Comparison of Measured Value and Known/Expected Value

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Standard Equivalency Test

• How consistent is a measured value A with a known/expected/benchmark value B?
• Usually, if the measured value is within $2-\sigma$ of the expected value, then it is consistent
• Which $\sigma$ to use? Usually, it is $\sigma_A$, that is your uncertainty, if the expected value has no error
• Standard Equivalency Test:
• $|A-B| < 2 \sigma_A$
Both Measured and Standard Values Have Uncertainties

• If B also has uncertainty, then the comparison needs to include both errors

• $|A-B| < 2 \sqrt{(\sigma_A^2 + \sigma_B^2)}$
Propagation of Errors – Addition and Subtraction

• Square of the uncertainty of the sum or quantities is the sum of the square of the uncertainties

• $\sigma_{AB}^2 = (\sigma_A^2 + \sigma_B^2)$
Propagation of Errors – Multiplication and Division

• Relative uncertainty in a product or quotient is the square root of the sum of the squares of the relative uncertainties

• \[ \sigma_{A*B}^2/(A*B)^2 = ((\sigma_A/A)^2 + (\sigma_B/B)^2) \]

• Multiplication or division by a constant (k) doesn’t change the relative error

• \[ \sigma_{kA} = k \sigma_A \]
Propagation of Errors – Exponent

• Correlated errors in A and A will not allow the product rule for calculation of error on \( A^2 \)
• Relative error on A raised to an exponent n is exponent times the relative error
• n can be any number, not just integers!
• \( \sigma_A^n/A^n = n \sigma_A^n/A^n \)
• So, \( \sigma_A^n = n A^{n-1} \sigma_A \)
More complicated Cases – Chain Rule

• Apply the above rules, treating each function as a variable and keep applying the rules until the individual variables are obtained

• This is just like in Calculus

• \( f(A, B) \approx f(A_{\text{exp}}, B_{\text{exp}}) + \frac{\partial f}{\partial A}|_{A_{\text{exp}}, B_{\text{exp}}} + \frac{\partial f}{\partial A}|_{A_{\text{exp}}, B_{\text{exp}}} + \frac{\partial f}{\partial B}|_{A_{\text{exp}}, B_{\text{exp}}} \)

• Hence, \( \sigma(f) = \sqrt{(\frac{\partial f}{\partial A})^2 \sigma^2(A) + (\frac{\partial f}{\partial B})^2 \sigma^2(B)} \)

• All of the error propagation rules can be derived through calculus