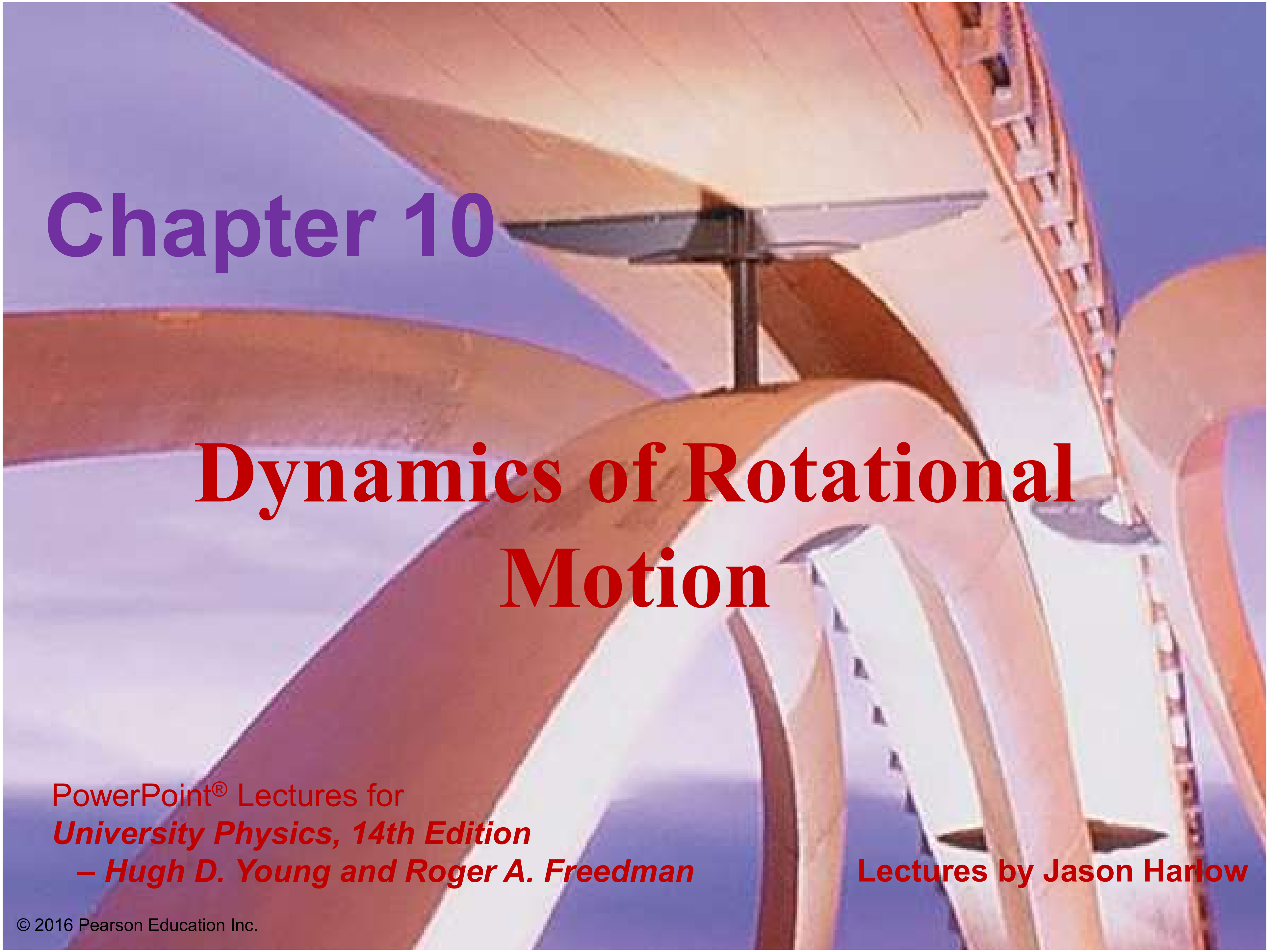


Chapter 10

Dynamics of Rotational Motion

PowerPoint® Lectures for
University Physics, 14th Edition
– Hugh D. Young and Roger A. Freedman

Lectures by Jason Harlow

Learning Goals for Chapter 10

Looking forward at …

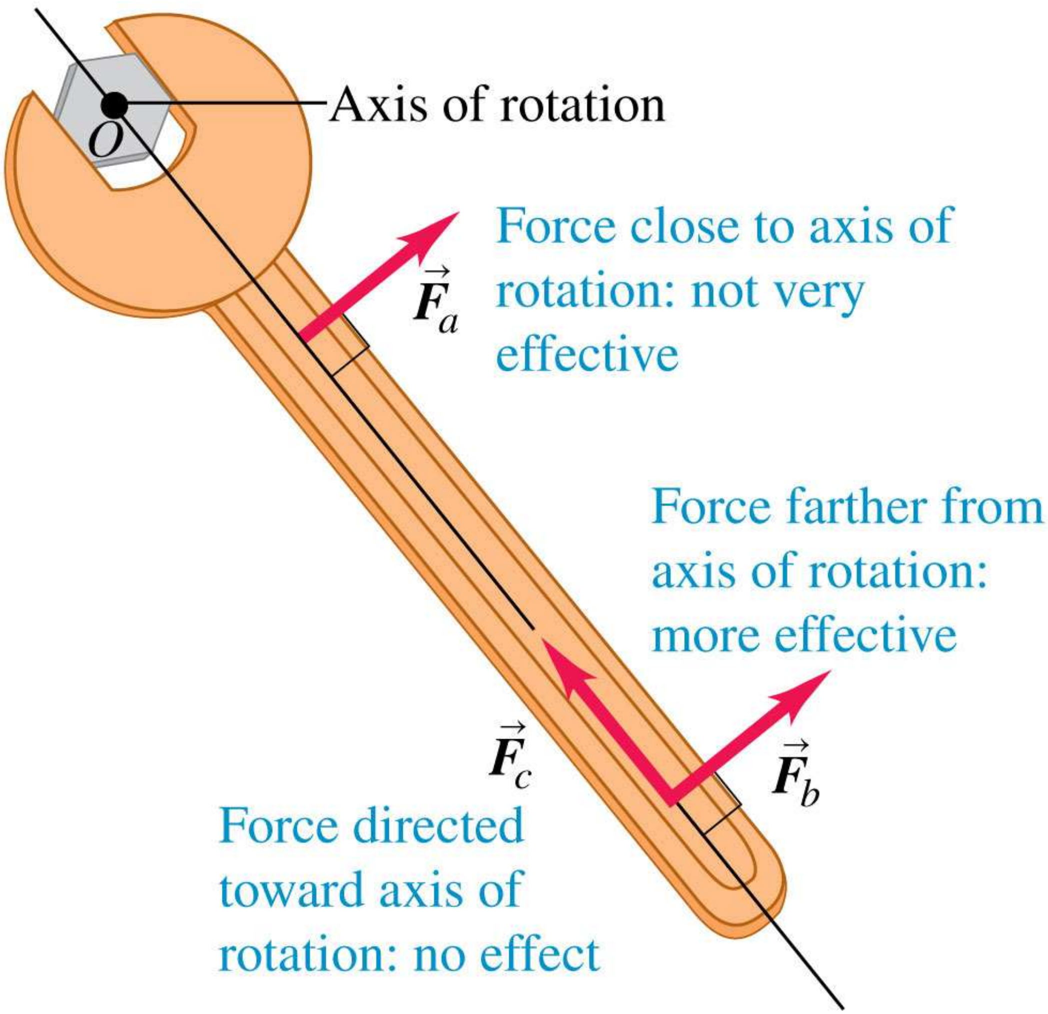
- what is meant by the torque produced by a force.
- how the net torque on a body affects the body’s rotational motion.
- how to analyze the motion of a body that both rotates and moves as a whole through space.
- how to solve problems that involve work and power for rotating bodies.
- how the angular momentum of a body can remain constant even if the body changes shape.

Introduction

- These jugglers toss the pins so that they rotate in midair.
- What does it take to start a stationary body rotating or to bring a spinning body to a halt?
- We'll introduce some new concepts, such as torque and angular momentum, to deepen our understanding of rotational motion.

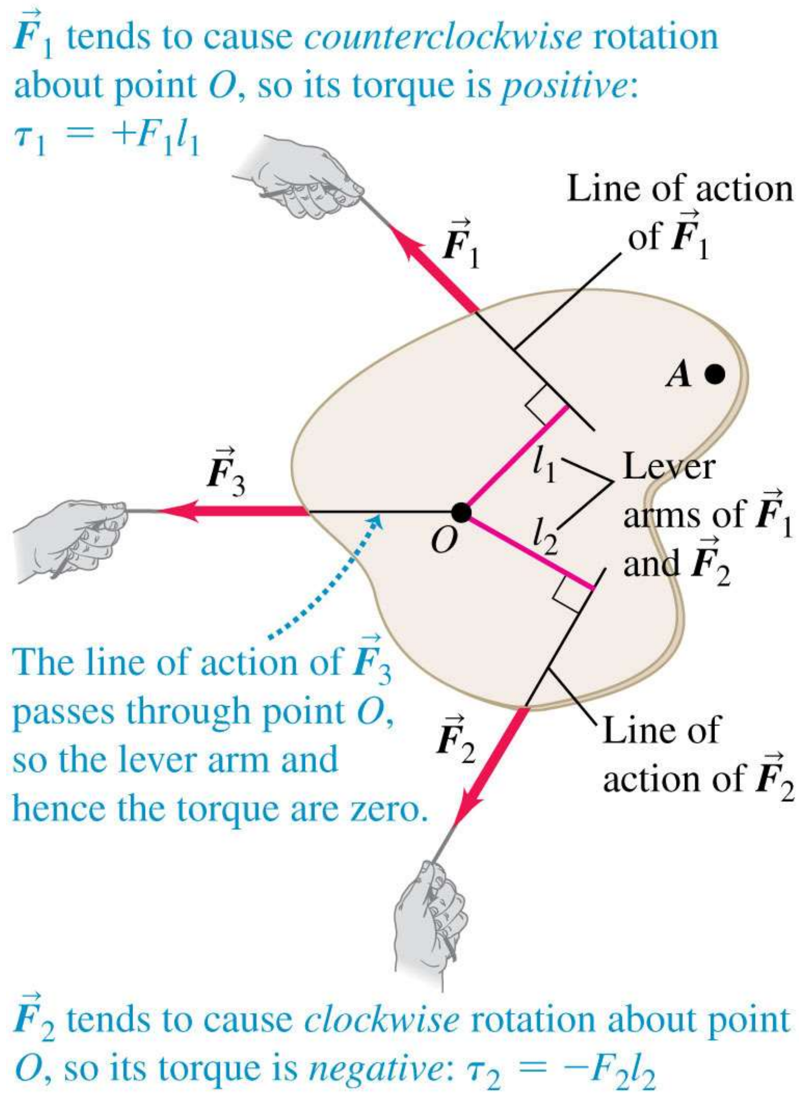
Loosen a bolt

- Which of the three equal-magnitude forces in the figure is most likely to loosen the bolt?

