Introduction to Projectile Motion

Description: Conceptual questions about projectile motion and some easy calculations. (uses applets)

Learning Goal:

To understand the basic concepts of projectile motion.

Projectile motion may seem rather complex at first. However, by breaking it down into components, you will find that it is really no different than the one-dimensional motions that you have already studied.

One of the most often used techniques in physics is to divide two- and three-dimensional quantities into components. For instance, in projectile motion, a particle has some initial velocity \( \vec{v} \). In general, this velocity can point in any direction on the \( xy \) plane and can have any magnitude. To make a problem more manageable, it is common to break up such a quantity into its \( x \) component \( v_x \) and its \( y \) component \( v_y \).

Consider a particle with initial velocity \( \vec{v} \) that has magnitude 12.0 m/s and is directed 60.0 degrees above the negative \( x \) axis.

Part A

What is the \( x \) component \( v_x \) of \( \vec{v} \)?

Express your answer in meters per second.

ANSWER:

\[ v_x = -6.00 \text{ m/s} \]

Part B

What is the \( y \) component \( v_y \) of \( \vec{v} \)?

Express your answer in meters per second.

ANSWER:

\[ v_y = 10.4 \text{ m/s} \]

Breaking up the velocities into components is particularly useful when the components do not affect each other. Eventually, you will learn about situations in which the components of velocity do affect one another, but for now you will only be looking at problems where they do not. So, if there is acceleration in the \( x \) direction but not in the \( y \) direction, then the \( x \) component of the velocity will change, but the \( y \) component of the velocity will not.
Part C

Look at this applet. The motion diagram for a projectile is displayed, as are the motion diagrams for each component. The \( x \)-component motion diagram is what you would get if you shined a spotlight down on the particle as it moved and recorded the motion of its shadow. Similarly, if you shined a spotlight to the left and recorded the particle's shadow, you would get the motion diagram for its \( y \) component. How would you describe the two motion diagrams for the components?

ANSWER:

- Both the vertical and horizontal components exhibit motion with constant nonzero acceleration.
- The vertical component exhibits motion with constant nonzero acceleration, whereas the horizontal component exhibits constant-velocity motion.
- The vertical component exhibits constant-velocity motion, whereas the horizontal component exhibits motion with constant nonzero acceleration.
- Both the vertical and horizontal components exhibit motion with constant velocity.

As you can see, the two components of the motion obey their own independent kinematic laws. For the vertical component, there is an acceleration downward with magnitude \( g = 10 \text{ m/s}^2 \). Thus, you can calculate the vertical position of the particle at any time using the standard kinematic equation \( y = y_0 + v_0 t + \frac{1}{2} a t^2 \). Similarly, there is no acceleration in the horizontal direction, so the horizontal position of the particle is given by the standard kinematic equation \( x = x_0 + v_0 t \).

Part D

How long \( t_g \) does it take for the balls to reach the ground? Use \( 10 \text{ m/s}^2 \) for the magnitude of the acceleration due to gravity.

Express your answer in seconds to two significant figures.

**Hint 1. How to approach the problem**

The balls are released from rest at a height of 5.0 m at time \( t = 0 \text{ s} \). Using these numbers and basic kinematics, you can determine the amount of time it takes for the balls to reach the ground.

ANSWER:

\[ t_g = 1.0 \text{ s} \]

This situation, which you have dealt with before (motion under the constant acceleration of gravity), is actually a special case of projectile motion. Think of this as projectile motion where the horizontal component of the initial velocity is zero.

Part E

Imagine the ball on the left is given a nonzero initial speed in the horizontal direction, while the ball on the right continues to fall with zero initial velocity. What horizontal speed \( v_x \) must the ball on the left start with so that it hits the ground at the same position as the ball on the right? Remember that the two balls are released, starting a horizontal distance of 3.0 m apart.
Express your answer in meters per second to two significant figures.

**Hint 1. How to approach the problem**

Recall from Part B that the horizontal component of velocity does not change during projectile motion. Therefore, you need to find the horizontal component of velocity $v_x$ such that, in a time $t_g = 1.0 \text{ s}$, the ball will move horizontally $3.0 \text{ m}$. You can assume that its initial $x$ coordinate is $x_0 = 0.0 \text{ m}$.

