(40) Positive charge $Q$ is distributed uniformly along the positive $x$-axis from $x=0$ to $x=a$. A negative charge $q$ is located on the $x$-axis at $x = a + r$, a distance $r$ to the right of the end of $Q$.

(a) (25) Calculate the $x$ and $y$ components of the electric field produced by the charge distribution $Q$ at points on the $x$-axis where $x > a$.

(b) (5) Calculate the force (magnitude and direction) that the charge distribution $Q$ exerts on $q$.

(c) (10) Find the magnitude of this force in the specific limit when $r \gg a$. Explain your result physically.

\[ E = \frac{kQ}{a} \int_{0}^{a} \frac{dx}{(a+r-x)^2} = \frac{kQ}{a} \int_{b}^{b-q} \frac{dz}{z^2} = \frac{kQ}{a} \left( \frac{1}{b-a} - \frac{1}{b} \right) = \frac{kQ}{a} \left( \frac{1}{b-a} - \frac{1}{a+r} \right) = \frac{kQ}{a} \left( \frac{1}{a+r} \right) \]

\[ E_x = \frac{kQ}{r(a+r)} \]

\[ E_y = -\frac{kQ|q|}{r(a+r)} \]

\[ \vec{F} = -\frac{kQ|q|}{r^2} \]

When the size of the charge distribution becomes so thin, the distance to observation point, this charge distribution should produce the same effect as the linear charge.
\( e) \quad P_{\text{net}} = \int_S \mathbf{E} \cdot d\mathbf{A} = S \int_0^a \mathbf{E} \cdot \mathbf{r} \frac{di}{dt} + \frac{\mu_0 n i}{R} \int_0^a \mathbf{B} \cdot d\mathbf{A} = \frac{dU_L}{dt} \)

(60) A very long solenoid with \( n \) turns per unit length and a radius \( a \) carries a current \( i \) that is increasing at a constant rate of \( di/dt \).

a) (10) Find the magnetic field inside the solenoid.

b) (13) Find the induced electric field inside the solenoid.

c) (13) Find the Pointing vector inside the solenoid.

d) (14) Find the magnetic energy stored in a length \( l \) of the solenoid and the rate at which the energy is increasing as a function of time.

e) (10) Show that the energy stored in a current-carrying solenoid can be thought of as entering through the cylindrical walls of the solenoid. Hint: Calculate the total rate at which electromagnetic energy is flowing into the solenoid through the solenoid walls and compare it with a result of part d.

\( a) \quad \mathbf{B} = \mu_0 n i \mathbf{i} \rightarrow \text{the left} \)

\( b) \quad \oint \mathbf{E} \cdot d\mathbf{l} = -\frac{d}{dt} \oint \mathbf{B} \cdot d\mathbf{l} = -n \mu_0 \mathbf{r} \frac{di}{dt} \)

\( E = -\mathbf{E} \cdot \frac{d\mathbf{B}}{dt} = -\frac{\mu_0 n i}{2} \mathbf{r} \frac{di}{dt} \) \text{ counter-clockwise}

\( \text{According to Lenz law } \mathbf{E} \text{ should point to increase of the flux } \rightarrow \text{ to produce } \mathbf{B} \text{ field opposite to } \mathbf{B} \)

c) \( \mathbf{S} = \mathbf{E} \times \mathbf{B} \)

\( \mathbf{S} = \frac{\mu_0 n^2 i \frac{di}{dt}}{2} \)

d) \( U_B = \frac{B^2}{2\mu_0} , \quad U_L = U_B \cdot \text{Vol} \cdot \text{mag} = \frac{B^2}{2\mu_0} \pi a^2 = \frac{\mu_0 n^2 \pi a^2}{2} \)

\( \langle \mathbf{v} \rangle U_L = \frac{L \mathbf{i}^2}{2} , \quad L = \frac{\mu_0 n^2 \pi a^2}{\mathbf{E}} = \mu_0 n^2 \pi a^2 \left( \mathbf{e} \times \mathbf{i} \right) \)

\( \mathbf{P} = \frac{d}{dt} \mathbf{U}_{\mathbf{L}} = \mu_0 n^2 \pi a^2 \mathbf{i} \frac{di}{dt} \)
4. \( \lambda = \frac{\lambda}{n} = \frac{c}{\sqrt{K K_m}} = \frac{3 \times 10^8}{6 \times 10^6} = 2 \text{ m} \), \( L = L_n \).

