Hubble’s Law
V. M. Slipher (1875-1969) was an astronomer who worked at Lowell Observatory in Flagstaff, Arizona. In 1909 he began studying the spectrum of the Andromeda Nebula. He found that that object was approaching us at a speed of 300 km/sec. This was the largest Doppler shift measured.
Radial velocities of nebulae measured by Slipher:

<table>
<thead>
<tr>
<th>NGC</th>
<th>velocity (km/sec)</th>
</tr>
</thead>
<tbody>
<tr>
<td>221</td>
<td>-300</td>
</tr>
<tr>
<td>224</td>
<td>-300</td>
</tr>
<tr>
<td>598</td>
<td>~zero</td>
</tr>
<tr>
<td>1023</td>
<td>+200 roughly</td>
</tr>
<tr>
<td>1068</td>
<td>+1100</td>
</tr>
<tr>
<td>3031</td>
<td>+ small</td>
</tr>
<tr>
<td>3115</td>
<td>+400 roughly</td>
</tr>
<tr>
<td>3627</td>
<td>+500</td>
</tr>
<tr>
<td>4565</td>
<td>+1000</td>
</tr>
<tr>
<td>4594</td>
<td>+1100</td>
</tr>
<tr>
<td>4736</td>
<td>+200 roughly</td>
</tr>
<tr>
<td>4826</td>
<td>+ small</td>
</tr>
<tr>
<td>5194</td>
<td>+ small</td>
</tr>
<tr>
<td>5866</td>
<td>+600</td>
</tr>
<tr>
<td>7331</td>
<td>+300 roughly</td>
</tr>
</tbody>
</table>
We can compare these velocities with a three other velocities:

orbital speed of the Earth around the Sun ~ 30 km/sec

orbital speed of Sun around center of Galaxy ~ 220 km/sec

Escape speed from our Galaxy is

\[ (V_{\text{esc}})^2 = \frac{2 \, G \, M_{\text{Gal}}}{r_{\text{Gal}}} \]

With a mass of the Galaxy of \(2.5 \times 10^{12}\) solar masses and a radius of 25,000 parsecs, the escape speed is about 930 km/sec.
In 1914 Slipher delivered a paper on the radial velocities of 15 spiral nebulae. Some had Doppler shifts of 1100 km/sec. Astronomers who heard his speech did not know what to make of it, but they knew it would prove to be significant. They gave him a standing ovation.

Slipher had at his disposal only the 24-inch refractor at Lowell Observatory. In 1917 a much larger telescope was inaugurated – the 100-inch telescope at Mount Wilson, California. Observations with the 100-inch soon proved that the nebulae studied by Slipher were “island universes” -- other galaxies like our Milky Way.
Edwin Hubble (1889-1953) was one of the most important astronomers of the first half of the 20th century.

In 1924, using observations of Cepheid variables in the Andromeda Nebula, he proved that it was a giant spiral galaxy like the Milky Way Galaxy.

In 1929 he discovered the expansion of the universe.
Hubble's 1929 diagram showing the radial velocities of galaxies vs. their distances. Note that the slope is about 500 km/sec/Mpc.
Hubble used as a “standard candle” the brightest stars he could detect in the galaxies he photographed. He assumed that they had absolute magnitudes comparable to the brightest supergiants in our Galaxy.

However, it turned out that Hubble's bright “stars” in other galaxies were groups of stars, so he was using the wrong absolute magnitudes. That is why the slope in his velocity-distance diagram is not correct.
By 1931 Hubble and Humason had determined distance to galaxies as far as 30 million parsecs, whose radial velocities were as large as 20,000 km/sec: 7 percent of the speed of light.

What we know as Hubble's Law relates the velocity of recession of galaxies with their distances:

\[ V = H_0 \, d \]

V is the velocity in km/sec. d is the distance in millions of parsecs (Megaparsecs). \( H_0 \) is known as the Hubble constant. It has units of km/sec/Mpc.
A few of the nearby galaxies are approaching us. But almost all the galaxies have redshifted spectra -- they are receding from us at a speed that is proportional to distance.
The linear velocity-distance relationship implies that the universe is expanding. Furthermore, no matter where you are in the universe, you would observe the nearest galaxies to be receding at the smallest velocities, and the more distant galaxies would recede with velocities proportional to distance.
A standard modern value of the Hubble constant is 72 +/- 8 km/sec/Mpc (Freedman et al. 2001). This was made from observations of Cepheids in nearby galaxies using the Hubble Space Telescope.

This value of $H_0$ also required observations of Type Ia supernovae, which are excellent standard candles.
What's the fastest speed you can go? The speed of light, of course. So, using Hubble's Law, how far would galaxies be if they were receding from us at the speed of light?

\[ V(\text{km/sec}) = 72 \ (\text{km/sec/Mpc}) \ d \ (\text{Mpc}) \]

Set \( V = \) speed of light = 300,000 km/sec. Then \( d = 4167 \ \text{Mpc} \). Since each parsec = 3.26 light-years, that's a distance of 13.58 billion lt-yrs.

Since those hypothetical galaxies have been sending us their light from 13.58 billion lt-yrs away, that light must be 13.58 billion years old. We are looking back in time 13.58 billion years.
Another way of looking at this is just to consider the Hubble constant:

\[ H_0 = 72 \text{ km/sec/Mpc} \]

The Hubble constant is measured in some units of distance per unit time, divided by some other unit of distance. In other words, it has units of 1/time. So, let us consider a different constant:

\[ \frac{1}{H_0} = \left(\frac{1}{72}\right) (\# \text{ km in a Mpc}) \times \text{sec} \]

This will have the units of time. In fact, we call \( \frac{1}{H_0} \) the *Hubble time*. 
One parsec = 206,265 AU times 149.6 million km/AU = 3.086 X 10^{13} \text{ km}.

One Megaparsec = one million parsecs, so 1 Mpc = 3.086 X 10^{19} \text{ km}.

Therefore,

\[ \frac{1}{H_0} = \frac{1}{72} \times 3.086 \times 10^{19} = 4.286 \times 10^{17} \text{ sec}. \]

That's a lot of seconds. Since 1 year \( \sim 3.1557 \times 10^7 \) sec, it follows that \( \frac{1}{H_0} \sim 13.58 \) billion years, exactly what we got from Hubble's Law and the speed of light.
Since the value of the Hubble constant always has some uncertainty associated with it, it is convenient to redefine the Hubble time as follows:

\[ T_H = \frac{9.778}{(H_0/100)} \]

If \( H_0 = 72 \) km/sec/Mpc, the Hubble time is 13.58 billion years.

The bottom line is that from an investigation of the rate of expansion of the universe, you directly get an estimate of the age of the universe.
Why is the Hubble time only an *estimate* of the age of the universe? Partly, because we don't know the Hubble constant with great accuracy.

Also, it turns out that one must consider the mean mass density of the universe. If the universe had just the right amount of matter to halt the general expansion, the age of the universe would be 2/3 of the Hubble time. Why is it smaller? Because the sum total of all the gravitational attraction of all the galaxies should be slowing down the expansion of the universe. This was our strong expectation until 1998 when two groups of astronomers discovered that the universe was actually accelerating in its expansion.