**ANSWER:**

$v_x = 3.0 \text{ m/s}$

You can adjust the horizontal speeds in this applet. Notice that regardless of what horizontal speeds you give to the balls, they continue to move vertically in the same way (i.e., they are at the same $y$ coordinate at the same time).

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**Exercise 3.8**

**Description:** A remote-controlled car is moving in a vacant parking lot. The velocity of the car as a function of time is given by $\vec{v} = (5.00 \text{ m/s}) - (0.0180 \text{ m/s}^3 \cdot t^2) \hat{i} + (2.00 \text{ m/s}) + (0.550 \text{ m/s}^2 \cdot t) \hat{j}$. (a) What is $a_x(t)$ the $x$-component...

A remote-controlled car is moving in a vacant parking lot. The velocity of the car as a function of time is given by $\vec{v} = [5.00 \text{ m/s} - (0.0180 \text{ m/s}^3 \cdot t^2) \hat{i} + [2.00 \text{ m/s} + (0.550 \text{ m/s}^2 \cdot t) \hat{j}$.

**Part A**

What is $a_x(t)$ the $x$-component of the acceleration of the car as function of time?

**ANSWER:**

- $a_x(t) = (-0.0360 \text{ m/s}^3) t$
- $a_x(t) = (-0.0180 \text{ m/s}^3) t$
- $a_x(t) = (0.0360 \text{ m/s}^3) t$

**Part B**

What is $a_y(t)$ the $y$-component of the acceleration of the car as function of time?

**ANSWER:**

- $a_y(t) = 2.00 \text{ m/s}^2$
- $a_y(t) = 0.550 \text{ m/s}^2$
- $a_y(t) = (-0.550 \text{ m/s}^2) t$
Part C
What is the magnitude of the velocity of the car at $t = 7.12\text{s}$?
Express your answer to three significant figures and include the appropriate units.
ANSWER:
$$v = \sqrt{(5 - 0.018t)^2 + (2 + 0.55t)^2} = 7.19 \frac{\text{m}}{\text{s}}$$

Part D
What is the direction (in degrees counterclockwise from +x-axis) of the velocity of the car at $t = 7.12\text{s}$?
Express your answer to three significant figures and include the appropriate units.
ANSWER:
$$\theta_v = \frac{\tan^{-1}\left(\frac{2+0.55}{5-0.018}\right) \cdot 180}{\pi} = 55.4^\circ$$

Part E
What is the magnitude of the acceleration of the car at $t = 7.12\text{s}$?
Express your answer to three significant figures and include the appropriate units.
ANSWER:
$$a = \sqrt{(-0.036)^2 + 0.55^2} = 0.607 \frac{\text{m}}{\text{s}^2}$$

Part F
What is the direction (in degrees counterclockwise from +x-axis) of the acceleration of the car at $t = 7.12\text{s}$?
Express your answer to three significant figures and include the appropriate units.
ANSWER:
$$\theta_a = 90 + \frac{\tan^{-1}\left(\frac{0.036}{0.55}\right) \cdot 180}{\pi} = 115^\circ$$

Exercise 3.9
Description: A physics book slides off a horizontal table top with a speed of $v$. It strikes the floor after a time of $t$. Ignore air resistance. (a) Find the height of the table top above the floor. (b) Find the horizontal distance from the edge of the table to...
A physics book slides off a horizontal table top with a speed of 1.25 m/s. It strikes the floor after a time of 0.380 s. Ignore air resistance.

**Part A**
Find the height of the table top above the floor.

**ANSWER:**

\[ y = \frac{9.80t^2}{2} = 0.708 \text{ m} \]

**Part B**
Find the horizontal distance from the edge of the table to the point where the book strikes the floor.

**ANSWER:**

\[ x = vt = 0.475 \text{ m} \]

**Part C**
Find the horizontal component of the book's velocity just before the book reaches the floor.

**ANSWER:**

\[ v_x = v = 1.25 \text{ m/s} \]

**Part D**
Find the vertical component of the book's velocity just before the book reaches the floor.

**ANSWER:**

\[ v_y = -9.80t = -3.72 \text{ m/s} \]

**Part E**
Find the magnitude of the book's velocity just before the book reaches the floor.

**ANSWER:**

\[ v = \sqrt{v^2 + (9.80t)^2} = 3.93 \text{ m/s} \]
Part F

Find the direction of the book's velocity just before the book reaches the floor.

