Mid 18th century, Kant and Wright suggested that the Milky Way is a finite system of stars. Turns out this is accurate.

Kant went on to suggest that the very faint “elliptical nebulae” might be similar collections of stars, well beyond the boundaries of the Milky Way. He called these Island Universes.
Charles Messier cataloged 103 “fuzzy” objects while looking for comets. This is still referred to as the Messier Catalog. Objects are given as “M#”, such as those below. It contains planetary nebulae, spiral and elliptical nebulae.
Charles Messier cataloged 103 “fuzzy” objects while looking for comets. This is still referred to as the **Messier Catalog**. Objects are given as “M#”, such as those below. It contains planetary nebulae, spiral and elliptical nebulae.
Cataloging “Nebulae”

Herschel expanded the cataloging of nebulae, succeeded by his son, Sir John Herschel, included objects in the Southern Hemisphere (viewed from South Africa).

Their “New General Catalog” (NGC) contains nearly 8000 objects, but the nature of these objects was an open question.
Edwin Hubble arranged a sequence of Galaxies on a tuning fork diagram. Originally he (incorrectly) hypothesized that galaxies evolved from the Left to the Right of this sequence.

Hubble went on in his paper “Extra-Galactic Nebulae” to propose that galaxies (nebulae) be classified as ellipticals, spirals, and irregulars. This is today known as the Hubble Sequence.
Hubble subdivided spiral sequences into Sa, Sab, Sb, Sbc, Sc (Scd, Sd), and SBa, SBab, SBb, SBbc, SBc (SBcd, SBd). Two characteristics dictate this (1) the bulge-to-disk ratio and (2) how tightly wound the spiral arms are. Spirals with high bulge-to-disk ratios ($L_{\text{bulge}}/L_{\text{disk}} > 0.3$) and tightly wound arms are the “a” subclass. The lower sequence has nuclear “bars”.
Hubble subdivided spiral sequences into Sa, Sab, Sb, Sbc, Sc (Scd, Sd), and SBa, SBab, SBB, SBbc, SBc (SBcd, SBd). Two characteristics dictate this (1) the bulge-to-disk ratio and (2) how tightly wound the spiral arms are. Spirals with high bulge-to-disk ratios ($L_{\text{bulge}}/L_{\text{disk}} > 0.3$) and tightly wound arms are the “a” subclass. The lower sequence has nuclear “bars”.

Irregulars: galaxies lacking organized structure.
Irregulars
Rotation Curves of Spiral Galaxies

Spiral galaxies *do rotate*.

Measure “rotation curve” by measuring doppler shifted light from spectral lines as a function of galacticentric distance.
Rotation curve for our Galaxy. Strange thing is.... rotation curve is flat beyond the Solar circle, $R_0 = 8.5$ kpc.

Rotation Curves of Spiral Galaxies


Vera Rubin (b1928)
Responsible for most of the work on the “galaxy rotation rate” problem.
Rotation Curves of Spiral Galaxies

Let Mass of Galaxy have a constant surface density, $\Sigma$, for $r < R$. Velocity is then just from Newton’s Laws:

$$\frac{v^2}{r} = \frac{GM(r)}{r^2}$$

with

$$M(r) = \Sigma A = \Sigma \pi r^2$$

yields

$$\frac{v^2}{r} = \frac{G\Sigma\pi r^2}{r^2}$$

Solving for $v$, gives:

$$v = \sqrt{G\Sigma\pi r} \propto r^{0.5}$$

for $r < R$

For $r > R$, we have:

$$\frac{v^2}{r} = \frac{GM}{r^2} = \frac{G\Sigma\pi R^2}{r^2}$$

Solving for $v$, gives:

$$v = \sqrt{G\Sigma\pi R^2 r^{-1}} \propto r^{-0.5}$$

for $r > R$
Rotation Curves of Spiral Galaxies

$\nu \sim \text{constant } (r^0)$

$v = \sqrt{G \Sigma \pi r} \propto r^{0.5}$

$v = \sqrt{G \Sigma \pi R^2 r^{-1}} \propto r^{-0.5}$
You can work out what the matter density profile should be to match the observed rotation curves of galaxies. Assume it is spherical:

Consider a spherical shell of radius $r$ and thickness $dr$. The mass in the shell is $dM_r$

Take Newton’s laws for the force acting on a particle (a star) in this shell.

\[
\frac{v^2}{r} = \frac{GM_r}{r^2}
\]

rearranging

\[
M_r = \frac{v^2 r}{G}
\]

and differentiating

\[
\frac{dM_r}{dr} = G^{-1}v^2
\]