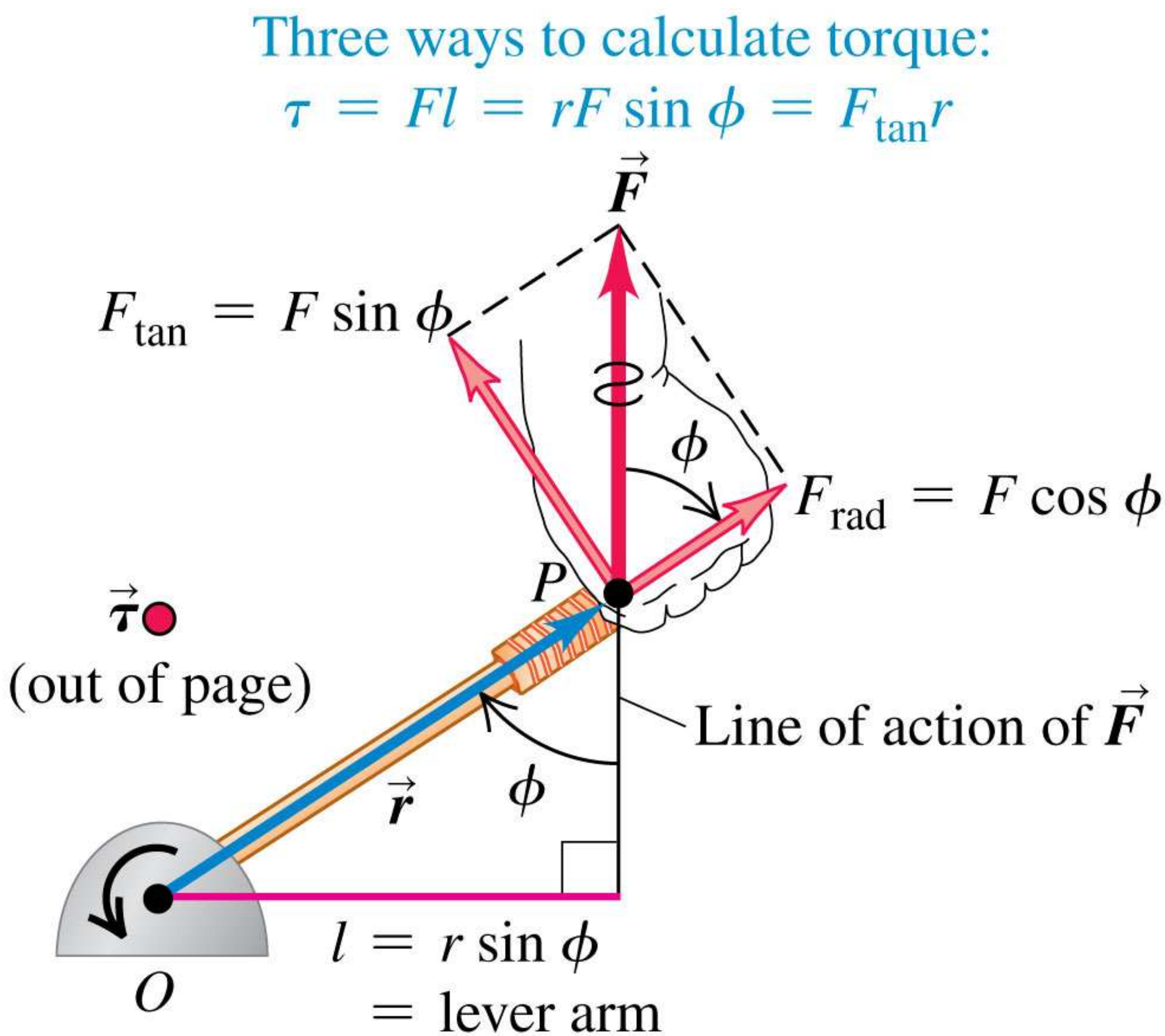


Torque

- The **line of action** of a force is the line along which the force vector lies.
- The **lever arm** for a force is the perpendicular distance from O to the line of action of the force.
- The torque of a force with respect to O is the product of the force and its lever arm.

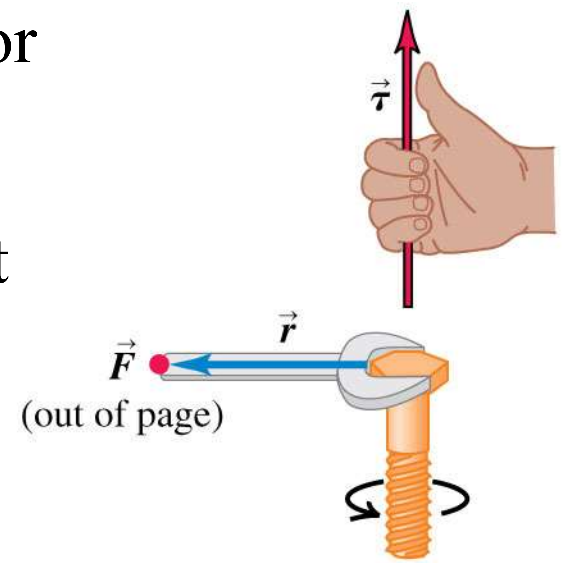


Three ways to calculate torque



Torque as a vector

- Torque can be expressed as a vector using the vector product.
- If you curl the fingers of your right hand in the direction of the force around the rotation axis, your outstretched thumb points in the direction of the torque vector.



Torque vector due to force $\vec{F}$ relative to point O → $\vec{\tau} = \vec{r} \times \vec{F}$

$\vec{r}$: Vector from O to where $\vec{F}$ acts

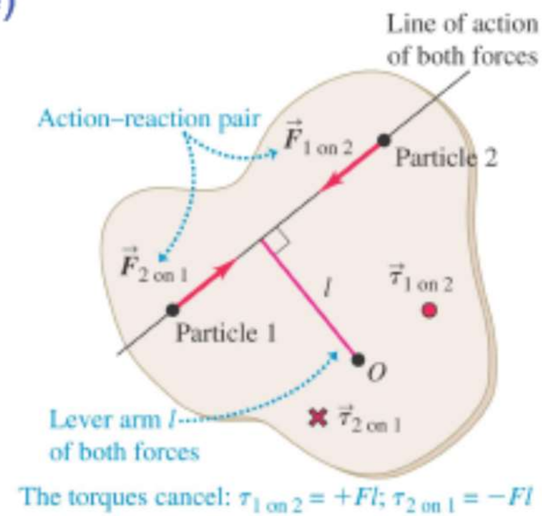
$\vec{F}$: Force $\vec{F}$

Newton's 2nd Law in angular sense: $\Sigma\,\tau = I\alpha$ is just like Σ F = ma

Moment of inertia AROUND the same PP (O in the figure)

$$\sum \tau_z = I\alpha_z$$

Sum of torques on a rigid object AROUND a PP (O in the figure)



The label z is to remind you that the actual torque and angular velocity vector point perpendicular to the x-y plane defined by the force and the leverage arm

This angular equation is IN ADDITION to any linear force which moves the center of mass but as long as the axis of rotation is fixed we do not need to worry about the linear motion. This changes when the rotating object is also moving

Torque and angular acceleration for a rigid body

- The rotational analog of Newton's second law for a rigid body is:

Rotational analog of Newton's second law for a rigid body:

$$\underbrace{\sum \tau_z}_{\text{Net torque on a rigid body about } z\text{-axis}} = \underbrace{I}_{\text{Moment of inertia of rigid body about } z\text{-axis}}\;\underbrace{\alpha_z}_{\text{Angular acceleration of rigid body about } z\text{-axis}}$$

- Loosening or tightening a screw requires giving it an angular acceleration and hence applying a torque.

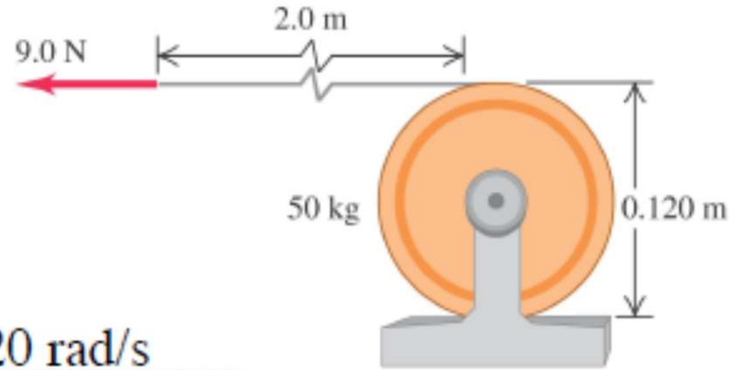
Flywheel problem from Ch 9 (using work energy theorem)

The cable is wrapped around a cylinder. If it unwinds 2.0 m by pulling it with a force of 9.0 N and it starts at rest, what is its final angular velocity and velocity of the cable? (use work energy theorem)

$$W_{total} = KE_f - KE_i$$

$$F\Delta x = \frac{1}{2}I\omega^2 - 0$$

$$\omega = \sqrt{\frac{2F\Delta x}{I}} = \sqrt{\frac{4F\Delta x}{mR^2}} = 20 \text{ rad/s}$$

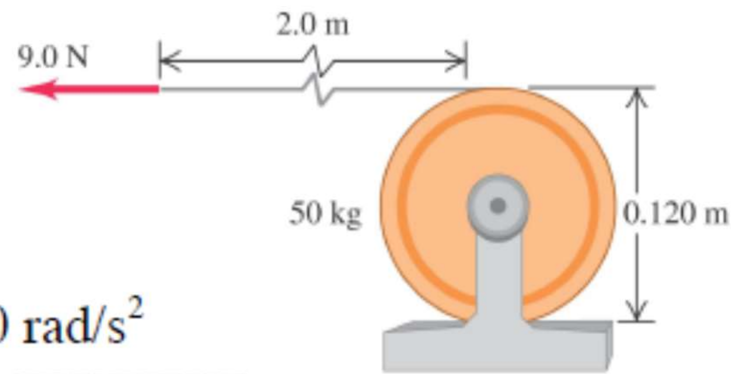


$$\text{v} = R\omega = (20 \text{ rad/s})(0.060 \text{ m}) = 1.2 \text{ m/s}$$

Flywheel problem using torque (using work energy theorem)

The cable is wrapped around a cylinder. If it unwinds 2.0 m by pulling it with a force of 9.0 N and it starts at rest, what is its final angular velocity and velocity of the cable? (use work energy theorem)

$$I = \frac{1}{2}MR^2$$



$$\tau = FR = (9.0\ \mathrm{N})(0.06\ \mathrm{m})$$

$$\tau = I\alpha \Rightarrow \alpha = \frac{\tau}{I} = \frac{2FR}{MR^2} = \frac{2F}{MR} = 6.0\ \mathrm{rad/s^2}$$

Use α to get acceleration of the cable:

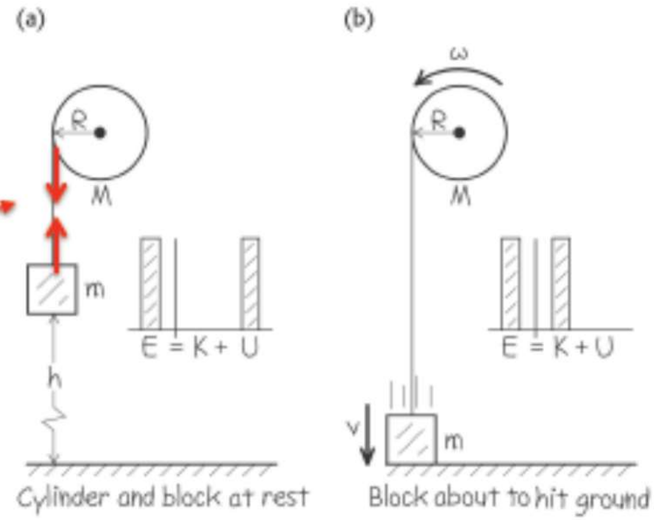
$$a_{tan} = R\alpha = (0.06\mathrm{m})(6.0\ \mathrm{rad/s^2}) = 0.36\ \mathrm{m/s^2}$$

Then use kinematics $\quad v^2 = v_0^2 + 2a_{tan}(x - x_0)$

$$v = \sqrt{0 + 2(0.36\ \mathrm{m/s^2})(2\ \mathrm{m})} = 1.2\ \mathrm{m/s}$$

What is the velocity of the block when it hits the ground?

