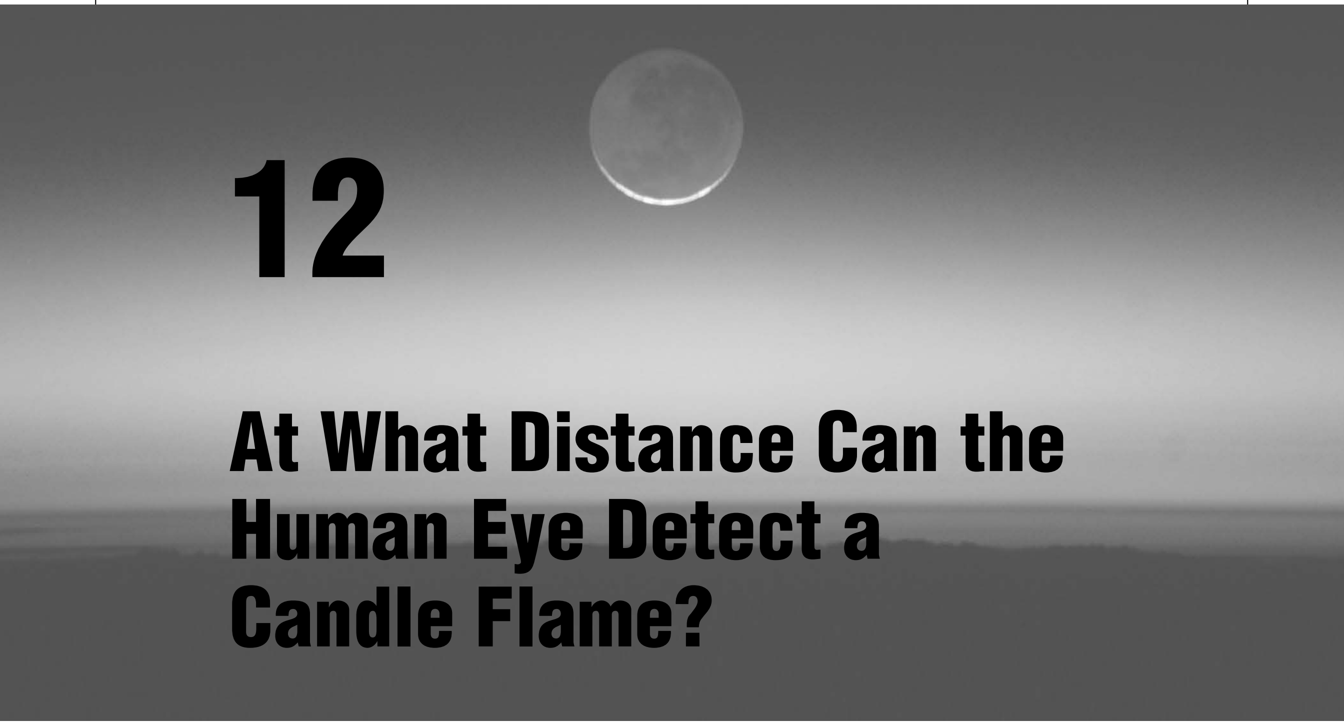

12

At What Distance Can the Human Eye Detect a Candle Flame?

In 2015, there was a Centrum Silver TV advertisement for vitamins, narrated reassuringly by Martin Sheen, which claimed that the unaided human eye can detect a candle flame at a distance of 10 miles.[1] Web searches on the question posed in the title of this chapter suggest that the correct answer might be 3 miles, or as far as 30 miles! Clearly, we can do better by considering this as a problem of astronomical detectability. Some data would be of considerable help too.

Let us start with a slightly different question. At what distance would a candle flame be comparable in brightness to the brightest stars in the sky? Let's consider a star of apparent magnitude 0, such as Vega. Naked eye experiments by various college students and myself indicated that the distance is greater than 150 yards, possibly as far as 400 yards.

I collaborated with Don Carona, the director of Texas A&M's Physics Observatory, to take some hard data. We used an SBIG uncooled CCD camera of 35 mm aperture and focal length 100 mm (see Figure 12.1). Each pixel covers 7.4 by 7.4 arc seconds on the sky, which is large compared to the size of an image of a star. From manufacturer specifications, we know that the quantum efficiency (QE) of the camera rises from zero in the near-ultraviolet, peaks at roughly 480 nm (4800 Å), and tails off to zero at a wavelength of about 1 micron in the near-infrared.

