Frequency Analysis
and
A New Class of Variable Stars

Kevin Krisciunas
Credit: L. Eyer & N. Mowlavi (10/2007)
Consider an idealized variable star having a constant amplitude and a single period of 2.5 days. If we could observe it every 0.1 day, this would be the light curve.
The Discrete Fourier Transform of such a dataset would give a power spectrum like this. The correct frequency would be recovered.
But if we are observing such a star at a single site, we cannot observe 24 hours a day, and some of the time it is cloudy. Real observations might be like this:
We still recover the correct frequency, but we also obtain two *aliases* of the correct frequency. Aliases can be diminished by multi-longitude observing or continuous observing from Antarctica or with a satellite.
For stars with one principal period of light variation aliasing is not so serious a problem. But if a star has multiple periods, even multi-longitude datasets are affected by aliases in the power spectrum.
In 1924 Nyquist and Shannon were working on telegraph communications at AT&T and formulated a sampling theorem: the number of independent pulses that could be put through a telegraph channel per unit time is limited to twice the bandwidth of the channel. As it pertains to variable star research, if you measure the brightness of a star every 1.00 days, you can recover all the periods of 2.00 days and longer. One speaks of the Nyquist frequency as a serious constraint.
If you have irregular sampling of data and some periods of frequent sampling, as often happens in astronomy, then you can recover periods shorter than $1/f_{\text{Nyq}}$.

One computational aid for such data is the Lomb-Scargle algorithm for irregularly spaced data.

Long, nearly continuous data runs (e.g. with MOST, CoRoT, Kepler, or during the Antarctic winter) avoid a lot of the problems with aliasing of data.
In 1912 Henrietta Leavitt (1868-1921) discovered a relationship between the maximum (and minimum) magnitudes and the periods of Cepheids in the Small Magellanic Cloud.
Location in the Hertzsprung-Russell Diagram of various types of pulsating stars.

Eyer and Mowlavi (2008)
Giant stars are 10-100 times the size of the Sun.

Supergiants are up to 1000 or more times the Sun’s size.
If the layers on the bottom can hold up the layers above, then an equilibrium situation can exist.
The same holds true for a star. If the weight of the layers above is balanced by the flow of energy from the core, the star can maintain hydrostatic equilibrium. But if there is a change of opacity, due to the ionization of helium for example, then the star can pulsate indefinitely.
Shapley (ca. 1916) showed that Cepheids are indeed pulsating stars.

The minimum size occurs at maximum luminosity.

Figure 14.5 Observed pulsation properties of δ Cephei.
Increase radiation pressure

Outer shell Expands and Cools

doubly ionized He (increases opacity)

Outer shell Contracts and Heats

Decrease radiation pressure

He recombines (decreases opacity)

κ mechanism for Cepheids
The **kappa mechanism**

“In a normal star, an increase in compression of the atmosphere causes an increase in temperature and density; this produces a decrease in the opacity of the atmosphere, allowing heat energy to escape more rapidly. The result is an equilibrium condition where temperature and pressure are maintained in a balance. However, in cases where the opacity increases with temperature, the atmosphere becomes unstable against pulsations. If a layer of a stellar atmosphere moves inward, it becomes denser and more opaque, causing heat flow to be checked. In return, this heat increase causes a build-up of pressure that pushes the layer back out again. The result is a cyclic process as the layer repeatedly moves inward and then is forced back out again.”
We all know Newton’s Law of Gravity:

\[ F = G \frac{m_1 \ m_2}{r^2} \]

Force has units of mass times acceleration, or \( \text{kg m} / \text{sec}^2 \). \( m_1 \) and \( m_2 \) are, of course, in kg and \( r \) is in meters. So the units of \( G \) are:

\[
[\text{kg m} / \text{sec}^2] \frac{m^2}{\text{kg}^2} = \frac{m^3}{\text{kg sec}^2}
\]

Density \( \rho \) has units of \( \text{kg} / \text{m}^3 \), so \( G\rho \) has units of

\[
\left\{ \frac{m^3}{[\text{kg sec}^2]} \right\} \frac{\text{kg}}{\text{m}^3} = \text{sec}^{-2}
\]

Thus, \((G\rho)^{-1/2}\) has units of \textit{seconds}. 
A star will pulsate as a Cepheid if the blueward loop from the giant branch extends into the instability strip.

(D. G. Turner et al. 2006, PASP)
Cepheid period-luminosity relations for stars in the Large Magellanic Cloud. JK: (Macri et al. 2010). BVI: Udalski et al. (1999).
Cepheids in nearby galaxies have been found using HST and ~12 epochs of photometry. Optimum sampling might be at moment 0, then 3.28, 7.24, 12.03, 17.89, 25.10, 34.03, 45.18, 59.24, 77.10 and 100.00 days later.

Range of mass of Cepheids is 4 to 12 solar masses.

Size range is roughly 10 to 200 solar radii.