The work done by the cable is zero since the two tension forces cancel each other out so energy is conserved



$$KE_i + PE_i = KE_f + PE_f$$

$$0 + mgh = \frac{1}{2}m\mathrm{v}^2 + \frac{1}{2}I\omega^2 + 0$$

$$0 + mgh = \frac{1}{2}m\mathrm{v}^2 + \frac{1}{2}\left(\frac{1}{2}MR^2\right)\left(\frac{\mathrm{v}}{\mathrm{R}}\right)^2 + 0$$

$$\mathrm{v} = \sqrt{\frac{2mgh}{(\mathrm{m}+\mathrm{M}/2)}}$$

Another look at the unwinding cable

What is the linear acceleration of the block?

These are two coupled objects; one rotates and the other moves linearly

For the rotating wheel we have:

$$\tau = I\alpha$$

$$TR = \frac{1}{2}MR^2\left(\frac{a}{R}\right) \quad \Rightarrow T = \frac{1}{2}Ma$$

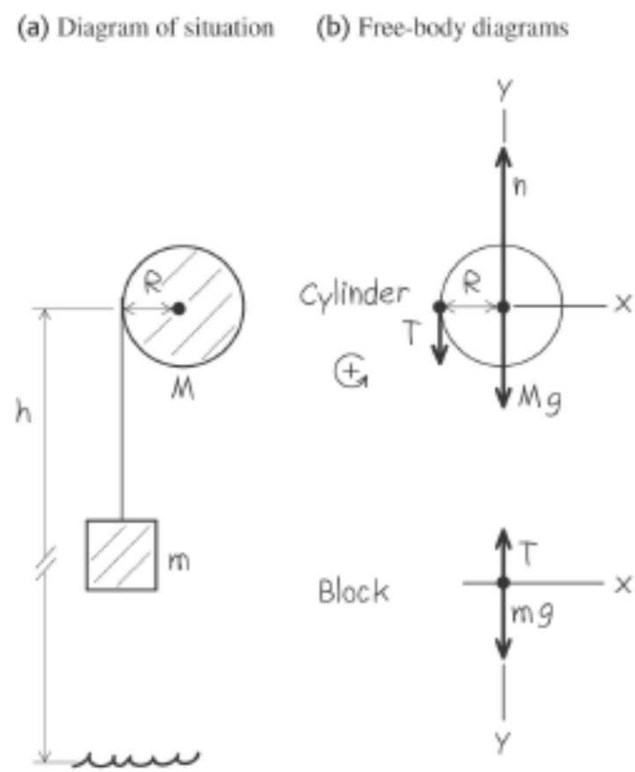
For the block we have:

$$mg - T = ma$$

Combine the two equations to get

$$mg - \frac{1}{2}Ma = ma$$

$$\Rightarrow a = \frac{mg}{m + M/2}$$

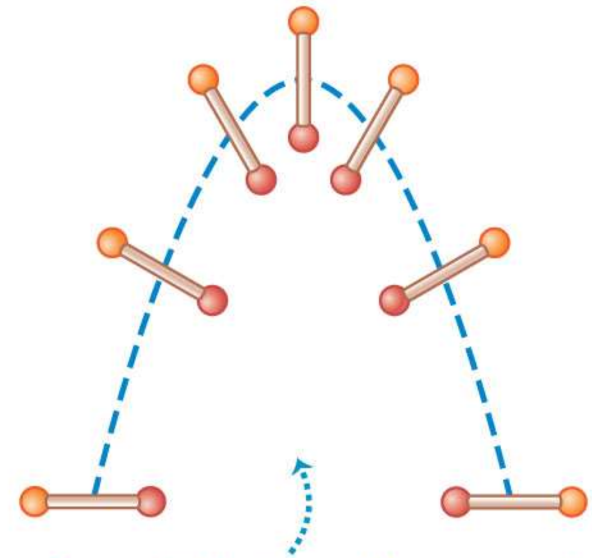


Rigid body rotation about a moving axis

- The kinetic energy of a rotating and translating rigid body is $K = 1/2\ Mv_{cm}^2 + 1/2\ I_{cm}\omega^2$.

There are two parts:

1. Motion of the CM (Center of Mass)
2. Rotation about the CM

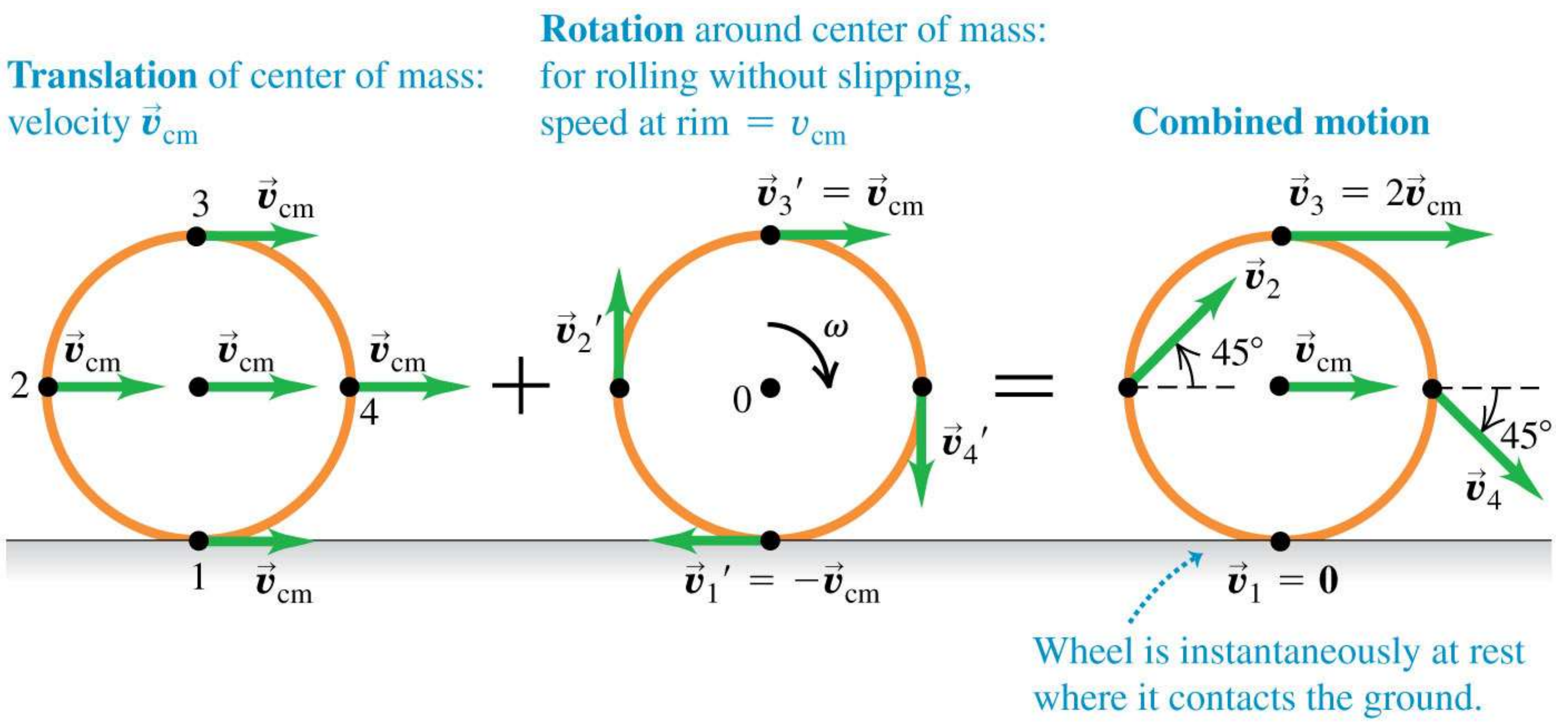


The motion of this tossed baton can be represented as a combination of ...

Rolling without slipping

- The motion of a rolling wheel is the sum of the translational motion of the center of mass plus the rotational motion of the wheel around the center of mass.

- The condition for rolling without slipping is $v_{cm} = R\omega$.



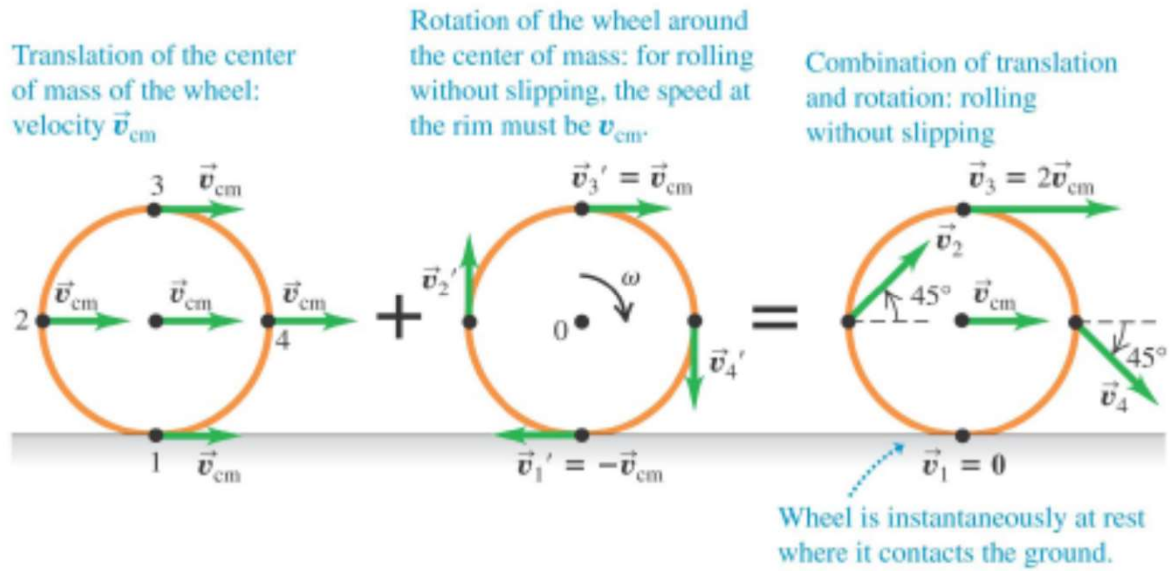
Rolling with and without slipping

When rolling without slipping then $v_{cm} = R\omega$ This is the condition to roll without slipping.

Then, if you are rolling without slipping the kinetic energy is

$$KE = \frac{1}{2}mv_{cm}^2 + \frac{1}{2}I_{cm}\omega^2 = \frac{1}{2}mv_{cm}^2 + \frac{1}{2}\frac{I_{cm}}{R^2}v^2$$

Also note that when one is rolling without slipping (i.e. rolling down an incline) the friction force is static so no work is done by it and energy is conserved in this case.



Consider the speed of a yo-yo toy

What is the speed of the Yo-yo at the bottom (use conservation of energy)

Why conservation of energy: the hand is not moving so it does no work on the system. You may be confused about the tension but keep in mind that it is an internal force so the sum of the upper and lower tension is zero.

