Astronomy 314 - Homework #4
Due Tuesday, April 5

1. Part a. The Rayleigh criterion, also known as Dawes’ limit, (p. 148 of Caroll and Ostlie) gives the theoretical resolution of a circular light gathering element:

\[ \theta = 1.22 \frac{\lambda}{D}, \]

where \( \theta \) is measured in radians, \( \lambda \) is the wavelength of light, and \( D \) is the diameter of the telescope, measured in the same units as the wavelength. For visual light \( \lambda = 550 \) nm. To convert \( \theta \) to arc seconds multiply by 206265.

Say the diameter of your pupil is 5 mm, and you have 20/20 vision. What is the theoretical resolution of your eye in arc seconds? Compare this to the angular diameter of Jupiter when at opposition from the Sun (distance = 4.2 AU). You may need some data in Appendix C.

Part b. Consider a telescope in low Earth orbit, 200 km above the surface of the Earth. What is the telescope diameter needed to theoretically resolve the letters in a car’s license plate (size = 6 cm) from that distance? Compare this to the diameter of the Hubble Space Telescope. In actual fact turbulence in the Earth’s atmosphere limits the resolution of remote sensing using satellites to about 0.5 meters.

2. Caroll and Ostlie, problem 10.3.

3. Caroll and Ostlie, problem 10.4 (a).

In this problem we consider that the protons moving around in the Sun’s core obey a Maxwell-Boltzmann distribution. A small fraction are going much faster than the mean speed. The question is, what temperature would be required for two protons to collide if we neglect quantum mechanical tunneling? First we equate the thermal energy to the kinetic energy:

\[ \frac{3}{2} kT = \frac{1}{2} \mu_p v_{rms}^2, \]

where \( \mu_p \) is the reduced mass of the proton-proton collision.

Assume that protons with 10 times the root-mean-square speed can overcome the Coulomb barrier.

Once you have an expression for the speed of these high velocity protons, equate the kinetic energy of the collision to the Coulomb potential energy:

\[ \frac{1}{2} \mu_p v^2 = \frac{Ke^2}{r}, \]
where $K = 8.99 \times 10^9 \text{ N m}^2/\text{C}^2$ and $e$ is the charge of the proton. Assume $r = 10^{-15} \text{ m}$ for the distance. What temperature do you obtain? How many times larger or smaller than the Sun’s central temperature is this?

4. Caroll and Ostlie, problem 10.17. The boundary conditions are $D(0) = 1$ and $D'(0) = 0$. (We will discuss polytropes in class.)

5. Show that

$$D_5(\xi) = \frac{1}{\sqrt{1 + \xi^2/3}}$$

satisfies the Lane-Emden equation for $n = 5$. (You might want to use the variable $X$ in place of $\xi$.) The curious thing about this solution is that the density reaches zero at infinite distance, but the total mass is finite.