Assuming similar internal structure for all Cepheids,

\[ P \text{ (period of pulsation)} = \text{constant} / (G \langle \rho \rangle)^{1/2}. \]

How much longer is the period of pulsation of a 12 solar mass Cepheid compared to a 4 solar mass Cepheid?

\[ \text{density} = \text{mass} / \left[ \frac{4}{3} \pi r^3 \right] \]

\[ P_{12} / P_4 \sim \left[ \frac{(4/10^3)}{(12/200^3)} \right]^{0.5} \sim 50. \] So, it makes sense that if the shortest period Cepheids have \( P = 2^\text{d} \), the longest period Cepheids have periods \( P \sim 100^\text{d} \).
Cepheids are arguably the most important variable stars. We understand the physical mechanism by which they pulsate, and the period-luminosity relation allows them to be used as *standardizable candles*. Since

\[ M = m + 5 - 5 \log d_{\text{pc}} \]

if we can correct the apparent magnitudes for any dust extinction along the line of sight and we know the absolute magnitude in some band, we can determine the distance. The *Hubble Space Telescope* can detect Cepheids out to a distance of \(~25\) Megaparsecs.
RR Lyrae stars are Population II variable stars with masses of ~1 solar mass that have fainter luminosities and shorter periods than the fastest Cepheids. They have mean absolute magnitudes of about $M_V \sim +0.7$ (much fainter than Cepheids, but still useful as standard candles).

RRab stars – periods of ~0.5 day, amplitude ~1 magnitude pulsating in the fundamental radial mode ($P_0$)

RRc stars – periods of 0.3 to 0.4 days, amplitude 0.5 mag pulsating in the first overtone ($P_1$)

RRd stars – exhibit both fundamental and first overtone pulsations (often $P_1 \sim 0.74$ to 0.75 $P_0$). (Analogy to musical notes....)
The Sloan Digital Sky Survey covered ~10,000 square degrees of the sky in 5 filters.
From observations at two epochs it is possible to identify RR Lyr candidates. They would be to the right of the 5-sided box (which is the location of blue horizontal branch stars).

Color-magnitude diagram of globular cluster M 15 (Krisciunas, Margon, & Szkody 1998)
Color-color diagram showing known RR Lyr stars in M 15 (KK et al. 1998)
These 6 RR Lyrae candidates identified by Beth Willman were confirmed to be RR Lyr stars by K. Krisciunas.

b2774 is a “Blazhko star” (it has cycle to cycle changes in the amplitude; Blazhko, 1907, Astron. Nachrichten, 175, 325).
MACHO ID 82.8410.55
P = 12.346721 hours

(D. Welch)
Chadid et al. (2009) discussed the first RR Lyrae stars discovered by the CoRoT mission. They write:

“The number of monoperiodic RR Lyrae stars is very low…” One star showed harmonic components up to 34th order with a clear decrease of the harmonic amplitudes up to 13th order.
Stellar oscillations can be very complex, but all of the quiverings and pulsations fall into patterns that are described by “harmonic indexes.”

Two of these indexes are called \( l \) and \( m \). They represent the numbers of nodes (planes where no variation occurs) that characterize a given mode of oscillation. The \( l \) index tells the total number of nodal planes slicing the star’s surface. The \( m \) value gives the number of these that contain the star’s axis. The computer-generated balls above the line diagrams show how various parts of the star’s surface move, perhaps brightening and dimming, for oscillations having an \( l \) value of 6 and various values of \( m \). At left, none of the nodal planes contain the star’s axis so \( m = 0 \). At center, six planes still cut the star but three are oriented vertically, so \( m = 3 \). At right, all six planes are vertical so \( m = 6 \).

The horizontal planes are motionless. But the vertical ones rotate around the star, either clockwise or counterclockwise; the direction is designated by a positive or negative value for \( m \). The third and last harmonic index, \( k \), represents nodes in the radial direction. These nodes are concentric spheres inside the star, not visible here. Courtesy National Solar Observatory.

Non-radial pulsational modes are characterized by the number of segments that the star is divided into and whether the dividing planes are coincident with the star’s axis of rotation, or perpendicular to it.
Fig. 16.—Illustration of the formation of distortions in the line profiles of a rapidly rotating nonradially oscillating star. This is a velocity map of the star with the resultant line profile shown below. The width of the line profile has been scaled to match the diameter of the star, thus reflecting the mapping which occurs between position across the disk and position across the line profile. The darkest shaded regions correspond to material moving away from the observer, while the lightest regions represent material moving toward the observer. Contours are drawn every 5 km s$^{-1}$.

(Vogt & Penrod, 1983)
Fig. 15.—Theoretical profiles of He I λ6678 of ζ Oph for various modes of nonradial oscillation. All profiles are shown at the same phase.
Fig. 1.—Observations of the He I $\lambda$6678 line of $\zeta$ Oph on 1980 June 29 and 1980 July 1 UT
If a star is pulsating non-radially with $l = 4$ or less, it is divided into large enough sections that it can exhibit photometric variations and radial velocity variations.

