

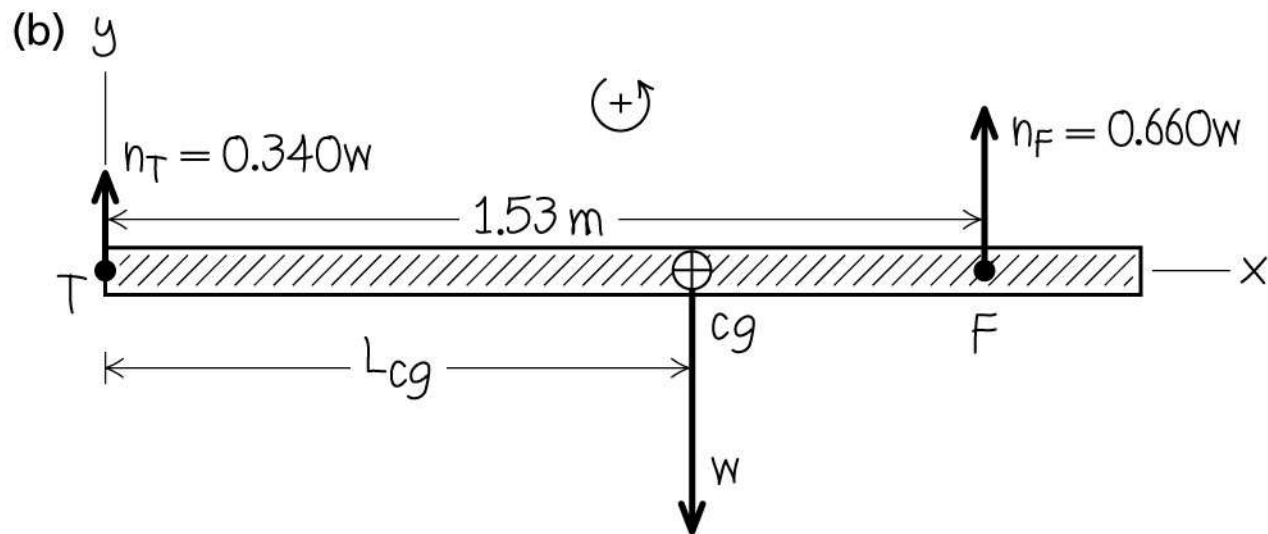
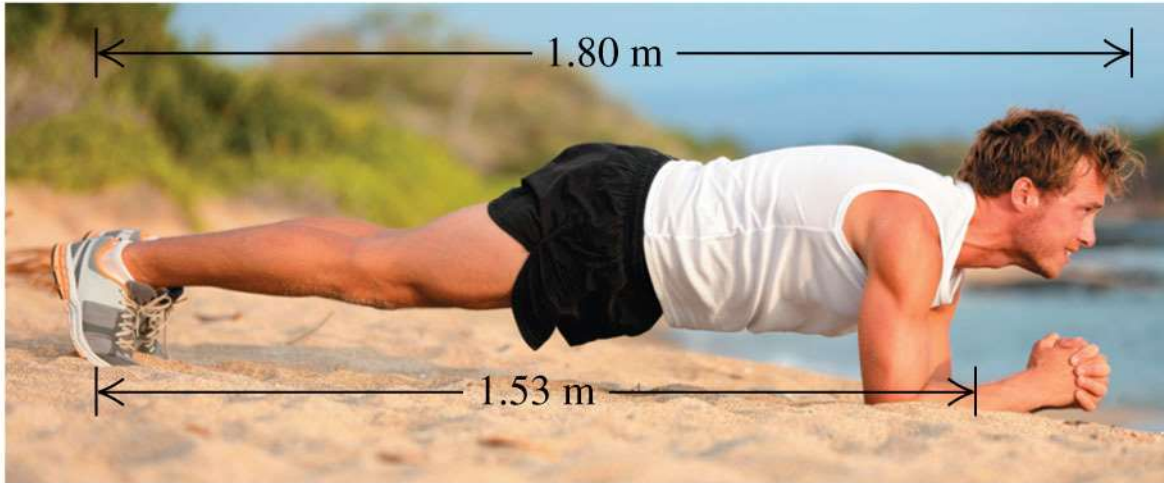
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- *Execute* the solution as follows:
  1. Write equations expressing the equilibrium conditions. Remember that  $\sum F_x = 0$ ,  $\sum F_y = 0$ , and  $\sum \tau_z = 0$  are separate equations.
  2. To obtain as many equations as you have unknowns, you may need to compute torques with respect to two or more reference points.
- *Evaluate* your answer: Check your results by writing  $\sum \tau_z = 0$  with respect to a different reference point. You should get the same answers.



