1. (34) The long straight wire AB in Fig. 1. carries a current $I$. It is placed on the axis of the conducting tube carrying a current $2I$ in the direction opposite to the current in the wire. The rectangular loop whose long edges are parallel to the wire carries a current $0.1I$.

1. (11) Determine the net force exerted on the loop by the magnetic field.

2. (11) Determine the flux of magnetic field through the loop.

3. (12) Suppose a current in the tube increases at a rate $dI/dt$. Find an induced emf in the loop. Would it increase or decrease the original current in this loop?

\[
1. \quad B = \frac{\mu_0 I}{2\pi r}
\]

\[
\oint F = \oint B \cdot dl = F_1 - F_2 + \text{left}
\]

\[
\Phi = \frac{1}{2\pi} \int B \, dr = \frac{\mu_0 I^2}{2\pi} \left( \frac{1}{a} - \frac{1}{a+d} \right)
\]

\[
\Phi = \frac{\mu_0 I^2}{2\pi} \ln \left( \frac{a+d}{a} \right)
\]

3. \( \varepsilon = -\frac{d\Phi}{dt} = -\frac{\mu_0 I^2}{2\pi} \ln \left( \frac{a+d}{a} \right) \frac{dI}{dt} \); According to Zwik's law, since the induced emf is opposite to the original current, the flux increases and the induced emf is directed opposite to it.

\[
\frac{\partial}{\partial t} (\mu_0 I^2) \ln \left( \frac{a+d}{a} \right) = \frac{\mu_0 I^2}{2\pi} \left( \frac{1}{a} - \frac{1}{a+d} \right) \]

be opposite to \( B_{tot} \) \( \varepsilon \) decreases the original current.
II. (32) An L-C circuit consists of an inductor with \( L = 0.03 \, \text{H} \) and a capacitor of \( C = 0.03 \, \text{F} \). The initial current in the inductor is 0.1 mA and initial charge on the capacitor is zero.

1. (8) What is the maximum energy stored in the inductor?

2. (8) What is the maximum charge stored in the capacitor?

3. (8) What is the current, \( i(t) \), at the moment \( t = \frac{T}{2} \) where \( T \) is the period of oscillation?

4. (8) What is the charge at the moment when current is equal to 0.05 mA?

I. First way to solve the problem is based on energy conservation law

\[
\frac{q_0^2}{2C} + \frac{L_i^2}{2} = \frac{L_i^2}{2}, \quad U_{\text{max}} = \frac{L_i^2}{2} = \frac{3 \times 10^{-2} \, 10^{-5}}{2} = 1.5 \times 10^{-5}
\]

2. \( \frac{L_i^2}{C} = \frac{Q^2}{C} \Rightarrow Q = I \sqrt{LC} = \frac{I_0}{\sqrt{2}} C = 10^{-4} \times 3 \times 10^{-2} \, \text{C} = 3 \times 10^{-6} \, \text{C} \)

3. at \( t=0 \) we have \( i_0 = I \) hence in a half of period \( t = -\frac{T}{2} \)

4. \[
\frac{q(t)^2}{2C} + \frac{L_i^2(t)}{2} = \frac{L_i^2}{2} \Rightarrow \frac{q(t)^2}{2} = \frac{L_i^2}{2} - \frac{L_i^2}{4} = \frac{3L_i^2}{4}
\]

\[
i(t) = \frac{1}{2}
\]

\[
q(t) = I \sqrt{C} \frac{\sqrt{3} \, Q}{2} = \frac{\sqrt{3} \, 2 \times 10^{-6}}{2} \approx 2.6 \times 10^{-6} \, \text{C}
\]

II. The second way to solve this problem is based on general solution

\[
q(t) = Q \cos(\omega t + \phi) \quad \text{and initial}
\]

\[
i(0) = C \omega Q \sin(\omega t + \phi)
\]

which leads to \( \phi = \frac{\pi}{2} \), \( \omega Q = i_0 \) i.e.

\[
q(t) = \frac{Q}{2} (\cos(\omega t + \phi) - 1)
\]

1. \( U_{\text{max}} = \frac{L_i^2}{2} = \frac{3 \times 10^{-2} \times 10^{-5}}{2} = 1.5 \times 10^{-5}
\]

2. \( Q = \frac{I_0}{\omega} \sqrt{LC} = 10^{-4} \times 3 \times 10^{-2} \approx 3 \times 10^{-6} \)

3. \[
q(t) = Q \sin(\omega t + \phi)\]

\[
q(t) = Q \sin(\omega t + \phi)\]

\[
q(t) = Q \sin(\omega t + \phi)\]

\[
q(t) = Q \sin(\omega t + \phi)\]
III.(34) z-polarized electro-magnetic wave with a frequency 10 MHz propagates in x-direction in air. A circular loop of wire is used as a radio antenna.

1. (9) What is an optimal orientation of this antenna?

