Ptolemy’s model of the Moon’s motion implied that its distance varies by nearly a factor of two, implying that its angular size should also vary by nearly a factor of two. We present an analysis of 100 naked eye observations of the Moon’s angular size obtained over 1145 days, showing regular variations of at least 3′. Thus, ancient astronomers could have shown that a key implication of Ptolemy’s model was wrong. In modern times we attribute the variation of distance of the Moon to the combined effect of the ellipticity of the Moon’s orbit and the perturbing effect of the Sun on the Earth-Moon system. We show graphically how this affects the ecliptic longitudes and radial distance of the Moon. The longitude and distance “anomalies” are correlated with the Moon’s phase. This is illustrated without any complex equations or geometry.

1 The Angular Size of the Moon

We have known since the time of Aristarchus (3rd century B.C.) that the Moon is about 60 Earth radii distant, and its angular diameter is roughly half a degree. The mean angular size is actually 31.1 arc minutes.

In the *Almagest* Ptolemy gives the correct range of the angular size of the Moon, about 4 arc minutes. His values of the minimum and maximum angular size are about 2 arc minutes too large. The ancient Babylonians and Greeks were more concerned with the direction towards the Moon, not its physical distance. Ptolemy’s model of the motion of the Moon implied that its distance ranged from 33.55 R⊕ (at first/third quarter) to 64.17 R⊕ (at full/new Moon), nearly a factor of two. Since the angular size of the Moon in radians is just equal to its diameter divided by its distance, Ptolemy’s model implies that the angular size of the Moon should also vary by nearly a factor of two.

Ptolemy’s model was the most popular model of the Moon’s motion until the time of Tycho Brahe (1546-1601), which is curious, because even a casual observer of the Moon would notice that the Moon’s angular size does not vary by a factor of two. Even more curious is the scarcity of actual measurements that have come down to us from ancient and medieval times. To my knowledge only Levi ben Gerson (1288-1344) and Ibn al-Shatir (1304-1375/6) made such measures. Levi claims to have measured the angular size of the Moon many times and found about the same value at full/new Moon as at first/third quarter, in contradiction to Ptolemy’s model (Goldstein 1985).

An obvious question arises. Is it possible to measure regular variations of the angular size of the Moon with the naked eye and derive the eccentricity of the Moon’s orbit? I fashioned a device that slides up and down a yardstick calibrated in millimeters. By eye I fit the Moon into a 6.2 mm hole. Simple geometry gives the angular size, with one proviso. The pupil of the eye is comparable in size to the hole for sighting the Moon. And the eye has a lens. I was not using a small (∼1 mm) sight at the eye end.

Consider a 91 mm disk viewed at a distance of 10 meters. It has the same angular size as the mean angular size of the Moon. I found that if I scale my raw angular sizes by 1.17 (from the measurements of a 91 mm disk), I obtain the correct mean angular size of the Moon. Many of my 20 year-old students have obtained correction factors between 0.9 and 1.0, while others get values as small as 0.7 or as large as 1.3. An understanding of this is beyond the scope of the present paper. Let us be content that my own scale factor eliminates a source of systematic error related to my measurements.

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1 See footnotes 7 and 8 of Krisciunas (2009).
Figure 1. Phased naked eye observations of the angular size of the Moon. Upper diagram: individual data from April 21, 2009, through June 9, 2012. Lower diagram: averages for binned data. Modern period finding software gives a period of 27.5042 ± 0.0334 days, which compares well with the official modern value of 27.55455 days.
Figure 2. Deviation of ecliptic longitude of Moon (in degrees) compared to a perfectly circular orbit, for the year 2012. We used information from the 2012 edition of the Astronomical Almanac.