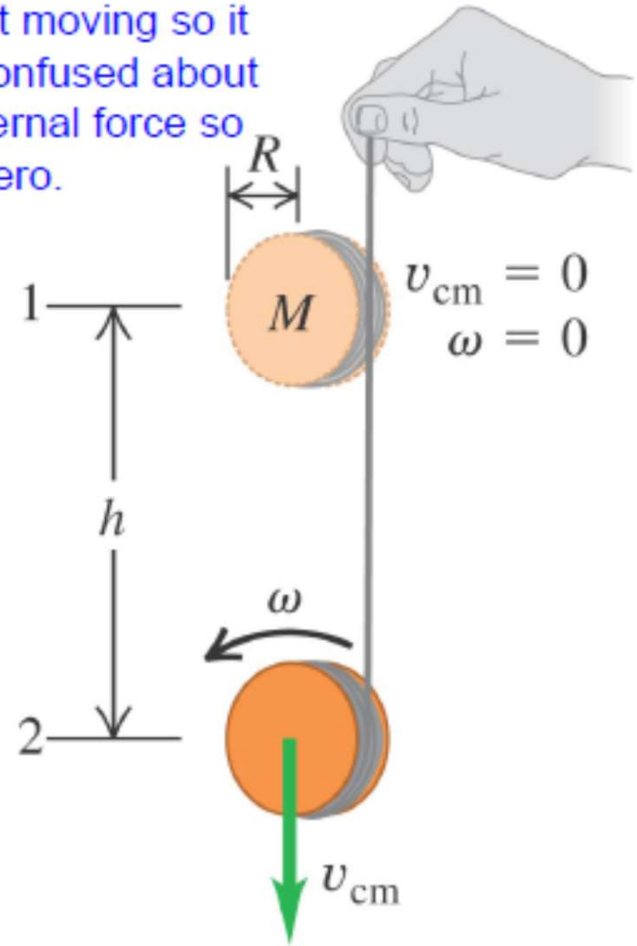
$$E_i = E_f$$

$$0 + Mgh = \frac{1}{2}Mv_{cm}^2 + \frac{1}{2}I_{cm}\omega^2$$

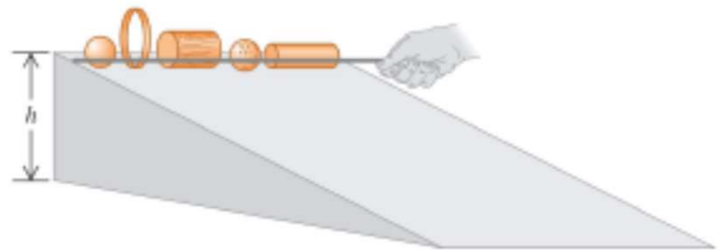
$$0 + Mgh = \frac{1}{2}Mv_{cm}^2 + \frac{1}{2}\left(\frac{1}{2}MR^2\right)\left(\frac{v_{cm}}{R}\right)^2$$

$$Mgh = \frac{3}{4}Mv_{cm}^2$$

$$\Rightarrow v_{cm} = \sqrt{\frac{4}{3}gh}$$



The race of objects with different moments



Let's figure out which circular object has the largest velocity when it reaches the bottom of the ramp. They roll without slipping so energy is conserved.

$$E_i = E_f$$

$$0 + Mgh = \frac{1}{2}Mv_{cm}^2 + \frac{1}{2}I_{cm}\omega^2$$

$$0 + Mgh = \frac{1}{2}Mv_{cm}^2 + \frac{1}{2}I_{cm}\left(\frac{v_{cm}}{R}\right)^2 = \frac{1}{2}Mv_{cm}^2 + \frac{1}{2}\frac{I_{cm}}{MR^2}v_{cm}^2$$

$$Mgh = \frac{1}{2}(1 + \frac{I_{cm}}{MR^2})Mv_{cm}^2$$

$$\Rightarrow v_{cm} = \sqrt{\frac{2gh}{1 + \frac{I_{cm}}{MR^2}}}$$

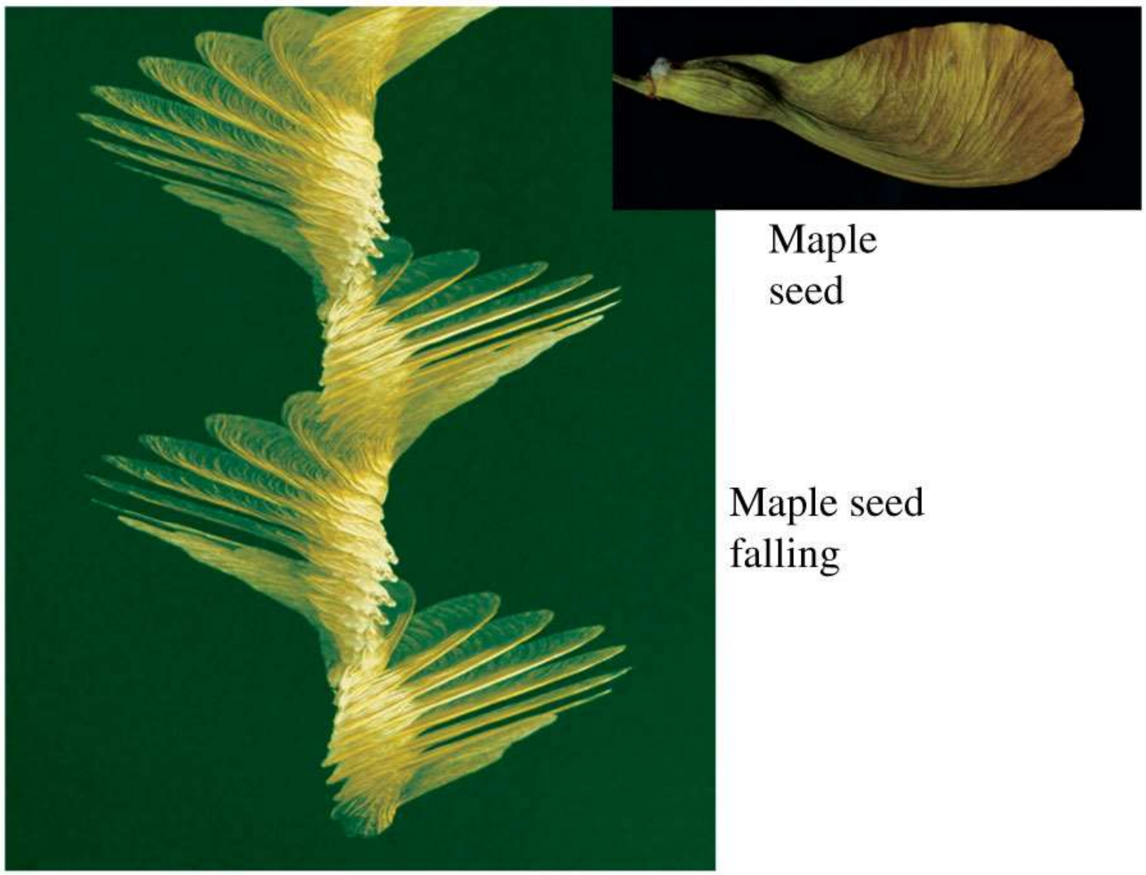
$$v_{cm}^{sphere} = \sqrt{\frac{2gh}{1 + \frac{2}{5}}} = \sqrt{\frac{10gh}{7}}$$

$$v_{cm}^{ring} = \sqrt{\frac{2gh}{1+1}} = \sqrt{gh}$$

$$v_{cm}^{cylinder} = \sqrt{\frac{2gh}{1 + \frac{1}{2}}} = \sqrt{\frac{4gh}{3}}$$

Combined translation and rotation

- Airflow around the wing of a maple seed slows the falling seed to about 1 m/s and causes the seed to rotate about its center of mass.



Rolling *with* slipping

- The smoke rising from this drag racer's rear tires shows that the tires are slipping on the road, so v_{cm} is *not* equal to $R\omega$.

Combined translation and rotation: dynamics

- The acceleration of the center of mass of a rigid body is:

Net external force on a body $\sum \vec{F}_{\text{ext}} = M\vec{a}_{\text{cm}}$ Total mass of body; Acceleration of center of mass

- The rotational motion about the center of mass is described by the rotational analog of Newton's second law:

Net torque on a rigid body about z-axis through center of mass $\sum \tau_z = I_{\text{cm}}\alpha_z$ Moment of inertia of rigid body about z-axis; Angular acceleration of rigid body about z-axis

- This is true as long as the axis through the center of mass is an axis of symmetry, and the axis does not change direction.

The yo-yo (again)

This time let's calculate the acceleration

The linear Newton's equation read:

$$y: \quad \sum F_y = Ma_{cm-y}$$

$$Mg - T = Ma_{cm}$$

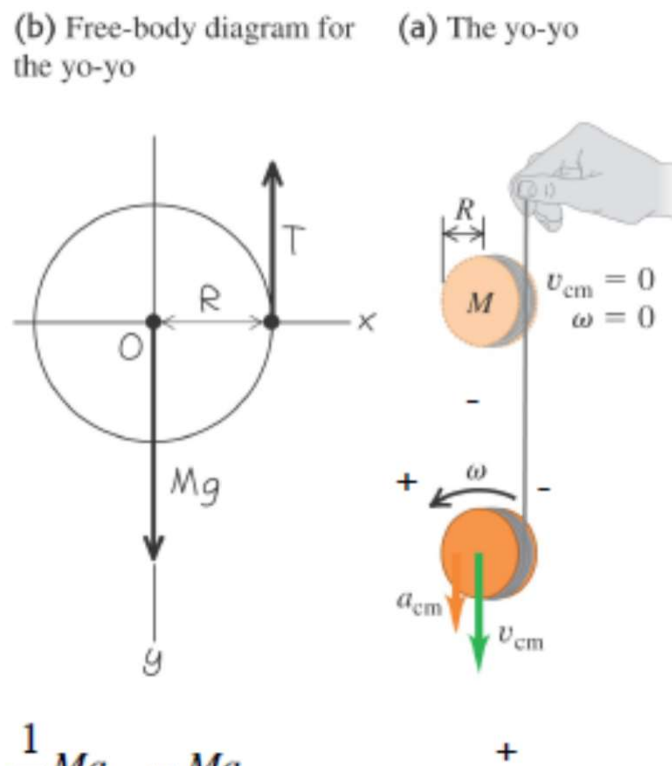
The rotational Newton's equation read:

$$\theta: \quad \sum \tau_z = I\alpha_z$$

$$\tau_{Mg} + \tau_T = I\alpha_z$$

$$0 + RT = \frac{1}{2}MR^2\frac{a_{cm}}{R}$$

$$T = \frac{1}{2}Ma_{cm}$$

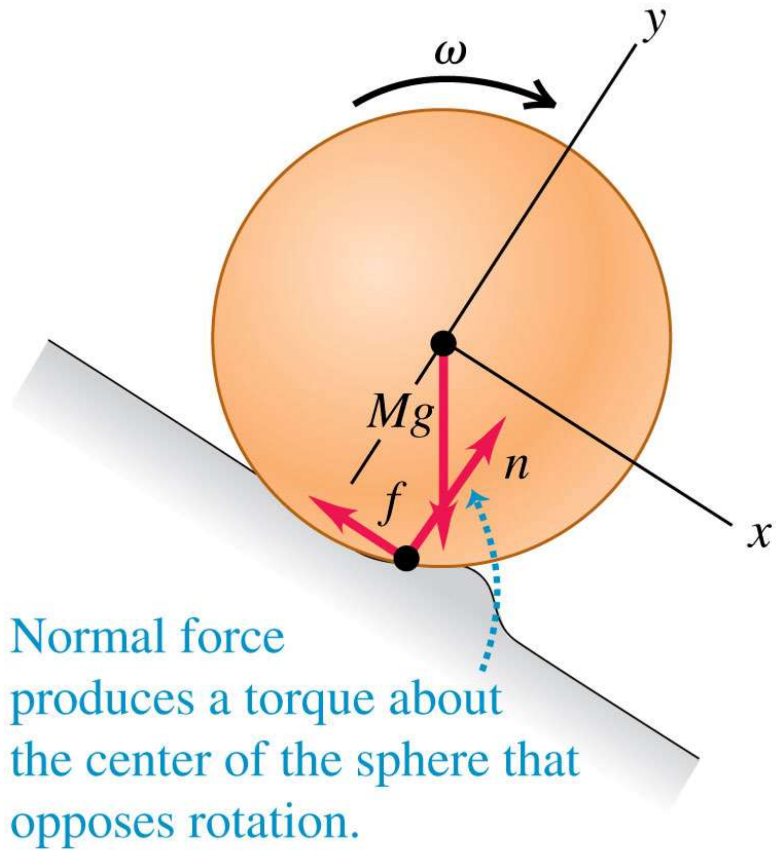


$$\Rightarrow \quad Mg - \frac{1}{2}Ma_{cm} = Ma_{cm}$$

$$a_{cm} = \frac{2}{3}g$$

Rolling friction

- We can ignore rolling friction if both the rolling body and the surface over which it rolls are perfectly rigid.
- If the surface or the rolling body deforms, mechanical energy can be lost, slowing the motion.