On 4 and 6 October 2014, we took sets of five exposures of Vega under clear sky conditions, with *nominal* exposure times of 1 to 10 milliseconds.

Figure 12.1 The SBIG camera is the small tube on the upper right. The other two telescopes are just finder scopes. *Photo courtesy of Don Carona.*

Analysis of the number of counts versus exposure time (Figure 12.2) reveals two things. The true exposure times are about 1.7 milliseconds longer than the nominal exposure times. Otherwise, the counts would not be zero at zero exposure. Also, there is some raggedness in the data owing to scintillation in the Earth's atmosphere. This is a consequence of having such a small telescope aperture.

On 6 October 2014 (UT), we also set up a candle flame at a distance of 338 m. To our *eyes* the candle flame and Vega appeared of comparable brightness, but we found that we saturated the CCD for nominal exposures of 6 milliseconds and longer. This means that the CCD sees the candle flame as a brighter source.

We corrected for the systematic error in the exposure times and combined all the Vega observations on the two nights. Then, using the standard astronomical image reduction package IRAF (Image Reduction and Analysis Facility), we determined the instrumental magnitudes of Vega and the candle flame. The candle flame was measured with the CCD camera to be 2.423 ± 0.060 magnitudes brighter than Vega, even though they looked comparable to the unaided eye.

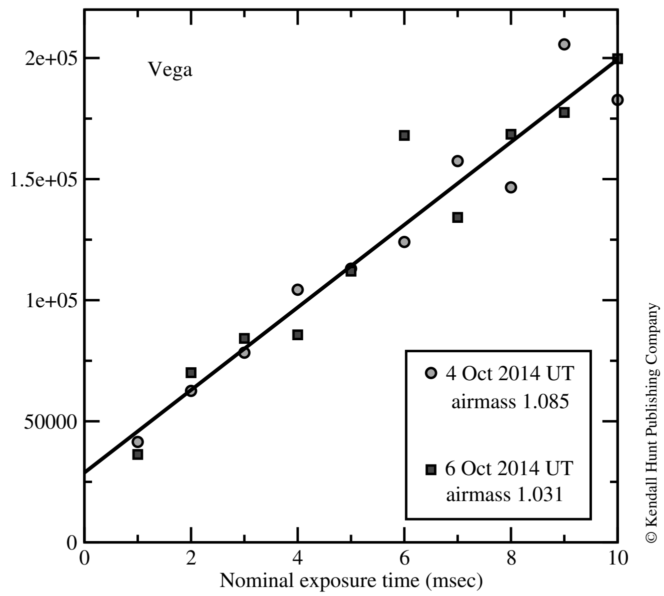

Figure 12.2 Net counts over sky versus nominal exposure time. We plot averages of five integrations per point.
Source: Kevin Krisciunas.

Since each magnitude corresponds to a factor of the fifth root of 100 (~2.51189) in brightness, the CCD camera obtained a signal from the candle flame at 338 m that was $2.51189^{2.423} = 9.315$ times brighter than Vega. This is the fundamental observational fact derived from our CCD measurements of Vega and the candle flame.

To understand this, we need to consider the spectral energy distribution of a black body[2] as a function of wavelength:

$$u_\lambda(T) = \frac{8\pi hc}{\lambda^5} \frac{1}{e^{hc/\lambda k_B T} - 1} \tag{12.1}$$

As described in the penultimate section of chapter 11, a solid, liquid, or dense gas will emit light. Whether that is primarily ultraviolet, visual, infrared or some other kind of light depends on the object's temperature. Suffice it to say that the light given off by a star's photosphere can be approximated by a black body spectrum.

We want to consider the number of *photons* we detect from two black bodies (Vega and the candle flame), so we divide u by the energy of the photons (hc/λ) and use a simple computer program to calculate this every 10 Angstroms for a range of wavelength λ.

From Vega's spectrum or photometric color, we can approximate its spectral energy distribution as a black body with a temperature of 10,000 K. This is not strictly true, as hydrogen absorption lines diminish the integrated flux of Vega at the blue end of the optical range of wavelengths. The temperature of a candle flame is roughly 1400 K, so it is primarily an infrared light source whose intensity falls off rapidly from red to orange to yellow at optical wavelengths.

