Measuring the Moon’s angular size  
(due October 29)

Angles, angular sizes, and angular separation

The apparent positions and separations of objects in the sky are not determined by the linear distance between two objects, but by their angular separation. For example, the angular distance from the horizon to directly overhead, the zenith, is 90°. Not only can we measure the angular separation between two objects, but we can also measure the angular size of an object such as the Moon, which is about 0.5°.

Since the Moon or the Sun isn’t always available to use as a basis to measure angular sizes, here are a few handy rules that you should remember. With your arm outstretched one finger-width is 2°, one fist-width is 10°, one hand span 20°. This technique works because an angle depends on the ratio of the linear size (d) of the object to the distance (D) to the object. Since the ratio of finger size to arm size is approximately the same for different people, most people have a finger width of ~ 2° no matter what the length of their arms.

When you start to measure angular sizes, you will soon realize that there is a relation between the angular size (a, in degrees) of an object and the distance to that object. If the linear size of an object (in meters, say) is much smaller than its distance (in meters, once again), then the appropriate “small angle formula” is:

\[
a(\text{deg}) = 57.3°(d/D) .
\] (1)

If angle a is smaller than one degree, we might express it in terms of arc minutes. Since 1 degree equals 60 arc minutes,

\[
a(\text{arcmin}) = 3438'(d/D) .
\] (2)

If angle a is greater than 10 degrees, then it becomes necessary to use a trigonometric formula:

\[
a(\text{deg}) = 2 \times \tan^{-1}(d/2D) ,
\] (3)
where \( \tan^{-1} \) is the arc tangent function on your calculator.
Similar triangles

Consider plane triangles ABC and ADE in Fig. 1. Since BC is parallel to DE, these two triangles are shaped exactly the same. The interior angles of one triangle are identical to the corresponding interior angles of the other. Also, the lengths of the sides are proportional. In other words, BC/DE = AB/AD = AC/AE.

Angle CAB is the same as angle EAD. We can use the properties of similar triangles to measure the angular size of the Moon. The Moon’s diameter is 3476.4 km and its mean distance is 384,401 km. Using Eq. 2, it follows the Moon’s mean angular size is 31.09 arc minutes.

The cross-staff

The cross-staff is a simple device, used for centuries before the invention of the telescope to determine the angular separation between two objects, or the angular size of a single object. If you are measuring a single object, the cross-staff can be used to determine its linear size or distance. The cross-staff consists of a yardstick and a crosspiece attached at a right angle. All you have to do is sight an object along the stick and slide the crosspiece back and forth (or change your own distance from the object) until the apparent or angular size of the crosspiece (or some portion of it) matches the apparent or angular size of the distance object.

If $D'$ is the “working length” of the cross-staff and $d'$ is the diameter of a sighting hole, then the angular size of the hole is $a' = 57.3^\circ (d'/D')$. But this value of $a'$ can also be the angular size of the Moon, for example. If you know the distance $D$ to the Moon, you can derive its diameter $d$. In other words, the cross-staff works according to the principle of similar triangles:

$$\frac{d'}{D'} = \frac{d}{D}$$

Since we are only going to measure the angular size of the moon, we can make a very simple device.
Figure 2. A Moon measuring device, made of the bottom of a box of pancake mix. It slides up and down a yardstick calibrated in inches on one side and millimeters on the other.

Take a small empty rectangular box like one that had cake mix, pancake mix, corn bread mix, or cream of wheat in it. Cut off the bottom inch of the box. That’s what you want to keep. Then cut a slot at the bottom of the middle of each of the longer sides. Make these slots just the same size as the height and width of a yardstick that you happen to have. An example is shown in Fig. 2. If done right, you will have something that slides snugly up and down a yardstick.

Finally, take a thin piece of cardboard about 1 by 2 inches and, using a hole punch, make a nice, neat hole about 1 1/4 inches from the middle of one of the shorter sides. Tape this to the box bottom that slides up and down the yardstick.