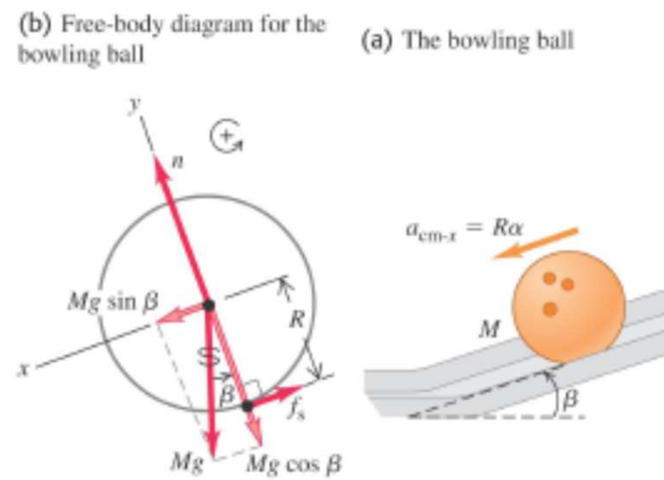


Consider the acceleration of a rolling sphere

The linear Newton's equations read:

$$x: \quad \sum F_x = ma_{cm-x} \qquad Mg\sin\beta - F_{f-stat} = ma_{cm}$$

$$y: \quad \sum F_y = 0 \qquad F_N - Mg\cos\beta = 0$$



The rotational Newton's equation read:

$$\theta: \quad \sum \tau_z = I\alpha_z$$

$$\tau_{F_N} + \tau_{Mg} + \tau_{fric} = I\alpha_z$$

$$0 + 0 + RF_f = \frac{2}{5}MR^2\frac{a_{cm}}{R} \qquad \Rightarrow Mg\sin\beta - \frac{2}{5}Ma_{cm} = Ma_{cm}$$

$$F_f = \frac{2}{5}Ma_{cm} \qquad a_{cm} = \frac{5}{7}g\sin\beta$$

Work in rotational motion

- A tangential force applied to a rotating body does work on it.

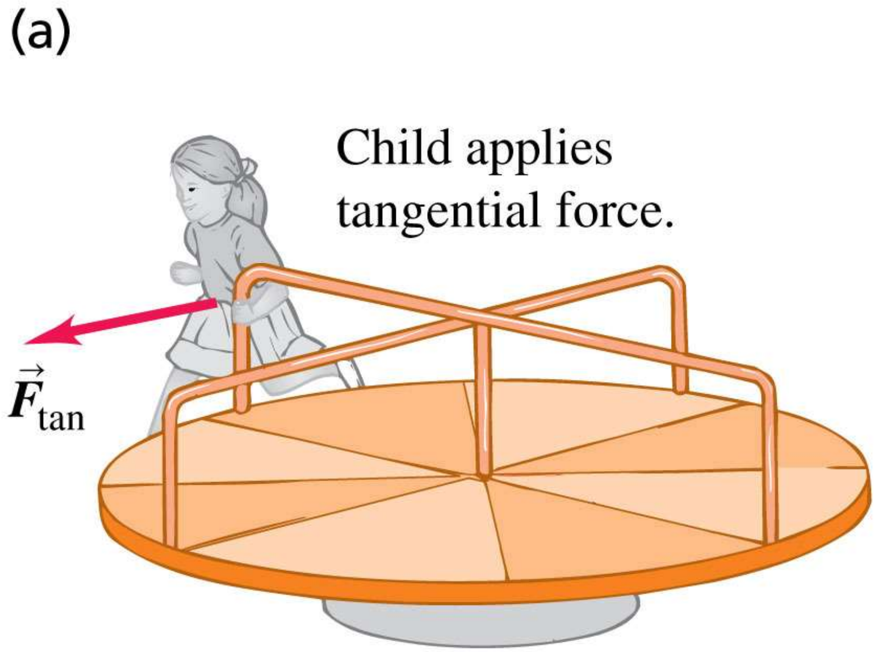
Work done by a torque τ_z

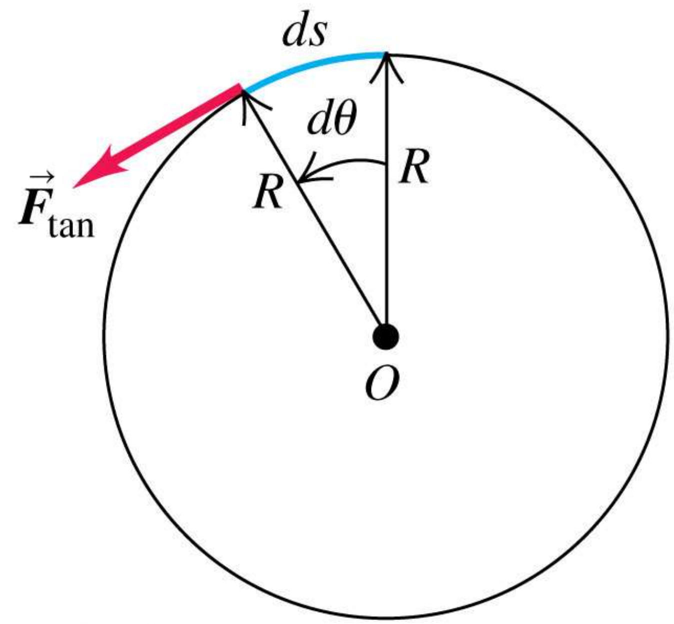
$$W = \int_{\theta_1}^{\theta_2} \tau_z \, d\theta$$

Upper limit = final angular position

Integral of the torque with respect to angle

Lower limit = initial angular position





Work and power in rotational motion

- The total work done on a body by the torque is equal to the change in rotational kinetic energy of the body, and the power due to a torque is:

Power due to a torque acting on a rigid body

$$P = \tau_z \omega_z$$

Torque with respect to body's rotation axis

Angular velocity of body about axis

- When a helicopter's main rotor is spinning at a constant rate, positive work is done on the rotor by the engine and negative work is done on it by air resistance.

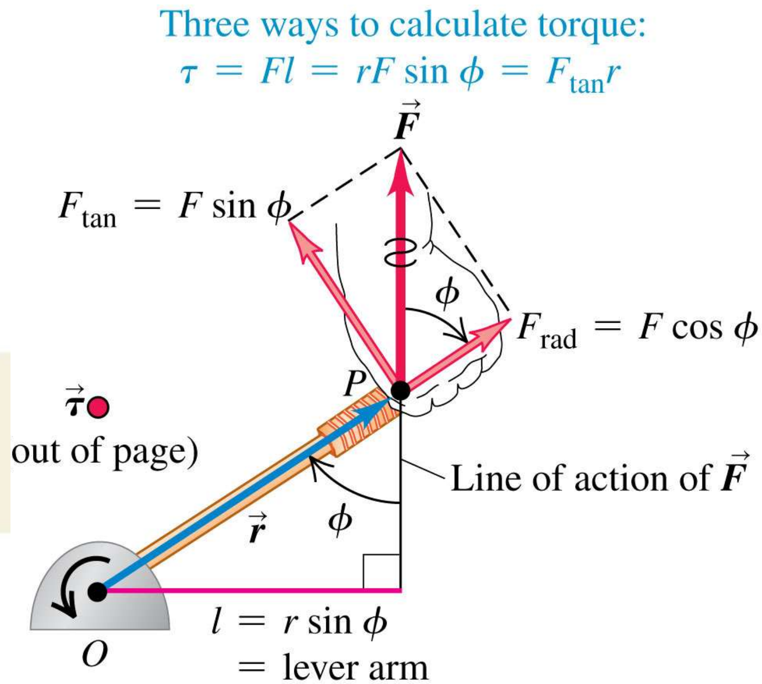
- Hence the net work being done is zero and the kinetic energy remains constant.

Recap of Ch 10 so far

- Force applied on a lever arm produces a torque:

Torque vector due to force $\vec{F}$ relative to point O → $\vec{\tau} = \vec{r} \times \vec{F}$ ← Force $\vec{F}$

Vector from O to where $\vec{F}$ acts → $\vec{r}$



- Like 2nd law shows ***F=ma*** , this τ produces an angular acceleration

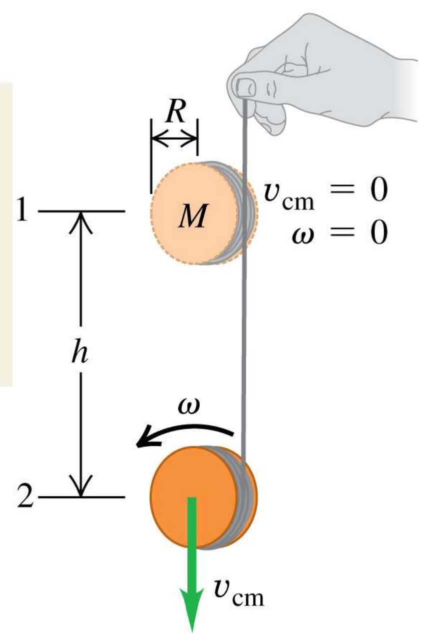
Rotational analog of Newton's second law for a rigid body:

Net torque on a rigid body about z-axis → $\sum \tau_z = I\alpha_z$

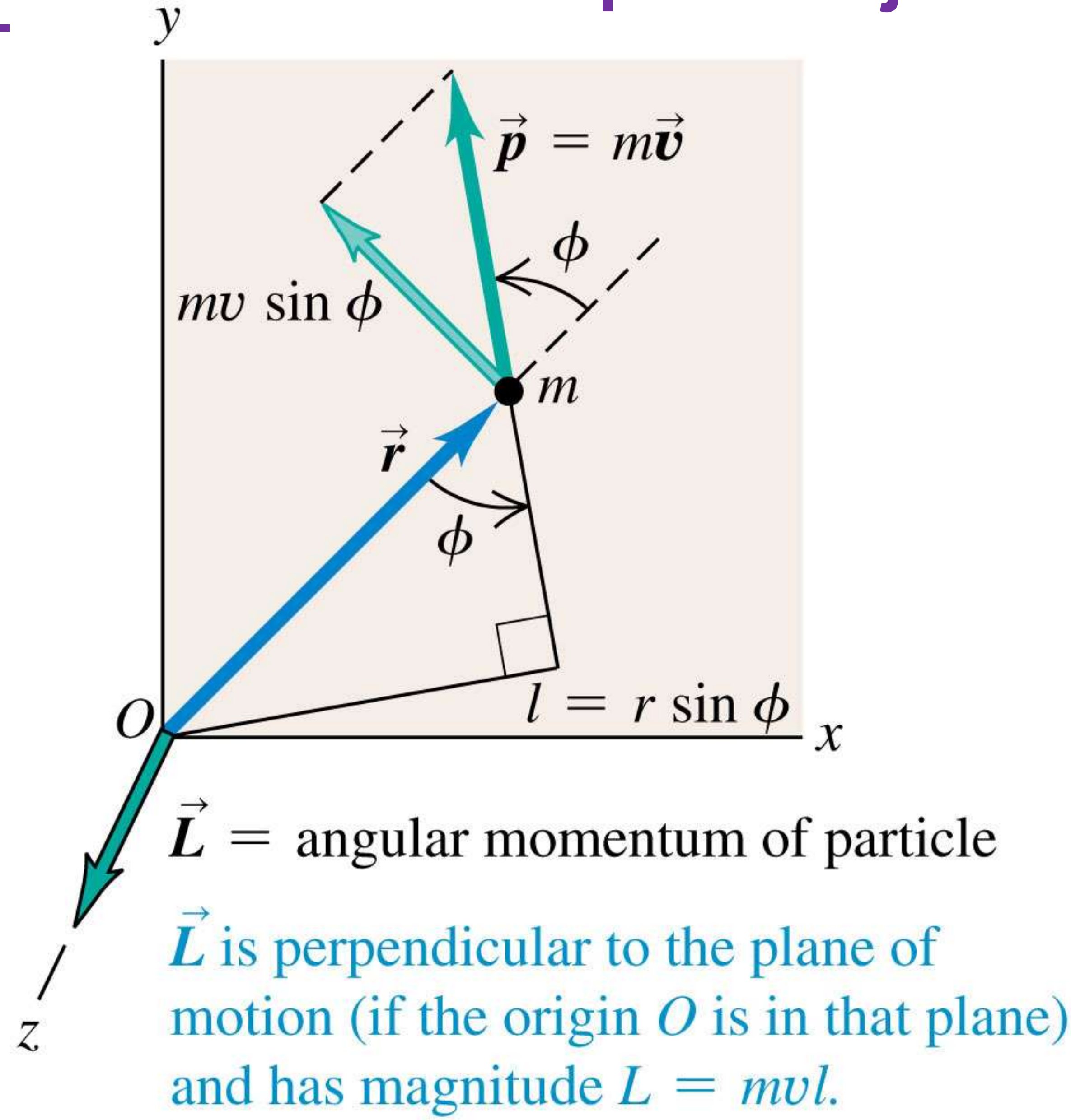
Moment of inertia of rigid body about z-axis → I