Figure 12.3 shows two curves. One is the photon number density of Vega multiplied by the camera's quantum efficiency curve. The other is the corresponding photon number density of a 1400 K, black body, multiplied by the camera's QE curve and scaled by 6.016×10^6, a scale factor needed to produce the *observed* ratio of signals from Vega and the candle flame.

Next, we must consider the luminosity functions of the human eye,[3] basically the filter response of our eye and retina (Figure 12.4). For daytime (photopic) vision we may approximate the response of the eye and retina to peak at 5550 Å, with a bell-shaped curve of half-width 500 Å. The response of the eye and retina under low light conditions is a different "filter" that peaks at 5070 Å, and is slightly asymmetrical.

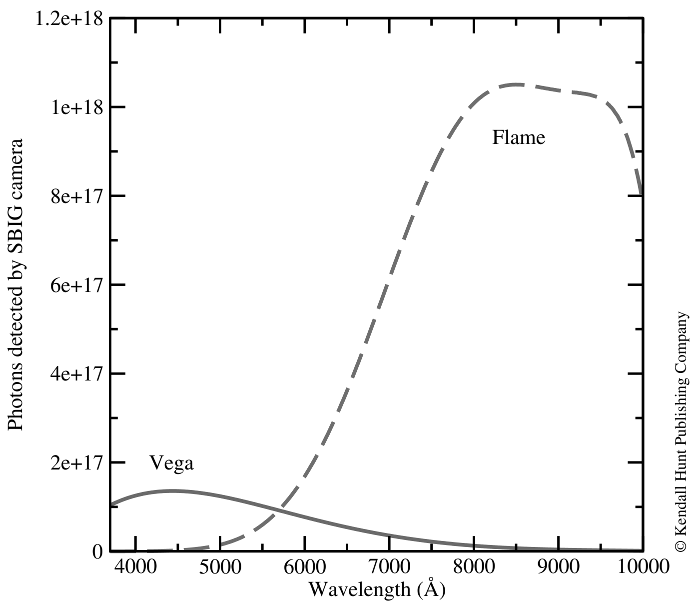

Figure 12.3 Photon number density for Vega (10,000 K black body, from Planck's law), multiplied by the SBIG QE curve, and for candle flame (1400 K black body), multiplied by the SBIG QE curve and scaled by 6.016×10^6 to give an observed ratio of signals of 9.315, as derived from CCD observations.
Source: Kevin Krisciunas.

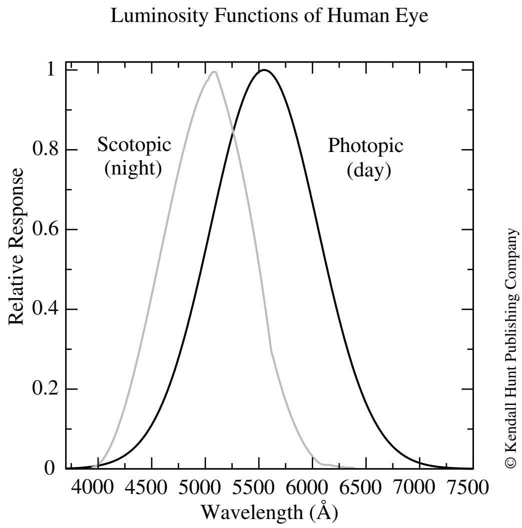

Figure 12.4 Relative response of the eye and retina for low light level conditions (scotopic curve) and for daytime (photopic) conditions.
Source: Kevin Krisciunas.

Returning to the question of the distance of the candle flame such that it would appear equal to Vega in brightness according to our eyes, we take the photon number density of a 10,000 K black body versus wavelength, multiply it by the phototopic luminosity curve of the eye, and add up the signal from 3700 to 10,000 Å. For the candle flame we take the photon number density of a 1400 K black body, multiply it by the photopic luminosity curve of the eye, scale it by 6.016×10^6, and integrate from 3700 to 10,000 Å. The ratio (flame divided by Vega) is ~1.344. (See top half of Figure 12.5.) Since light intensity decreases proportional to $1/d^2$, if the candle flame had been at 338 m times the square root of 1.334, or 392 m, it would have exactly matched the brightness of Vega to our eyes.