To get the maximum flux one should place a loop at the nodes of \( \vec{E} \) (where \( B \) is max).

\[ S_{\text{max}} = 2 N \frac{X_n}{\lambda} \]

(60) Y-polarized electromagnetic wave with a frequency 6 MHz propagates in z-direction in air. You have an antenna in the form of a circular loop of wire with a diameter 1m to detect this wave.

1. (15) How should you orient your antenna in space to provide a maximum induced current in it?

2. (15) Suppose you oriented an antenna in this optimal position. The resistance of the antenna is 1 Ohm and the electric power input into the antenna is 50W. What is an amplitude of the magnetic field in the wave?

3. (15) What is the total power of the source producing this wave, if you know that this source is 1 km away from the point where you measured the field and it emits uniformly in all the directions?

4. (15) Suppose you settle the wave of the same frequency between two metallic plates. The space between plates is filled by the dielectric gas with a dielectric constant, \( K=4 \), and relative magnetic permeability \( K_m=1 \). The distance between the plates is 50m. Where between the plates should you place the circular loop antenna in order to get a maximum response in it?

\[ \vec{E} = \hat{y} \text{ E} \]

\[ \vec{B} = \hat{z} \text{ B} \]

1. Loop should be placed in x plane (i.e., \( \perp \) to \( \vec{B} \)) to provide \( \lambda \)-max flux of \( \vec{B} \).

2. \( \overrightarrow{\mathbf{P}} = \vec{c} = \overrightarrow{\mathbf{E}} \times \overrightarrow{\mathbf{B}} \)

\[ \overrightarrow{S} = \rho \overrightarrow{d} \]

\[ \rho = \frac{\left( \frac{d}{2} \right)^2 \omega \beta_{\text{max}}^2}{2} \]

\[ \beta_{\text{max}} = \sqrt{\frac{2 \rho R}{\omega \left[ \frac{\epsilon}{(2)} \right]^2}} = \sqrt{2.5 \times 10^{12}} \]

\[ I = \frac{1}{\mu_0} \frac{c}{2} \]

\[ I = \frac{9 \times 10^{-8} \times 3 \times 10^8}{2.5 \times 10^{-9} \times 4 \times 10^{-15}} = 9 \text{ W} \]

\[ P = I \times \overrightarrow{S} \cdot \overrightarrow{d} = 9 \times 3 \times 10^3 \times 10^3 = 1.08 \times 10^8 \text{ W} \]
(40) Two slits paced 0.1mm apart are placed 1m from a screen and illuminated by coherent light of wavelength 640nm.

1) What is the distance on the screen from the center of the interference pattern to the position where the intensity of the light is equal to the sum of intensities produced by each slit?

2) Suppose a very thin sheet of plastic (n=1.6) covers one slit. The center point of the screen, instead of being a maximum, is dark. What is the (minimum) thickness of the plastic?

\[ E = \sqrt{2} E_0 \Rightarrow E = \sqrt{2} \]

\[ k = \frac{n \lambda}{L} \]

\[ \frac{2 \lambda}{d} \sin \theta = \frac{m \lambda}{d} \]

\[ \theta = \frac{4 \pi}{\lambda} \]

\[ d = \frac{\lambda}{2(n-1)} = \frac{640 \text{ nm}}{2 \cdot 0.6} = 533 \text{ nm} \]

The closest position are: \( \pm 16 \text{ mm} \)