# Figure 11.8

(a)



# Feedback

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I am still confused on the situation where a sign is held static with two scenarios, one with the length of the sign one half of the other. I am not sure how you would calculate which tension force is greater

# A “teeter-totter” balancing torques

What is the maximum mass of the person so the plank does not turn?

$$\sum F_x = 0 \quad 0 = 0$$

$$\sum F_y = 0 \rightarrow F_{N1} + F_{N2} - mg - Mg = 0$$

$$\sum \tau = mg(L/2 - D/2) - Mg(D/2) + F_{N1}D = 0$$

For toppling  $F_{N1} = 0$  so

$$mg(L/2 - D/2) - Mg(D/2) + 0 = 0$$

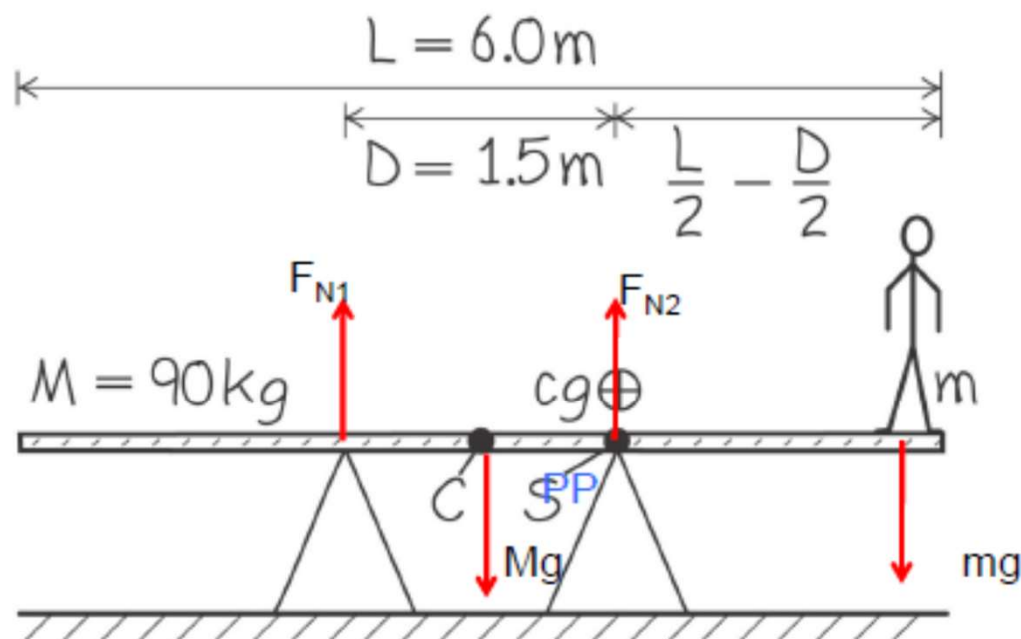
$$\Rightarrow m = \frac{M(D/2)}{(L/2 - D/2)}$$