Express your answer as an angle measured below the horizontal

ANSWER:

\[
\theta = \tan^{-1}\left(\frac{9.80\text{ m/s}}{180\text{ m/s}}\right) = 71.4^\circ \text{ below the horizontal}
\]
How high is this point?

**Express your answer using three significant figures.**

**ANSWER:**

\[ h = \frac{v_0^2}{2g} = 13.4 \text{ m} \]

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**Part C**

How much time (after it is thrown) is required for the football to return to its original level?

**Express your answer using three significant figures.**

**ANSWER:**

\[ t_2 = \frac{2v_0}{g} = 3.31 \text{ s} \]

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**Part D**

How does this compare with the time calculated in part (a).

**Express your answer using three significant figures.**

**ANSWER:**

\[ \frac{t_2}{t_1} = 2.00 \]

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**Part E**

How far has it traveled horizontally during this time?

**Express your answer using three significant figures.**

**ANSWER:**

\[ x = \frac{v_0 \cdot 2v_0}{g} = 62.5 \text{ m} \]

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**Exercise 3.28**

**Description:** The radius of the earth's orbit around the sun (assumed to be circular) is \(1.50 \times 10^8\) km, and the earth travels around this orbit in 365 days. (a) What is the magnitude of the orbital velocity of the earth in m/s? (b) What is the radial...

The radius of the earth's orbit around the sun (assumed to be circular) is \(1.50 \times 10^8\) km, and the earth travels around this orbit in 365 days.
Part A
What is the magnitude of the orbital velocity of the earth in m/s?
ANSWER:
2.97 x 10^4 m/s

Part B
What is the radial acceleration of the earth toward the sun?
ANSWER:
5.91 x 10^{-3} m/s^2

Part C
What is the magnitude of the orbital velocity of the planet Mercury (orbit radius = 5.79 x 10^7 km, orbital period = 88.0 days)?
ANSWER:
4.78 x 10^4 m/s

Part D
What is the radial acceleration of the Mercury?
ANSWER:
3.95 x 10^{-2} m/s^2

Exercise 3.29
Description: A Ferris wheel with radius 14.0 m is turning about a horizontal axis through its center (the figure). The linear speed of a passenger on the rim is constant and equal to v. (a) What is the magnitude of the passenger's acceleration as she passes...

A Ferris wheel with radius 14.0 m is turning about a horizontal axis through its center (the figure). The linear speed of a passenger on the rim is constant and equal to 7.92 m/s.
Part A

What is the magnitude of the passenger's acceleration as she passes through the lowest point in her circular motion?

ANSWER:

\[ a = \frac{v^2}{r} = 4.48 \text{ m/s}^2 \]

Part B

What is the direction of the passenger's acceleration as she passes through the lowest point in her circular motion?

ANSWER:

- towards the center
- outwards the center

Part C

What is the magnitude of the passenger's acceleration as she passes through the highest point in her circular motion?

ANSWER:

\[ a = \frac{v^2}{r} = 4.48 \text{ m/s}^2 \]

Part D

What is the direction of the passenger's acceleration as she passes through the highest point in her circular motion?
Part E

How much time does it take the Ferris wheel to make one revolution?

ANSWER:

\[ T = \frac{2\pi \cdot 14}{v} = 11.1 \text{ s} \]

Exercise 3.34

**Description:** Two piers, A and B, are located on a river: B is 1500 m downstream from A. Two friends must make round trips from pier A to pier B and return. One rows a boat at a constant speed of 4.00 km/h relative to the water; the other walks on the shore at a constant speed of 4.00 km/h. The velocity of the river is 2.80 km/h in the direction from A to B.
Part A

How much time does it take the walker to make the round trip?

Express your answer using two significant figures.

ANSWER:

$t = 45$ min

Part B

How much time does it take the rower to make the round trip?