From observations obtained in 2009 over 7 lunations, I was able to demonstrate regular variations of the angular size of the Moon (Krisciunas 2009). The uncertainty of my individual observations is about 1 arc minute, which is accurate enough to show monthly variations of 3-4 arc minutes. Thus, it was within the capabilities of the ancient Greeks not only to discover a serious problem with Ptolemy’s model of the motion of the Moon, but also to establish an approximately correct value for the variation of angular diameter and distance.

Using 100 observations\(^2\) obtained over 1145 days (or 39 lunations), my derived value of the anomalous month (perigee to perigee period) is $27.5042 \pm 0.0334$ days, which is 1.5 standard deviations less than the official value of 27.55455 days.\(^3\)

The measures of the Moon’s angular size, folded by our derived value of the length of the anomalous month, are shown in Fig. 1. Here I have used mathematical tools unavailable to pre-19th century astronomers, namely a period finding algorithm (Breger 1989) and a means of estimating the uncertainty of the derived period (Montgomery & O’Donoghue 1999). My value of the eccentricity of the Moon’s orbit is $0.039 \pm 0.004$, which is noticeably smaller than the official modern value of 0.0549.

\(^2\)The raw data can be obtained at: http://people.physics.tamu.edu/krisciunas/moon_ang.html

\(^3\)According to Toomer (1981, on p. 211), Hipparchus knew that the Moon returns to the same velocity 4573 times in 126,007 days, 1 hour. This gives an anomalous month of 27.55457 days.
Figure 3. Distance from the center of the Earth to the center of the Moon, for each day of the year 2012. Data are from the 2012 edition of the *Astronomical Almanac*.

### 2 Anomalies of the Moon’s Motion

What has been known about the Moon’s motion, from ancient times to the late 20th century, is summarized in a long and impressive article by Gutzwiller (1998). On pp. 601-602 he briefly describes the basics. “The Babylonians knew that the full moons could be as much as 10 hours early or 10 hours late [compared to uniform circular motion]; this is due to the eccentricity $\epsilon$ of the Moon’s orbit.” This is known as the first anomaly of the Moon’s motion. The Greeks would have just considered it the largest epicycle of the Moon’s orbit.

Gutzwiller continues, “[T]he Greeks wanted to know whether the Moon displays the same kind of speedups and delays in the half moons, either waxing or waning. The half moons can be as much as 15 hours early or late. With the Moon moving at an average speed of slightly more than 30′ per hour ... it may be as much as 5 deg ahead or behind in the new/full moons; but in the half moons, it may be as much as 7 deg 30′ ahead or behind its average motion. This new feature is known as *evection*.” This is the second anomaly of the Moon’s motion.

The first anomaly causes deviations from the mean longitude up to 6 deg 15′. The evection adds (sinusoidally) another 1 deg 15′. Consider it the second largest epicycle. Tycho Brahe discovered the third largest perturbation of the longitudes, known as the *variation*. It has an amplitude of 40′ and depends on twice the mean angular distance of the Moon from the Sun.

Tycho also discovered the “annual inequality,” which has an amplitude of 11′. It accounts for the Moon slowing down in its motion around the Earth when the Earth is near perihelion and speeding up when the Earth is near aphelion. Finally, Tycho discovered two inequalities relating to the Moon’s ecliptic latitudes.
Figure 4. Because of the Sun’s strong perturbation on the Moon’s motion, the Moon’s actual distance from the Earth deviates from the basic (slowly rotating) ellipse by up to 1.1 Earth radii. We show this graphically in polar coordinates for the year 2012. In each of these graphs the Sun is far off the right hand side of the paper. The new Moon, first quarter, full Moon, and third quarter are at the 3 o’clock, 12 o’clock, 9 o’clock, and 6 o’clock positions, respectively. The dashed circle has a radius of 2 R⊕, which gives us a scale for the perturbations plotted in polar coordinates.
In modern parlance we would say that these anomalies are smaller and smaller terms of a Fourier series that describes the Moon’s motion.