Angular acceleration of rigid body about z-axis → α_z

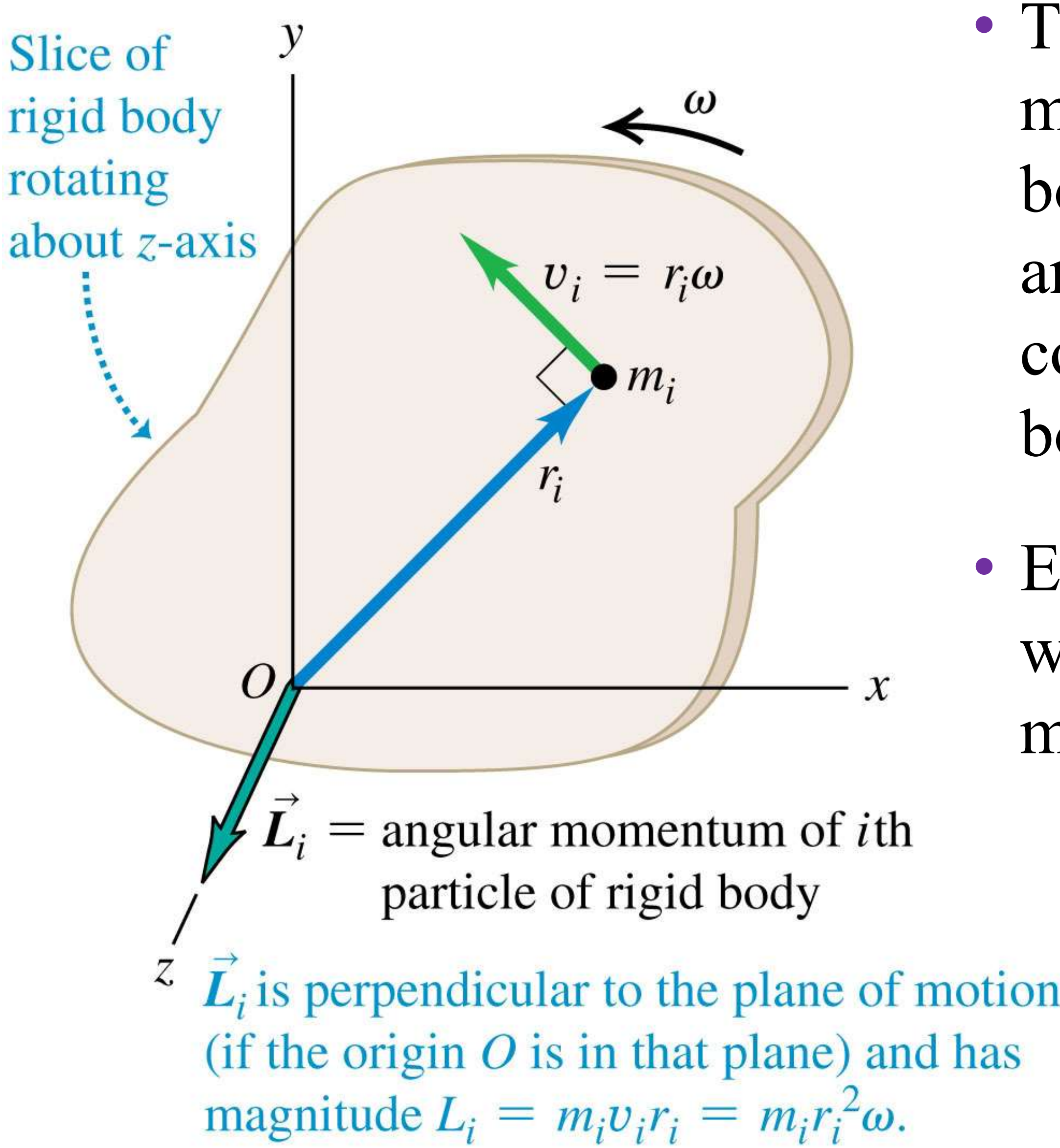
- We used this to solve many rotation problems, including translation + rotation



Angular Momentum of a point object



Angular momentum of a Rigid Body



- To find the total angular momentum of a rigid body rotating with angular speed ω, first consider a thin slice of the body.
- Each particle in the slice with mass m_i has angular momentum:

$$L_\mathrm{i} = m_i \, r_i^2 \, \omega$$

Angular momentum

- For a rigid body rotating around an axis of symmetry, the angular momentum is:

Angular momentum of a rigid body rotating around a symmetry axis

$$\vec{L} = I\vec{\omega}$$

Moment of inertia of body about symmetry axis

Angular velocity vector of body

- For any system of particles, the rate of change of the total angular momentum equals the sum of the torques of all forces acting on all the particles:

For a system of particles:

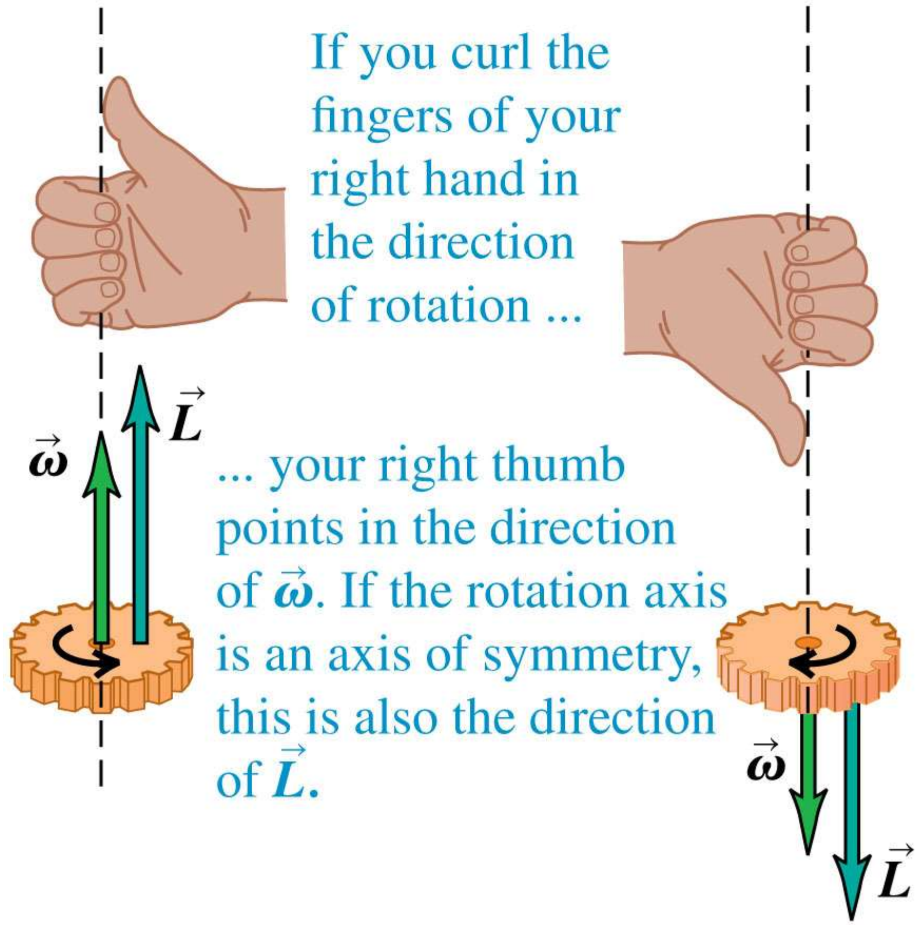
Sum of external torques on the system

$$\sum \vec{\tau} = \frac{d\vec{L}}{dt}$$

Rate of change of total angular momentum $\vec{L}$ of system

Angular momentum

- The angular momentum of a rigid body rotating about a symmetry axis is parallel to the angular velocity and is given by $\vec{L} = \vec{I}\omega.$



Angular Momentum

Recall that torque was defined as $\vec{\tau} = \vec{r} \times \vec{F}$

Similarly the angular momentum of a particle is

$$\boxed{\vec{L} = \vec{r} \times \vec{p} = \vec{r} \times m\vec{\mathrm{v}}}$$

Also, just as before for linear momentum we can show that the rate of change of the angular momentum on a particle is equal to the torque on that particle

$$\frac{d\vec{L}}{dt} = \frac{d}{dt}(\vec{r} \times m\vec{\mathrm{v}}) = \frac{d\vec{r}}{dt} \times m\vec{\mathrm{v}} + \vec{r} \times m\frac{d\vec{\mathrm{v}}}{dt} = \vec{\mathrm{v}} \times m\vec{\mathrm{v}} + \vec{r} \times m\vec{a} = 0 + \vec{r} \times \vec{F}$$

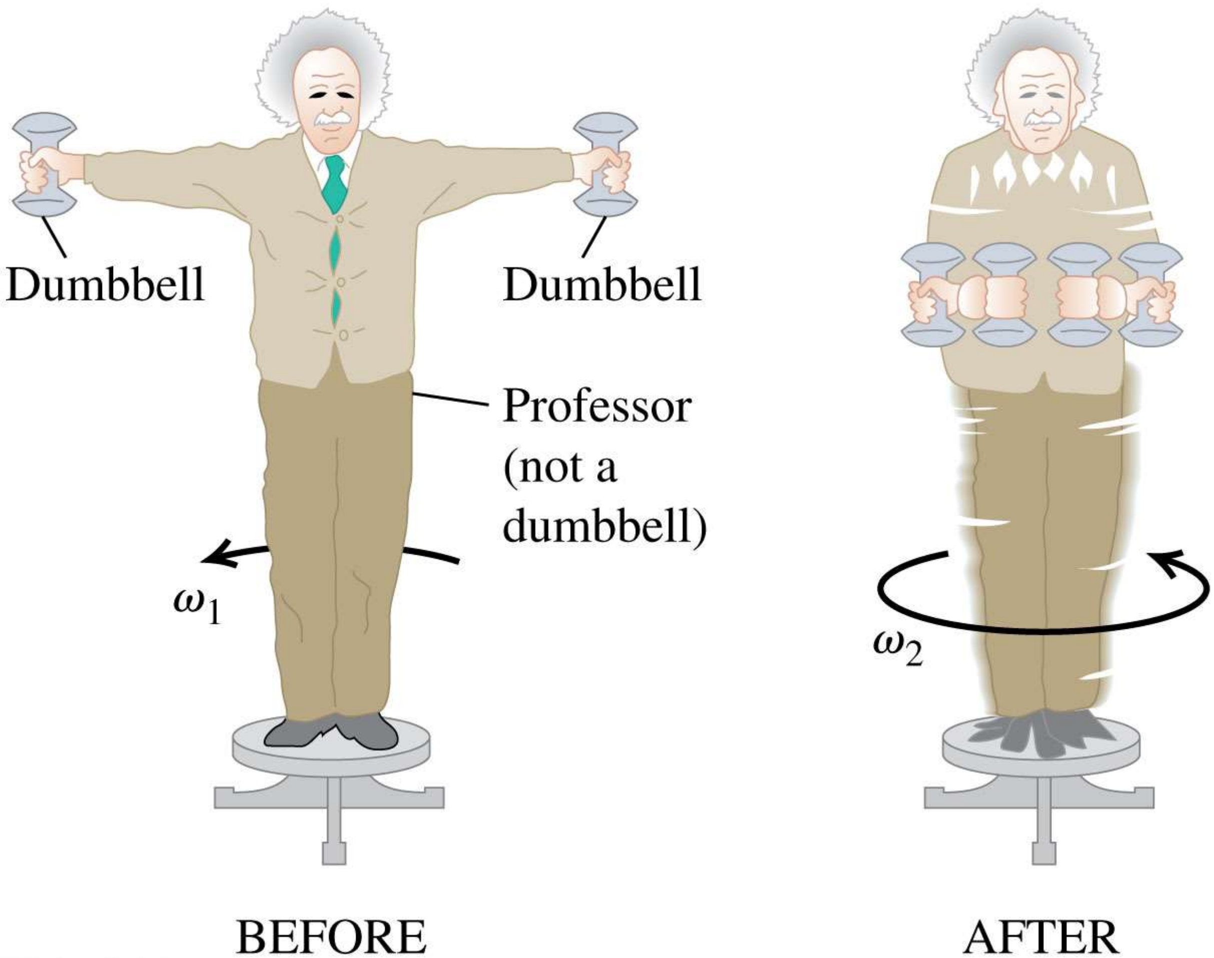
$$\Rightarrow \vec{\tau} = \frac{d\vec{L}}{dt}$$

