1. The absolute visual magnitude ($M_V$) of the Sun +4.8. Use the standard formula relating the absolute magnitude, apparent magnitude ($m_V$), and distance: $M_V = m_V + 5 - 5 \log_{10}(d)$. (a) Calculate the distance at which a star like the Sun would have an apparent magnitude of 6.5, 7.5, and 8.5. (b) Call the Sun-like stars between magnitude 6.5 and 7.5 the “7th magnitude stars” and the stars between magnitude 7.5 and 8.5 the “8th magnitude stars”. If these stars are uniformly distributed throughout space, show that there will be $10^{0.6} \approx 3.98$ times as many 8th magnitude stars compared to 7th magnitude stars.

2. The number of chocolate chips per cookie will obey a Poisson distribution:

$$f(x) = \frac{m^x e^{-m}}{x!}.$$  

If you want the probability that there are fewer than 2 chocolate chips per cookie to be less than 0.01, the mean value of chips per cookie ($m$) has to be at least how large? Method: start with some integral value of $m$. Calculate $P(x=0) + P(x=1)$. Is this sum close to 0.01? Try other values of $m$ (to the nearest 0.1) until you find the answer.

3. Consider the following function, which is defined from $x = 0$ to $x = a$.

$$f(x) = \frac{2}{a} \left( 1 - \frac{x}{a} \right).$$

(a) Make a drawing of the function.

(b) Show that $f(x)$ is a probability density function, namely that the integral of $f(x)$ from 0 to $a$ equals 1.

(c) Calculate the first moment of $f(x)$. This is also designated $E(x)$, or $\mu$ (the mean value).

(d) Calculate the second moment of $f(x)$. This is also designated $E(x^2)$.

(e) Obtain $Var(x) = \sigma^2$, which equals $E(x^2) - \mu^2$.

4. Consider the Gaussian probability density function:

$$f(x) = \frac{1}{\sigma \sqrt{2\pi}} e^{-\frac{(x-\mu)^2}{2\sigma^2}}.$$  

Show that the inflection points of the function (where the second derivative changes sign) occur at $\pm$ one standard deviation ($\sigma$) from the mean.