Details of Tycho’s lunar theory are considerably beyond the scope of the present article. The reader is directed to the appropriate parts of Victor Thoren’s excellent biography (Thoren 1990) and to Swerdlow (2009).

How can we most easily understand this behavior of the Moon? Consider the gravitational force between the Sun and the Moon when the Moon is 1 AU from the Sun. Call this $F_{\odot-Moon}$. It is equal to $4.36 \times 10^{20}$ Newtons. Let the gravitational force between the Earth and Moon when the Moon is at its mean distance of 384,400 km be $F_{\oplus-Moon} \approx 1.98 \times 10^{20}$ Newtons. The ratio is $\approx 2.2$. In other words, the Sun-Moon gravitational force is 2.2 times stronger, on average, than the Earth-Moon gravitational force. The Sun’s effect on the Moon’s motion must therefore be considerable!

In Fig. 2 we show the deviations of the Moon’s motion from uniform circular motion during the year 2012, illustrating graphically what we have quoted from Gutzwiller (1998). The Moon’s biggest deviations in longitude occur at first and third quarter. The deviations at full and new Moon are smaller owing to the evaporation.

In Fig. 3 we plot the Earth-Moon distance vs. day of year for 2012. We used values from the 2012 volume of the Astronomical Almanac. Note that the apogee distance has a very small range from orbit to orbit (63.3 to 63.7 $R_\oplus$), but the perigee distance ranges considerably, from 56 to 58 $R_\oplus$. If the Moon had an ideal, Keplerian elliptical orbit, its distance would range from 5.5 percent closer than the mean value to 5.5 percent larger than the mean. The effect of the Sun on the Moon’s orbit provides an unequal situation. At maximum distance the Moon is 5.8 percent further than the mean distance. At minimum distance the Moon is 7.3 percent closer than the mean distance.

In Fig. 4 we provide a graph for each month of 2012, showing the variation of distance of the Moon with respect to the Moon’s mean ellipse as a function of the difference of ecliptic longitudes of the Sun and Moon. In other words, in polar coordinates we show the perturbation in the radial direction due to the Sun’s effect on the Moon’s orbit. During 2012 the Moon’s orbit bulged out by as much as 1.1 $R_\oplus$ at third quarter during the first three months. Then it did so again during July, August, and September, but at first quarter. The radial distance was smaller than the mean ellipse at full Moon by 1.0 $R_\oplus$ from March through July, then was smaller by 0.9 to 1.0 $R_\oplus$ at new Moon from September through December.

3 Discussion

We have demonstrated that it is possible to measure regular variations of the Moon’s angular size using naked eye observations. The results shown in Fig. 1 are consistent with Kepler’s First Law, but we have not proven that the Moon’s orbit is an ellipse. That would require much more accurate data. In fact, the Moon’s path around the Earth is not even a closed orbit. The line of apsides (which connects the apogee and perigee) rotates slowly in the same direction that Moon itself moves, with a period of 3232.6 days (about 8.85 years). And the effect of the Sun on the Moon’s motion causes multiple anomalies in the ecliptic longitudes and the radial distance that are correlated with the phase of the Moon.

It should be pointed out that while the Babylonians and ancient Greeks obtained positional measurements of the Moon that extended over centuries, they did not take data in the “modern” way. Starting with Tycho, astronomers have endeavored to identify and eliminate sources of systematic error, and have quantified and sought to reduce random errors.

Neugebauer (1975) pointed out, “In all ancient astronomy direct measurements and theoretical considerations are so inextricably intertwined that every correction at any one point affects in the most complex fashion countless other data, not to mention the ever present numerical inaccuracies and arbitrary rounding which repeatedly have the same order of magnitude as the effects under consideration. In the history of
the most causal of all empirical sciences, in astronomy, the search for causes is as fruitless as in all other historical disciplines.”

Because of the Moon’s brightness and availability every month, it has been an obvious candidate for study, from ancient to modern times. Its motion is anything but simple.

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**References**