$$\tau_{mg} = mg(L/2 - D/2)$$

$$\tau_{Mg} = -Mg(D/2)$$

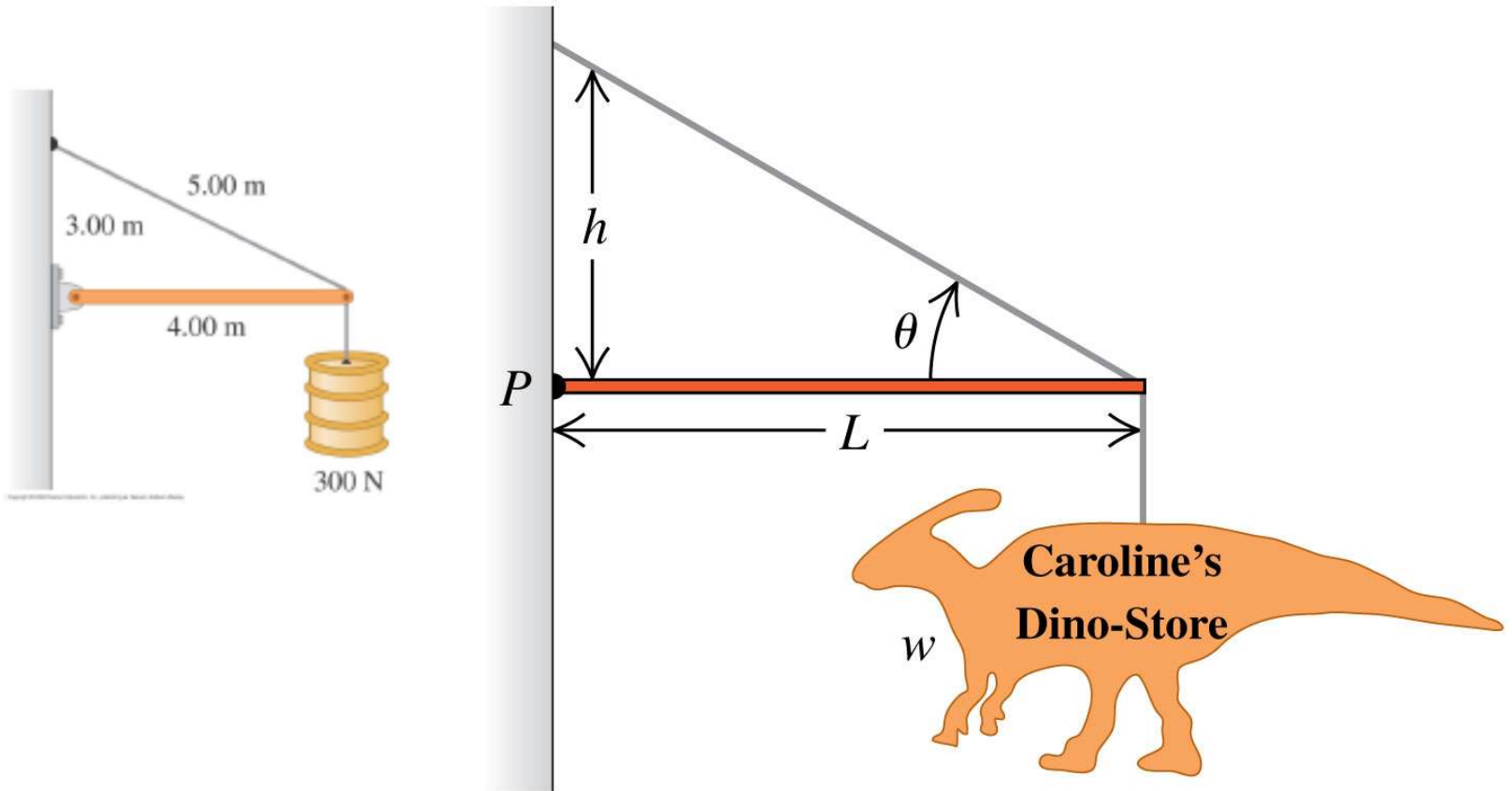
$$\tau_{FN1} = F_{N1}D$$

$$\tau_{FN2} = 0$$



# Problem 11.27

The horizontal beam in the figure weighs 150 N, and its center of gravity is at its center. Find the tension in the cable and on the beam at the hinge.



# The ladder against a wall

Sir Lancelot is trying to go up a castle wall on a ladder as shown in the picture. The wall is slippery (no friction) and he weights 800N. The ladder is 5.0 m long, wights 180 N and he stops one third of the way up.

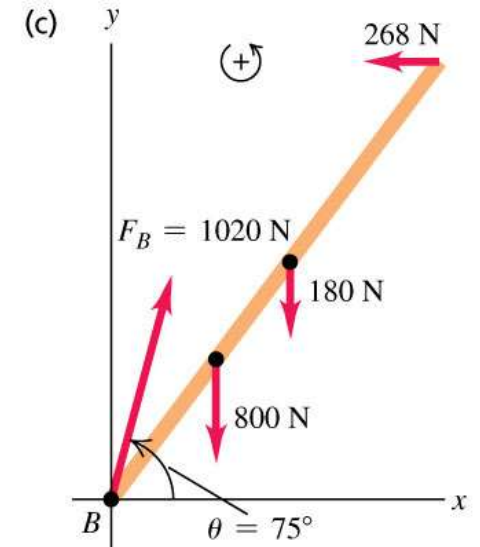
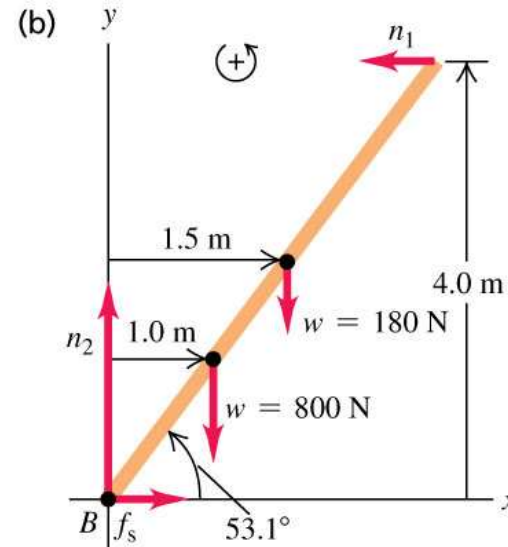
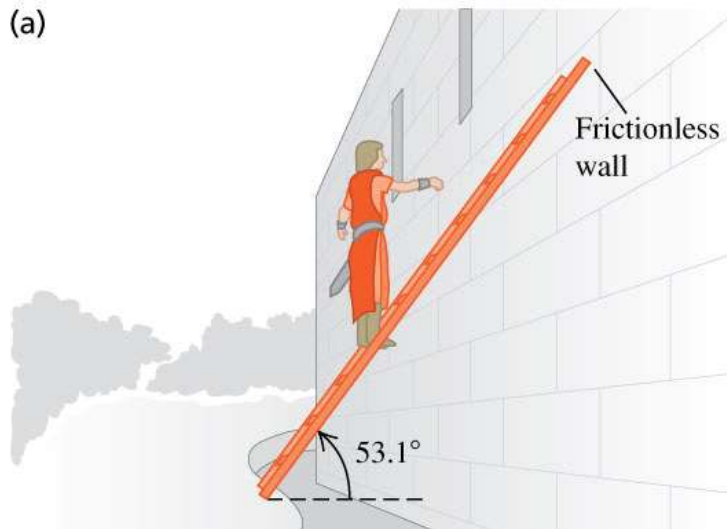
(a) Find the normal and frictional force on the ladder at its base