FOR A RIGID BODY ONE USES THE MOMENT OF INERTIA TO DEFINE THE ANGULAR MOMENTUM

$$\boxed{\vec{L} = I\vec{\omega}}$$

Conservation of angular momentum

- When the net external torque acting on a system is zero, the total angular momentum of the system is constant (conserved).



Conservation of angular momentum

- A falling cat twists different parts of its body in different directions so that it lands feet first. At all times during this process the angular momentum of the cat as a whole remains zero.

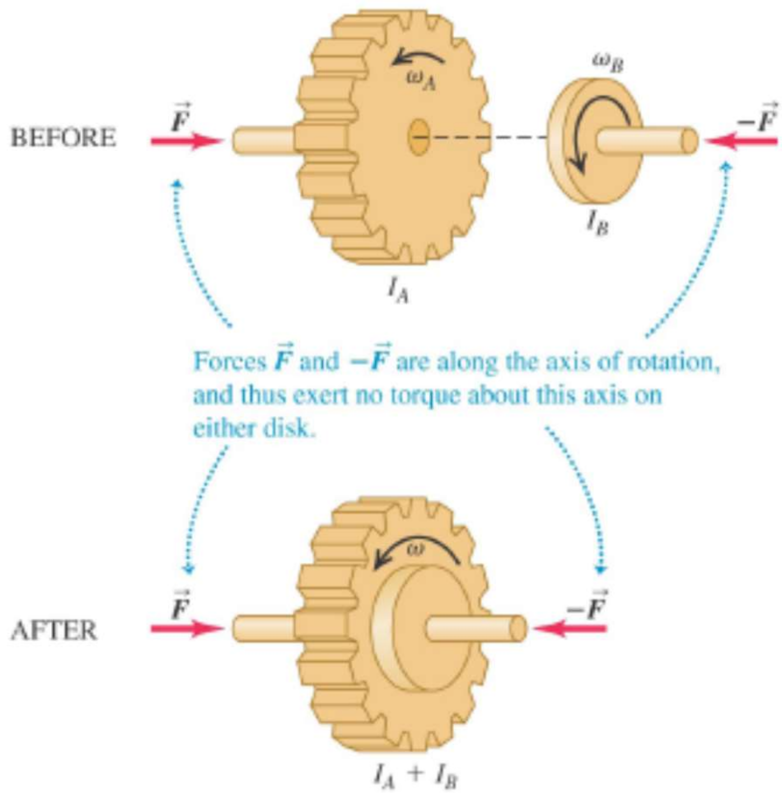
How a car's clutch work

The clutch disk and the gear disk is pushed into each other by two forces that do not impart any torque, what is the final angular velocity when they come together?

$$L_{z-before} = L_{z-after}$$

$$I_A\omega_A + I_B\omega_B = (I_A + I_B)\omega_{final}$$

$$\Rightarrow \quad \omega_{final} = \frac{I_A\omega_A + I_B\omega_B}{(I_A + I_B)}$$



Angular momentum conservation in collisions

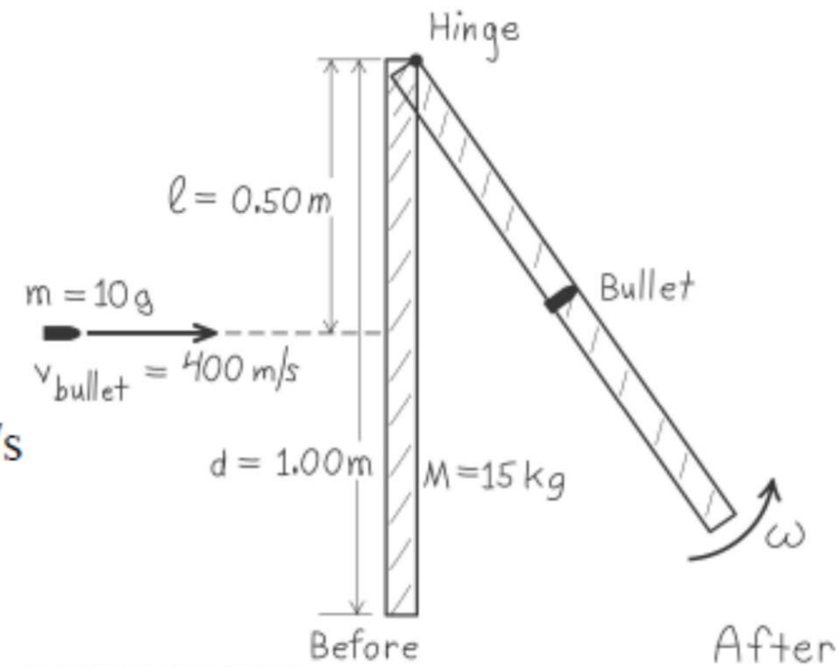
A door 1.00 m wide, of mass 15 kg, is hinged at one side so that it can rotate without friction about a vertical axis. It is unlatched. A police officer fires a bullet with a mass of 10 g and a speed of 400 m/s into the exact center of the door, in a direction perpendicular to the plane of the door. Find the angular speed of the door just after the bullet embeds intelf in the door.

$$L_{z-before} = L_{z-after}$$

$$m_B v_B l = (I_{door} + m_B l^2)\omega_f$$

$$m_B v_B l = (\frac{1}{3}Md^2 + m_B l^2)\omega_f$$

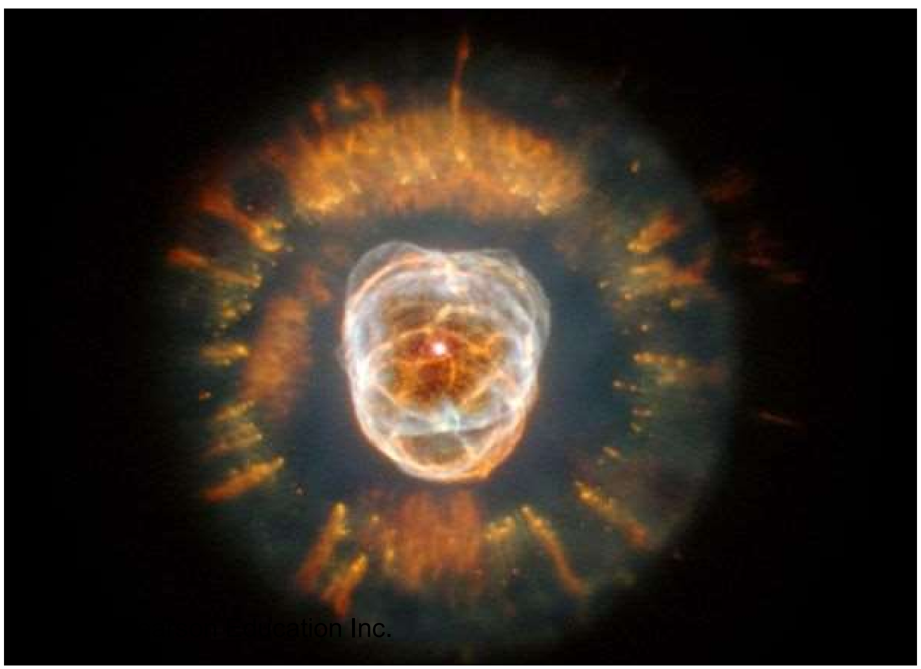
$$\Rightarrow \quad \omega_f = \frac{m_B v_B l}{(\frac{1}{3}Md^2 + m_B l^2)} = 0.4 \text{ rad/s}$$



Supernova and Neutron stars

Exercise 10.39

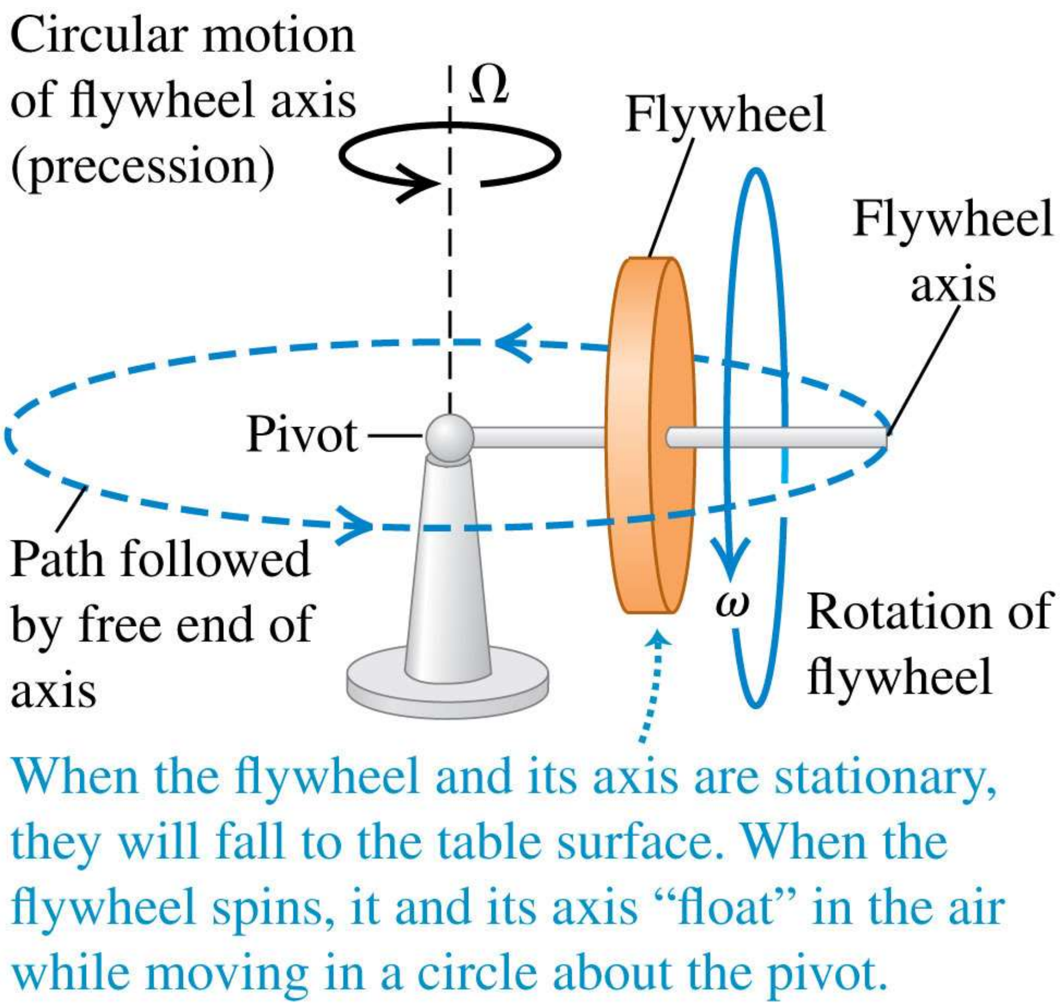
Under some circumstances, a star can collapse into an extremely dense object made mostly of neutrons and called a neutron star. The density of a neutron star is roughly 10^{14} times as great as that of ordinary solid matter. Suppose we represent the star as a uniform, solid, rigid sphere, both before and after the collapse. The star's initial radius was 6.0×10^5 km (comparable to our sun); its final radius is 17 km. If the original star rotated once in 32 days, find the angular speed of the neutron star.