2. (16) If 0.4 m-diameter antenna is 500 m from a radio source with a total power 2.75 MW, what is a maximum emf induced in the loop? (Assume that the source radiates uniformly in all directions.)

3. (9) Determine the frequency of the same electromagnetic wave when it enters into the medium with \( n = 2 \).

\[
E_0 = cB_0, \quad I = \frac{cE_0^2}{2\mu_0} \quad \Rightarrow \quad B_0 = \frac{2\overline{E_0}F_0}{c} = \sqrt{\frac{2P\mu_0}{4\pi F_0^2c}} = \sqrt{\frac{2 \cdot 2.75 \cdot 10^6 \cdot 3.14 \cdot 10^{-4}}{4 \cdot 3.14 \cdot 3 \cdot 10^9 \cdot 2.5 \cdot 10^5}} = 8.56 \cdot 10^{-8} T
\]

\[
B = B_0 \delta (\omega t - kx), \quad \omega = 2\pi f
\]

\[
E_{\text{in}} = \frac{I}{4}\frac{d^2}{dt^2} 2\pi F_0 B_0 = \frac{3.14}{4} (0.4)^2 \cdot 2.314 \cdot 10^9 \cdot 8.56 \cdot 10^{-8} = 0.645 V
\]

\[
f = 10 \text{ MHz}
\]
I. (33) Bar ab moves without friction on conducting rails as shown in Fig. 1.

a) (11) A constant uniform magnetic field with magnitude $B=0.5 \, T$ is directed into the plane of the page (Fig. 1a). You want to make an exercise machine out of this apparatus, in which the person exercises by pushing the bar back and forth with a root-mean-square speed of 4 m/s. What should be the circuit resistance $R$ if the person moving the bar is to do work at an average rate of 200W?

b) (22) A time-dependent uniform magnetic field $B=0.5 T(1+\frac{t}{\tau})$, where $\tau>0$, is directed into the plane of the page (Fig. 1b). A very long solid conducting cylindrical wire of diameter $D$ carrying a constant current $I$ is placed a distance $r_o$ to the right from the rectangular loop abcd.

(i) (11) Find $\tau$ at which the same magnitude of emf in this rectangular loop is produced as in part a).

(ii) (11) Find magnetic force produced by the loop abcd on the wire.
II. (33) In LC- circuit C = 18 microfarads, L = 1.8 H.

The initial charge on the capacitor is 0.4 mC and the initial current is 0.2 A.

1) Find a charge on the capacitor at the instant \( t = \frac{3\pi}{\omega} \), \( \omega = \frac{1}{\sqrt{LC}} \).

2) Find the maximum energy stored in the inductor.

3) When the current in the inductor is changing at a rate of 3.4 A/s, what is the charge on the capacitor?

\[
1. \quad \frac{3\pi}{\omega} = \frac{2\pi}{\omega} + \frac{\pi}{2} \rightarrow t = T + \frac{T}{2} \rightarrow q \left( \frac{3\pi}{\omega} \right) = -q_0 = -0.4 \text{ mC} \\
2. \quad U_{\text{max}} = \frac{q_o^2}{2C} + \frac{L \cdot i_o^2}{2} = \frac{1.8 \times 10^{-8}}{2 \times 18 \times 10^{-6}} + \frac{0.75 \times 4 \times 10^{-2}}{2} = 0.73 \times 10^{-2} J + 1.5 \times 10^{-2} J = 2.25 \times 10^{-2} J \\
3. \quad \frac{q}{C} = L \frac{di}{dt} \rightarrow q = L \cdot C \frac{di}{dt} = 0.75 \times 18 \times 10^{-6} \cdot 3.4 = 45.9 \text{ mC}
\]
III. (34) A very long solenoid with \( n \) turns per unit length and radius \( a \) carries a current \( i \) that is increasing at a constant rate of \( \frac{di}{dt} \).

a) (9) Find the magnetic field and induced electric field inside the solenoid at a distance \( r \) from the solenoid axis.

b) (8) Find the Pointing vector at this point.

c) (8) Find the magnetic energy stored in a length \( l \) of the solenoid and the rate at which that energy is increasing as a function of time.

d) (9) Show that the energy stored in a current-carrying solenoid can be thought of as entering through the cylindrical walls of the solenoid. Hint: Calculate the total rate at which electromagnetic energy is flowing into the solenoid through the solenoid walls and compare it with a result of part e.

\[
\begin{align*}
\text{a) } B &= \mu_0 n I \\
\oint \mathbf{E} \cdot d\mathbf{l} &= -\frac{d\Phi_B}{dt} = -\pi r^2 \frac{dB}{dt} \\
\mathbf{E} &= \frac{\mu_0 n}{2} \frac{d\mathbf{B}}{dt} = \frac{\mu_0 n I}{2} \frac{di}{dt} \\
\end{align*}
\]