For higher order non-radial pulsations, like the Sun’s 5 minute principal period, the hemisphere facing the observer is divided into too many sections to produce brightness variations. (The Sun is constant to 0.001 percent.)
γ Doradus stars were first identified as a new group of variable stars by

Cousins & Balona (S. Africa)

Mantegazza, Poretti, & Antonello (Italy)

Krisciunas & Guinan (USA)

R. Griffin (UK) obtained data with his radial velocity spectrometer.

Jaymie Matthews (Canada) pointed out that we were studying stars of almost the same spectral type and luminosity class, late A to early F-type stars on (or just above) the main sequence in the HR Diagram. The periods of variation are 0.4 to 3 days.
9 Aurigae has
RA ~ 5:06:40
DEC +51.6 deg
Photometry from Lithuania, Spain, New Mexico, Arizona, and Hawaii!
9 Aurigae shows two principal periods of 1.258 and 2.893 days. Later, Zerbi et al. (1997) found a third period of 1.302 days.

Krisciunas et al. (1995)
A 15 year run of differential photometry of 9 Aur and two comparison stars, obtained by Greg Henry with a robotic telescope in Arizona.
The principal period still is 1.258 days. G. Henry claims that this dataset indicates 4 other periods too.
Line profile variations of 9 Aurigae are correlated with the photometric changes and the radial velocity changes.
9 Aurigae: strong evidence for non-radial pulsations

Figure 8. Power spectrum of radial velocities shown in Fig. 7. The frequency $f_2$ and its one-day alias $1 + f_2$ are indicated.

Krisciunas et al. (1995)
Radial velocity variations of 9 Aur are primarily associated with the longer of the two principal photometric periods.

Figure 9. Folded plot of radial velocity data from the 1993/4 season, using epoch of zero phase and period derived from the photometry. The sinusoid shown is derived from the Fourier fit of the radial velocities. Qualitatively, the star's maximum rate of expansion corresponds to the time of minimum light for the $f_2$ sinusoid.

Krisciunas et al. (1995)
We concluded in 1995 that the only way to explain the behavior of 9 Aurigae was that it exhibits non-radial pulsations for which the restoring force was the gravity of the star. These are called **g-mode** pulsations. The time scale is *longer* than the fundamental radial pulsational period of a star with this density. They are driven by convective blocking at the base of their envelope convective zone (Grigahcène et al. 2010, ApJL).

Balona et al. (1996) found that γ Dor could be modeled with three non-radial modes with \((l,m) = (3,3), (1,1), \text{ and } (1,1)\).

Aerts & Krisciunas (1996) found that 9 Aur could be modeled with an \(l = 3, |m| = 1\) mode.
It was thought that the $\gamma$ Dor phenomenon was age related.

<table>
<thead>
<tr>
<th>cluster</th>
<th>$\gamma$ Dor</th>
<th>cluster age</th>
</tr>
</thead>
<tbody>
<tr>
<td>Pleiades</td>
<td>yes</td>
<td>78 Myr</td>
</tr>
<tr>
<td>NGC 2516</td>
<td>yes</td>
<td>137 Myr</td>
</tr>
<tr>
<td>M 34</td>
<td>yes</td>
<td>250 Myr</td>
</tr>
<tr>
<td>Hyades</td>
<td>no</td>
<td>625 Myr</td>
</tr>
</tbody>
</table>

But the situation is more complicated than we thought.
A 1.55 solar mass star does not exhibit $\gamma$ Dor variations until it is nearly 1 Gyr old. 600 million years later the pulsations stop. Then they resume 200 million years later. A 1.45 solar mass star pulsates non-radially for much of its first billion years, then stops.
δ Scuti stars are found in the Hertzsprung-Russell Diagram just below the RR Lyrae stars in the instability strip. They pulsate non-radially, but their periods are typically ~1 hour (shorter than the fundamental radial pulsational period). The restoring force of the pulsations is the acoustic pressure of the gas. These are the so called p-mode pulsations.
Grigahcine et al. (2010)
Observations with MOST, CoRoT, and Kepler have revealed hybrid $\gamma$ Dor/$\delta$ Sct pulsators. Grigahcène et al. (2010, ApJL) state that “there are practically no pure $\delta$ Scuti or $\gamma$ Doradus pulsators.”

What does this mean? High quality, uninterrupted photometry can reveal pulsational activity which is very difficult to prove from old style observing runs.
KIC 6462033
\[ f_1 = 0.92527 \text{ cycles/day} \]
\[ f_2 = 2.03656 \]
\[ f_3 = 1.42972 \]

(Ulusoy et al. 2014)
The stars on the left show $\delta$ Scuti and $\gamma$ Doradus pulsations. Below is the result of modeling of one of them.

Grigahcène et al. (2010)
Long period variables with large amplitudes are, of course, the easiest kind of periodic variables to identify. Short-period variables can be identified from single-site observations.

(Eyer & Mowlavi 2007)
Ways in which a star can vary up to 5 or 10%

1) ellipsoidal variable (tidally distorted by companion)
   variability happens on time scale of 1/2 orbital period

2) rotating spotted star

3) transit of object much smaller than the star

4) transit of companion star across N or S pole of primary

   1) through 4) would have a single period

5) non-radial pulsations (could have many periods)

6) microlensing (non-periodic) – magnification of light
   is independent of photometric band