Twice as big as Sun, but as small as campus perimeter

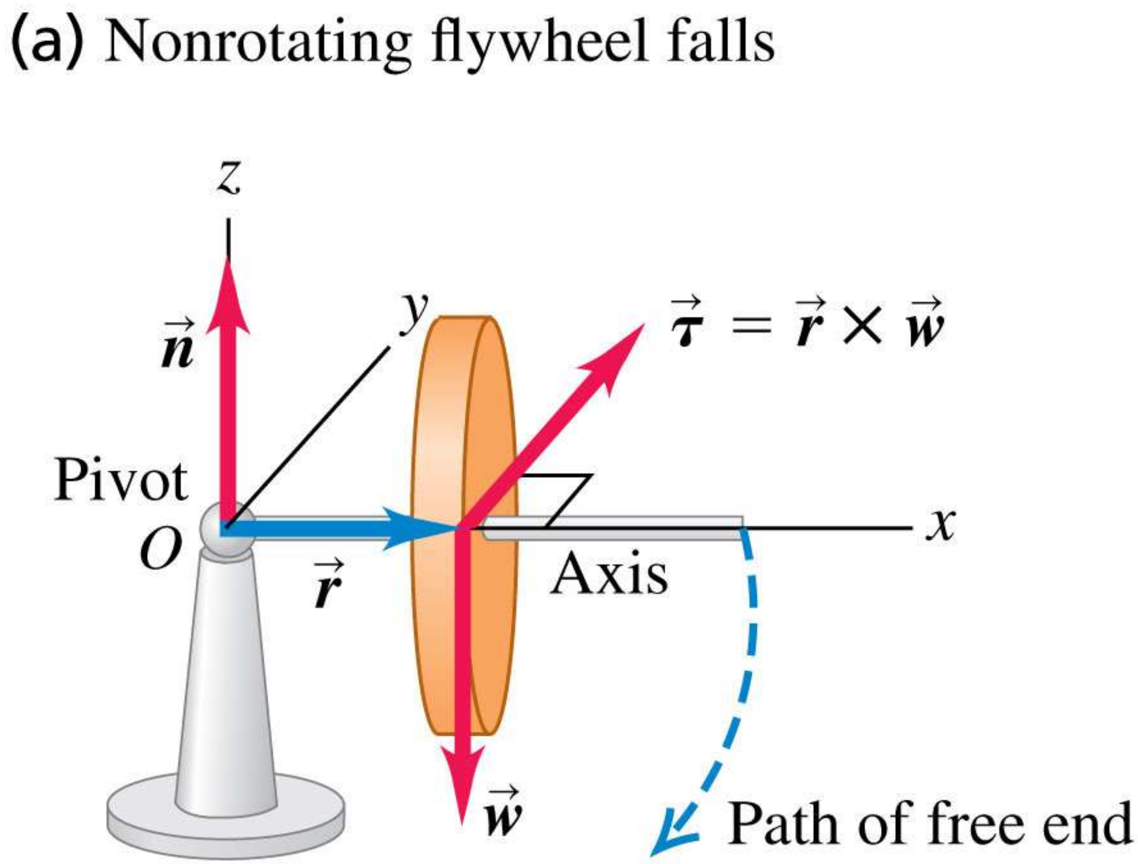
Gyroscopes and precession

- For a gyroscope, the axis of rotation changes direction.
- The motion of this axis is called **precession**.



Gyroscopes and precession

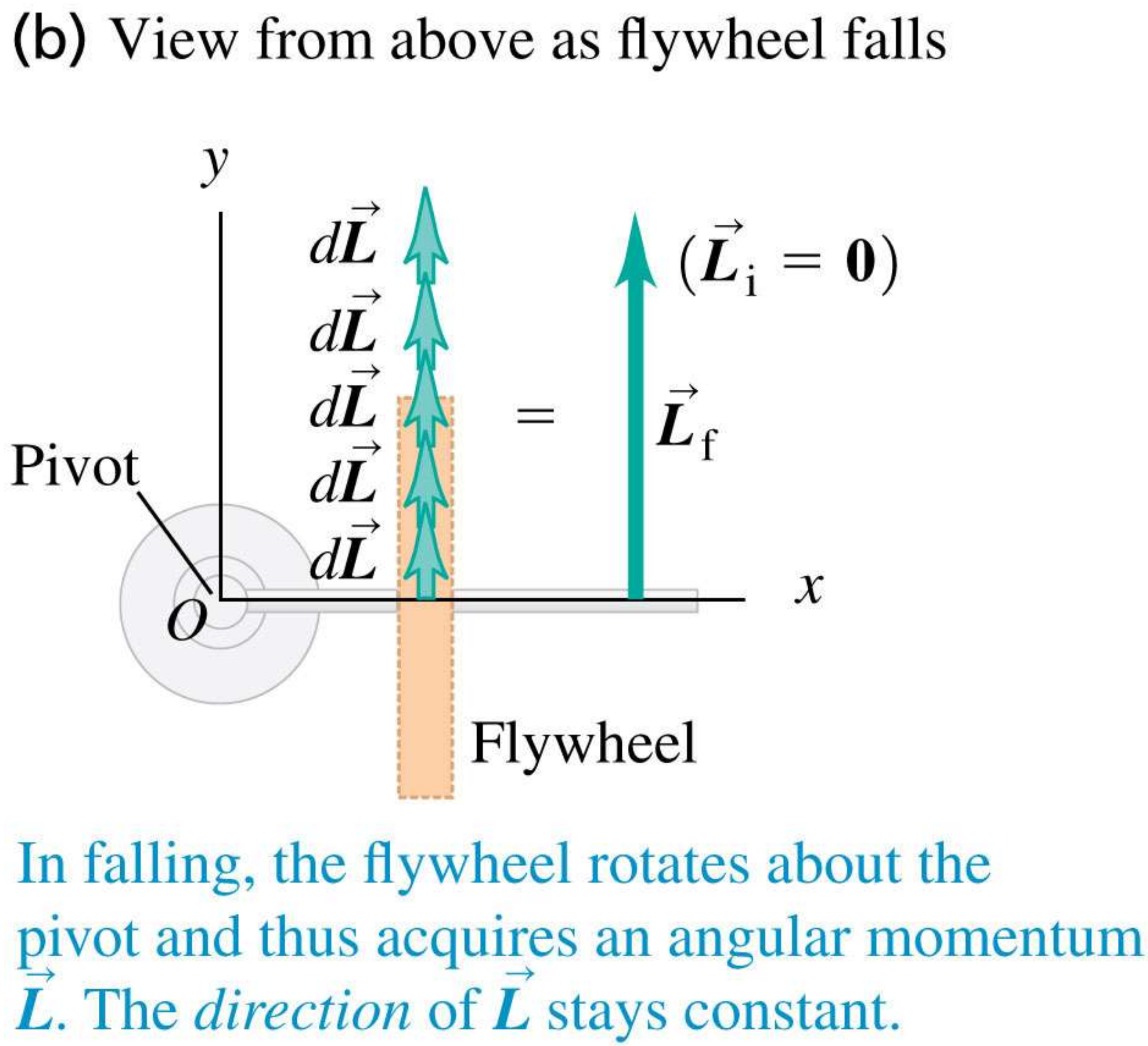
- If a flywheel is initially not spinning, its initial angular momentum is zero.



When the flywheel is not rotating, its weight creates a torque around the pivot, causing it to fall along a circular path until its axis rests on the table surface.

Gyroscopes and precession

- In each successive time interval dt, the torque produces a change in the angular momentum in the same direction as the torque, and the flywheel axis falls.

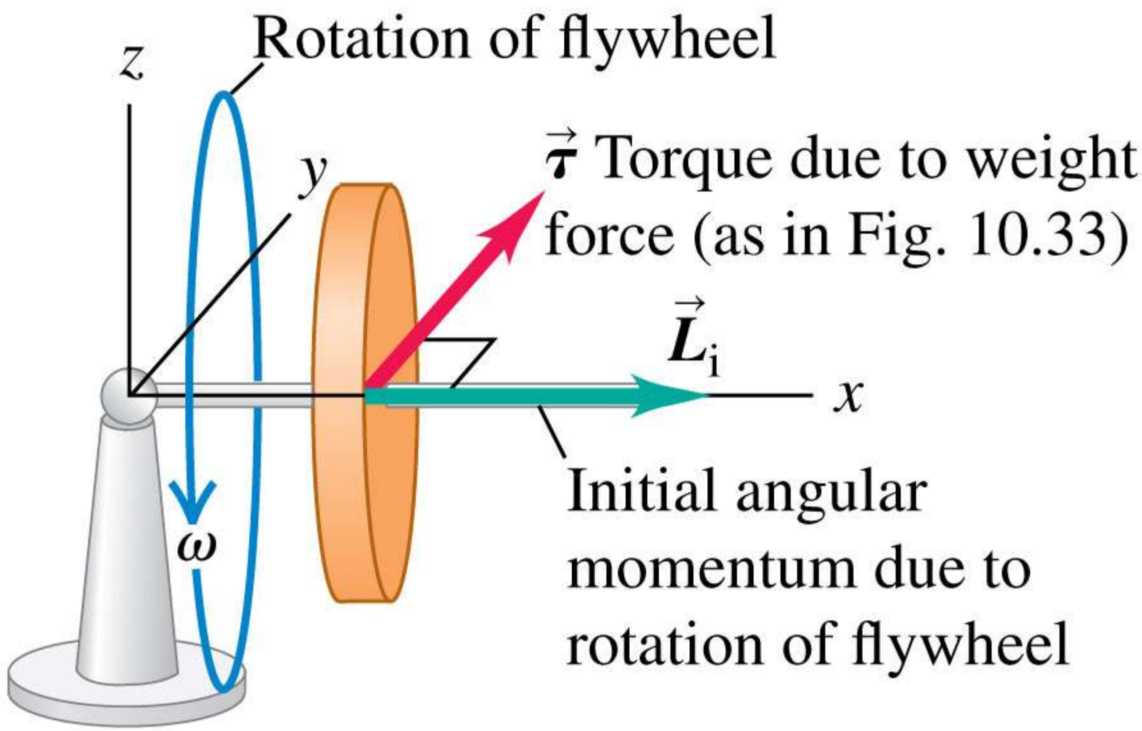


A rotating flywheel

- This flywheel is initially spinning, with a large angular momentum.

(a) Rotating flywheel

When the flywheel is rotating, the system starts with an angular momentum $\vec{L}_i$ parallel to the flywheel's axis of rotation.



A rotating flywheel

- Because the initial angular momentum is not zero, each change in angular momentum is perpendicular to the angular momentum.
- As a result, the magnitude L remains the same but the angular momentum changes its direction continuously.