To do

• Make a Moon measuring device, as described above.
• This project can and should be done with one (1) partner. But if you want to work alone, that’s OK too.
• Using a millimeter ruler, as accurately as you can measure the diameter \(d'\), in millimeters, of your 1/4 inch hole that will be used for sighting.
• Cut out a nice neat circular disk of diameter \(d = 91 \text{ mm}\). If it’s not exactly 91 mm that’s OK. If it is 92 mm or 90 mm (or whatever), just use that value for any subsequent calculation. What we’re doing is calculating a correction factor for measuring something about half a degree in diameter.
• Tape the 91 mm disk to a door or wall at your eye level, and measure off 10 meters (= 10,000 millimeters = 32 feet 9.7 inches) from the location of the disk. Put a piece of tape on the floor at distance \(D = 10 \text{ meters}\).
• Using Eq. 4, estimate how many millimeters along the ruler \( (D') \) the sighting hole must be to match the angular size of the 91 mm disk which is at a distance of 10 meters. Round off this number to the nearest millimeter.
• Now, put your toes just behind the 10 meter line, put one end of the yardstick up against your cheekbone under your better eye and slide the cross staff to the position where the angular size of the sighting hole matches, as accurately as possible, the angular size of the 91 mm circle 10 meters away. How many mm down the yardstick did the sighting hole have to be? Kevin Krisciunas found that his 6.2 mm sighting hole had to be 821 mm from his eye. Your eye could be very different, so that’s no guarantee you will find a similar value.
• Using Eq. 2, calculate the angular size of the 91 mm circle at a distance of 10 meters. Round this off to the nearest 0.1 arc minute. We’ll call this the \textit{true} angular size of the circle.
• Using Eq. 2, calculate the angular size (in arc minutes) of the sighting hole at the best-measured-distance-down-the-yardstick to match the angular size of the circle on the wall 10 meters away. We’ll call this the \textit{measured} angular size of the circle.
• Take the ratio of these two angular sizes (”true” divided by “measured”). We’ll call this “parameter C”. How close is the number to 1.000? What was Krisciunas’s value?
• Now go outside on a reasonably clear night when the Moon is up and measure the angular size of the Moon using the same sighting hole and yardstick. Make sure you write down the double date (like 3/4 February 2010), and the time. How close to 31 arc minutes did you get?
• If you multiply your measured angular size of the Moon by your “parameter C”, what is the corrected measurement of the angular size of the Moon?
• OK, now consider the data set given in Table 1. These are measures relating to the angular size of the Moon during September, October, and November of 2009, taken by Kevin Krisciunas.
• Assuming that his sighting hole was indeed 6.2 mm in size, calculate the observed angular sizes of the Moon using the data set. The first calculation has been done for you. What is the average of the values? How close to 31.1 arcmin is it?
• Given that Krisciunas found a systematic difference between the “expected” and the measured angular size of the 91 mm disk, shouldn’t a correction factor be applied (by multiplication) to the “observed” values in the table?
• Multiply the observed angular sizes in the table by some appropriate factor to get corrected values. Now, what is the average value? Close to the “right answer”?
• Make a graph of the corrected angular sizes of the Moon from the table vs. Julian Date. The X-axis could range from about 5090 to 5160 days. The true period of these variations is 27.55 days, which is known as the \textit{anomalistic month}. It is the time from the maximum angular motion of the Moon till the next time of maximum angular motion. (Today, we’d say from perigee to perigee.) This was known to the Babylonians more than 2500 years ago.
• Draw in a freehand sine wave through the data obtained during two cycles of the Moon’s orbit. What is the mean scatter of the data points from the “true” values in the last column of the table? Based on the corrected measures, what is the range of angular size of the Moon? The true eccentricity of the Moon’s orbit is 0.055, which means that the Moon’s distance ranges \pm 5.5 percent (to a first approximation).
Curiously, none of the great pre-telescopic observers (Hipparchus, Ptolemy, Tycho) made a set of observations like those presented here. Using very simple equipment, did they have the means to measure the range of the distance of the Moon?
Extra Credit

- Many people claim that the full or nearly-full Moon close to the horizon is considerably bigger in angular size than it is several hours later when it is high in the sky. Make and report observations and calculations that confirm or refute this.

- Make a series of observations of the Moon over the course of the semester and see if you can measure the range of its angular size. In other words, generate your own table of data like Table 1, along with a graph of your data showing the degree of success you achieved in demonstrating the varying angular size. The due date for this extra credit is December 6.

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<sup>a</sup> Julian Date minus 2,450,000.

<sup>b</sup> Time since the previous new Moon.

<sup>c</sup> Distance from the eye of the 6.2 mm sighting hole.

<sup>d</sup> Geocentric angular diameter of the Moon, interpolated from data given in the Astronomical Almanac.