Let the mass in the shell be

\[
dM_r = \rho dV = \rho (4\pi r^2) dr
\]

Then this leads to:

\[
\frac{dM_r}{dr} = G^{-1}v^2 = 4\pi r^2 \rho
\]
Solving for the density gives

\[
\frac{dM_r}{dr} = G^{-1}v^2 = 4\pi r^2 \rho
\]

\[
\rho(r) = \frac{v^2}{4\pi G r^2}
\]

A slight variation keeps the density from diverging at \( r \to 0 \):

\[
\rho(r) = \frac{\rho_0}{1 + (r/a)^2}
\]

This is the *Dark Matter* distribution in galaxies.
True for the Milky Way and others.
This is the *Dark Matter* distribution in galaxies. True for the Milky Way and others.

\[ \rho(r) = \frac{\rho_0}{1 + \left(\frac{r}{a}\right)^2} \]

Julio Navarro, Carlos Frenk, and Simon White in 1996 ran a series of cold-dark matter computer simulations, and they came up with a “Universal profile” used today:

\[ \rho(r) = \frac{\rho_0}{(r/a)(1 + r/a)^2} \]

This seems valid over an very large range of \( a \) and \( \rho_0 \). For the smallest galaxies to the largest galaxy clusters.
Rotation Curves of Spiral Galaxies

There is a relationship between the Maximum rotation velocity and the Galaxy’s Absolute Magnitude (Luminosity).

\[ \frac{V^2}{R} = \frac{GM}{R} \rightarrow M = \frac{V^2 R}{G} \]
There is a relationship between the Maximum rotation velocity and the Galaxy’s Absolute Magnitude (Luminosity).

This relation is now referred to as the Tully-Fisher Relation, after Brent Tully and Richard Fisher who first determined it in 1977.

They derived:

\[ M_B = -9.95 \log_{10} (v_{\text{max}}) + 3.15 \] (Sa)
\[ M_B = -10.2 \log_{10} (v_{\text{max}}) + 2.71 \] (Sb)
\[ M_B = -11.0 \log_{10} (v_{\text{max}}) + 3.31 \] (Sc)

As with other quantities, this relation is tightened when using Infrared magnitudes:

\[ M_H = -9.50(\log_{10} V_R - 2.50) - 21.67 \]

Elliptical Galaxies

M 87
Giant Elliptical
Mass ~ $3 \times 10^{12} \, M_\odot$
Size is ~10 x diameter of Milky Way
Elliptical Galaxies

Recall that the gravitational potential of a collection of points with total mass $M$ and radius $R$ is:

$$U = -\frac{3}{5} \frac{GM^2}{R}$$

And the Kinetic energy is:

$$K = \frac{1}{2} M v^2 = \frac{3}{2} M \sigma^2$$

Where $\sigma^2$ is the velocity dispersion (average velocity of all particles in all dimensions).

Using the Virial Theorem $2K + U = 0$, and solving for the velocity dispersion:

$$-3M \sigma^2 = -\frac{3}{5} \frac{GM^2}{R}$$

Solving for the Mass gives:

$$M_{\text{Virial}} \approx \frac{5R \sigma^2}{G}$$

What the heck is $\sigma$?

This is the velocity dispersion, which is the average of radial velocities in a galaxy.
Elliptical Galaxies

What the heck is $\sigma$?

This is the velocity dispersion, which is the average of radial velocities in a galaxy.

$$M_{\text{Virial}} \approx \frac{5R\sigma^2}{G}$$

$$\sigma_x = \frac{\Delta \lambda}{\lambda_0} C$$

Normally, measure velocity dispersion in only 1-degree of freedom, so the mass will be:

$$\left( M_{\text{Virial}} \approx \frac{3R\sigma_x^2}{G} \right)$$
Supermassive Blackholes in Galaxies

M 87
Giant Elliptical
Mass $\sim 3 \times 10^{12} M_\odot$
Size is $\sim 10 \times$ diameter of Milky Way
Supermassive Blackholes in Galaxies

M 87
Giant Elliptical
Mass $\sim 3 \times 10^{12} M_\odot$
Size is $\sim 10 \times$ diameter of Milky Way

Jet of material emitted by nucleus
Supermassive Blackhole in M87

Modeling gives Virial Mass of \((3.2+/-0.9) \times 10^9 \, M_\odot\) within \(r < 0.05'' = 3.5\) pc.

(This density is \(1.8 \times 10^7 \, M_\odot \, pc^{-3}\). Recall, the solar neighborhood has \(0.05 \, M_\odot \, pc^{-3}\).)