The faintest stars visible to the unaided eye under dark sky conditions have apparent magnitude V ~ 6.0. We adopt this as a practical limit. (I have actually reached apparent magnitude 6.3. Some keen-sighted observers, such as Brian Skiff (pers. comm.) and Stephen O'Meara (pers. comm.), have proven that they can see stars fainter than V = 8.0.) A star of magnitude zero is 2.51189^6 ~ 251.2 times brighter than a 6th magnitude star. One might think, then, that the limit of seeing a candle flame would be the square root of 251.2 (about 15.85) times 392 meters, or 6.2 km (3.86 miles). However, this neglects the fact that when we look at a bright star, there are enough photons to see the color of the star. We are using our photopic (day) vision, which uses the cones in our retina. When we look at the faintest star we can see, we use the rods in our retina, and scotopic (night) vision applies.

In Figure 12.5 we show the significance of using our daytime (photopic) vision versus our nighttime (scotopic) vision on Vega and a candle flame. Because the spectral energy distribution of the candle flame and corresponding photon energy density fall off rapidly at the short wavelength end, the 480 Å shift from day to night vision makes a significant difference in the results. To answer the question posed in our title, we need to determine how far a candle flame would have to be to appear of equal brightness to a 6th magnitude star with the spectral energy distribution of Vega. So we take the photon energy density of a 10,000 K black body for Vega, multiply it by the scotopic (night) luminosity function of the eye, and integrate from 3700 to 10,000 Å. We take the photon energy density of a 1400 K black body for the candle flame, multiply by the night luminosity function of the eye, scale by 6.016×10^6, and integrate from 3700 to 10,000 Å. Now the ratio of flame function to star function is 0.2312. A mythical creature whose bright light luminosity

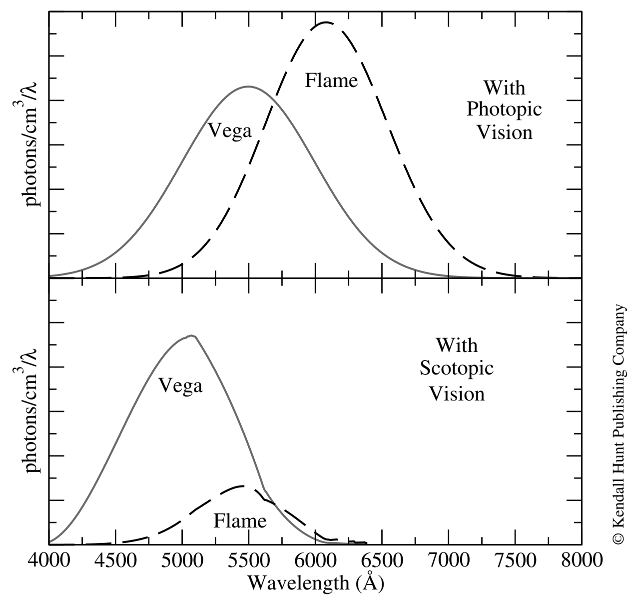

Figure 12.5 Top: relative brightness of candle flame at 338 m and Vega using daytime vision. Bottom: relative brightness of candle flame at 338 m and Vega using the night vision luminosity function of the eye.
Source: Kevin Krisciunas.

function is the same as our faint light luminosity function would observe the candle flame at 338 m to be comparable in brightness to Vega at 338 m times the square root of 0.2312, or about 162.5 m. A 6th magnitude star with the spectral energy distribution of Vega would be 15.85 times more distant, or 2576 m (roughly 1.60 miles). A candle flame situated at 10 miles (16093.5 m) would have an apparent brightness of V = 5 log (16093.5/162.5) ~ 9.98 mag. This is far beyond the capabilities of the most sensitive human eyes.

Thus, we have shown that a candle flame at roughly 2.6 km would have an apparent brightness comparable to a 6th magnitude star. Could the keenest human eyes on the planet see a candle flame at 10 miles? We have provided strong evidence that the answer is No, for it would be as faint as a star of apparent magnitude 10, and that would require a pair of 7 × 50 binoculars mounted on a tripod, even for experienced observers with good night vision.

Endnotes

1. Centrum Silver TV Spot, 'Your Eyes'. Accessed April 20, 2016, http://www.ispot.tv/ad/75pr/centrum-silver-your-eyes.
2. Planck's law. Accessed April 20, 2016, https://en.wikipedia.org/wiki/Planck%27s_law.
3. Luminosity function. Accessed April 20, 2016, https://en.wikipedia.org/wiki/Luminosity_function.