(b) Find the minimum coefficient of friction needed at the base for him not to slip all the way up

(a)



(a)

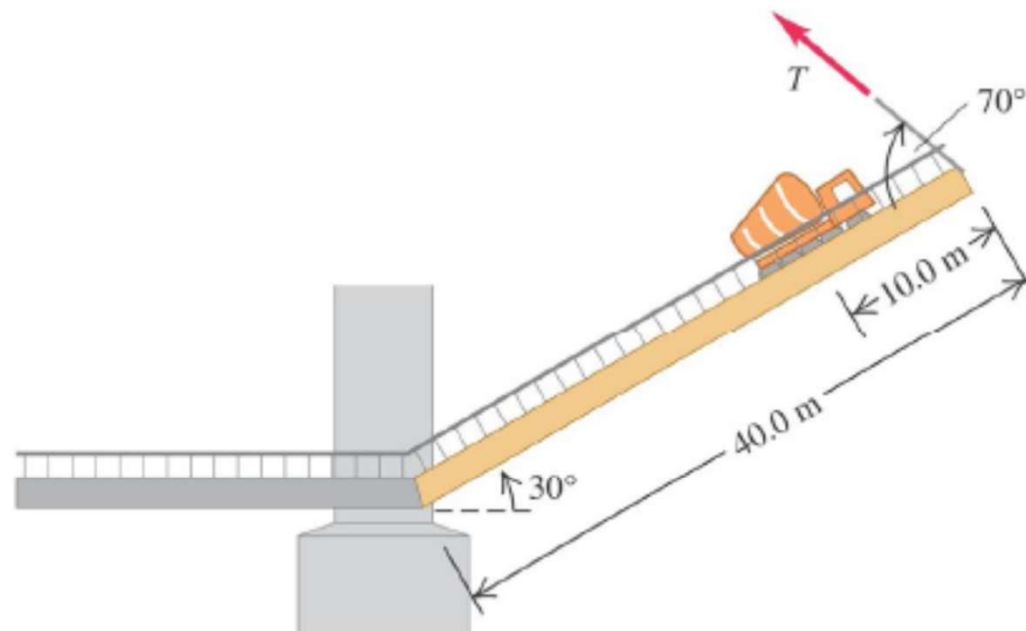




# Problem 11.52

A loaded cement mixer drives onto an old drawbridge, where it stalls with its center of gravity three-quarters of the way across the span. The truck driver radios for help, sets the handbrake, and waits. Meanwhile, a boat approaches, so the drawbridge is raised by means of a cable attached to the end opposite the hinge (the figure). The drawbridge is 40.0 m long and has a mass of 12,000 kg; its center of gravity is at its midpoint. The cement mixer, with driver, has mass 30,000 kg. When the drawbridge has been raised to an angle of 30 degrees above the horizontal, the cable makes an angle of 70 degrees with the surface of the bridge.

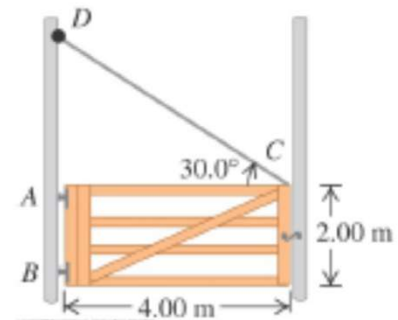
What is the tension  $T$  in the cable when the drawbridge is held in this position?



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# Problem 11.73

A gate 4.00 m wide and 2.00 m high weighs 450 N. Its center of gravity is at its center, and it is hinged at A and B. To relieve the strain on the top hinge, a wire CD is connected as shown in the figure. The tension in CD is increased until the horizontal force at hinge A is zero.



- What is the tension in the wire CD?
- What is the magnitude of the horizontal component of the force at hinge B?
- What is the combined vertical force exerted by hinges A and B?