Express your answer using two significant figures.

ANSWER:

$t = 88$ min

Exercise 3.38

Description: An airplane pilot wishes to fly due west. A wind of $v_w$ is blowing toward the south. (a) If the airspeed of the plane (its speed in still air) is $v_a$, in which direction should the pilot head? (b) What is the speed of the plane over the ground?

An airplane pilot wishes to fly due west. A wind of $82.0 \text{ km/h}$ is blowing toward the south.

Part A

If the airspeed of the plane (its speed in still air) is $310.0 \text{ km/h}$, in which direction should the pilot head?

Express your answer as an angle measured north of west.

ANSWER:
Part B

What is the speed of the plane over the ground?

ANSWER:

\[ v = \sqrt{v_a^2 - v_w^2} = 299 \text{ km/h} \]

Problem 3.52

Description: A movie stuntwoman drops from a helicopter that is 30.0 m above the ground and moving with a constant velocity whose components are 10.0 m/s upward and 15.0 m/s horizontal and toward the south. You can ignore air resistance. (a) What is the...

A movie stuntwoman drops from a helicopter that is 30.0 m above the ground and moving with a constant velocity whose components are 10.0 m/s upward and 15.0 m/s horizontal and toward the south. You can ignore air resistance.

Part A

What is the horizontal distance (relative to the position of the helicopter when she drops) at which the stuntwoman should have placed the foam mats that break her fall?

ANSWER:

\[ x = 55.5 \text{ m} \]

Part B

Draw \( x - t \) graph of her motion.

ANSWER:
Part C

Draw $y - t$ graph of her motion.

ANSWER:
Part D

Draw $v_x - t$ graph of her motion.

ANSWER:

Part E

Draw $v_y - t$ graph of her motion.

ANSWER:
Problem 3.56

Description: As a ship is approaching the dock at 45.0 cm/s, an important piece of landing equipment needs to be thrown to it before it can dock. This equipment is thrown at 15.0 m/s at 60.0 degree(s) above the horizontal from the top of a tower at the edge of the water, 8.75 m above the ship's deck (the figure).

Part A

For this equipment to land at the front of the ship, at what distance $D$ from the dock should the ship be when the equipment is thrown? Air resistance can be neglected.
Problem 3.68

Description: A rock is thrown from the roof of a building with a velocity $v_0$ at an angle of $\alpha_0$ from the horizontal. The building has height $h$. You can ignore air resistance. (a) Calculate the magnitude of the velocity of the rock just before it strikes the...

A rock is thrown from the roof of a building with a velocity $v_0$ at an angle of $\alpha_0$ from the horizontal. The building has height $h$. You can ignore air resistance.

Part A

Calculate the magnitude of the velocity of the rock just before it strikes the ground.

ANSWER:

$$v = \sqrt{v_0^2 + 2gh}$$

Problem 3.81

Description: An airplane pilot sets a compass course due west and maintains an airspeed of $v$. After flying for a time of $t$, she finds herself over a town a distance $x$ west and a distance $y$ south of her starting point. (a) Find the magnitude of the wind velocity. ...

An airplane pilot sets a compass course due west and maintains an airspeed of $223\, \text{km/h}$. After flying for a time of $0.520\, \text{h}$, she finds herself over a town a distance $119\, \text{km}$ west and a distance $20\, \text{km}$ south of her starting point.

Part A

Find the magnitude of the wind velocity.

ANSWER:

$$v = \sqrt{\left(\frac{v}{t}\right)^2 + \left(\frac{x-vt}{t}\right)^2} = 38.9\, \text{km/h}$$

Part B

Find the direction of the wind velocity.

Express your answer as an angle measured south of west

ANSWER:
Part C

If the wind velocity is 39 km/h due south, in what direction should the pilot set her course to travel due west? Use the same airspeed of 223 km/h.

Express your answer as an angle measured north of west

\[ \theta = \frac{\pi - \pi}{\pi} \cdot 180 = 81.4^\circ \text{ south of west} \]

\[ \theta = \frac{\pi - \pi}{\pi} \cdot 180 = 10.1^\circ \text{ north of west} \]

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