Physics 221: Exam 1, Spring-2010

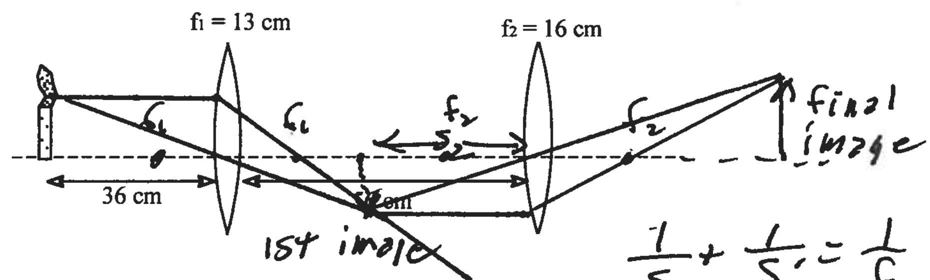
Name:____

Section. No:______________________________________

NOTE: Problems 1 and 2 are each worth 18 points and 3-6 are each worth 16 pts

1. A lighted candle is placed 36.0 cm in front of a converging lens of focal length $f_1 = 13$ cm which in turn is 56 cm in front of another converging lens of focal length $f_2 = 16.0$ (see Fig. below)
a) Draw a ray diagram to locate the final image
b) Now calculate the location of the final image after passing through both lenses
c) Is the final image real or virtual?
d) What is the magnification of the final image?
e) Is it erect or inverted?

a)



$$\frac{1}{S_1} + \frac{1}{S_1'} = \frac{1}{f_1}$$

$$\frac{1}{36} + \frac{1}{S_1'} = \frac{1}{13}$$

$$\frac{1}{S_1'} = \frac{1}{13} - \frac{1}{36} = 0.0491$$

$$S_1' \simeq 20.35\text{ cm}$$

$$m_1 = -\frac{S_1'}{S_1} = -\frac{20.35\text{ cm}}{36\text{ cm}}$$

$$= -0.565 \text{ (image inverted)}$$

$$\frac{1}{S_2} + \frac{1}{S_2'} = \frac{1}{f_2}$$

$$\frac{1}{35.65} + \frac{1}{S_2'} = \frac{1}{16}$$

b) $\frac{1}{S_2'} = 0.0344$, $S_2' = 29.03\text{cm}$ from 2nd lens

c) final image real

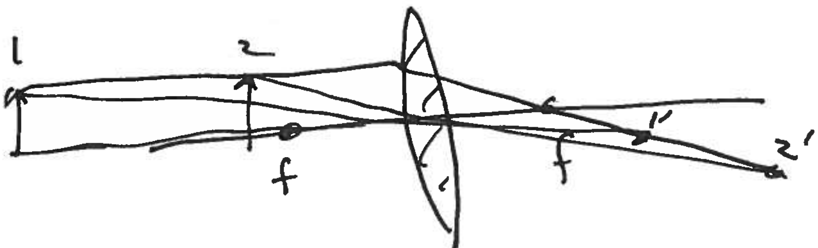
$$m_2 = -\frac{S_2'}{S_2} = -\frac{29.03}{35.65} = -0.814$$

inverts 1st image

d) $m_T = m_1 m_2 = 0.46$

e) erect since $m_T > 0$

2. An object is moving toward a converging lens of focal length f with a constant speed v_0 such that its distance s_0 from the lens is always greater than f.
a) Determine the velocity v_i of the image as a function of s_0, f, and v_0.
b) Which direction (towards or away from the lens) does the image move?
c) For what s_0 does the image's speed equal the object's speed?



$$\frac{1}{S_o} + \frac{1}{S'} = \frac{1}{f}$$

$$\frac{1}{S'} = \frac{1}{f} - \frac{1}{S_o}$$

$$\frac{1}{S'} = \frac{S_o - f}{f S_o}$$

$$S' = \frac{f S_o}{S_o - f}$$

differentiate re time.

$$-\frac{1}{S_o^2}\frac{dS_o}{dt} - \frac{1}{S'^2}\frac{dS'}{dt} = 0$$

$$\frac{V_o}{S_o^2} = -\frac{1}{S'^2} V_i'$$

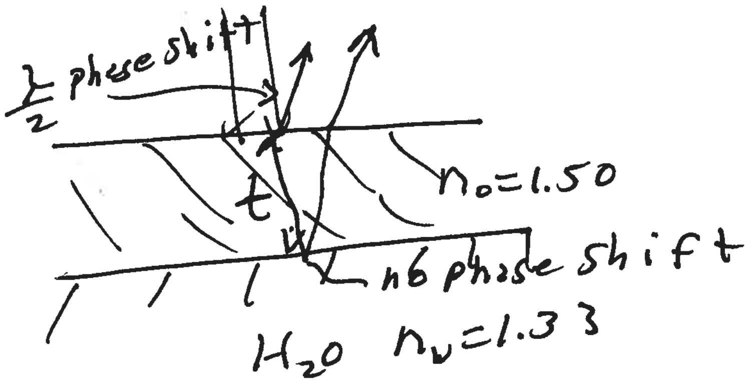
$$V_i' = -\frac{S'^2}{S_o^2} V_o = -\left(\frac{f}{S_o - f}\right)^2 V_o$$

b) image moves away from lens

c) speed equal when $\frac{f}{S_o - f} = 1$

or when $S_o = 2f$

3. A thin oil slick ($n_0 = 1.50$) floats on water ($n_w = 1.33$). When a beam of of white light strikes the film at normal incidence from air, the only enhanced reflected colors are red (650 nm) and violet (390 nm). From this information, deduce the (minimum) thickness of the oil slick.



For Const. Int. $P.D = 2t = (m+\frac{1}{2})\lambda_{medium}$

$$= (m+\frac{1}{2})\frac{\lambda_{vac}}{n_o}$$

$$2t = (m_{red}+\frac{1}{2})\frac{650\,nm}{1.5}$$

$$2t = (m_{violet}+\frac{1}{2})\frac{390}{1.5}$$

$$1 = \frac{(m_{red}+\frac{1}{2})\,650}{(m_{violet}+\frac{1}{2})\,390}$$

$$\therefore \frac{650}{390} = \frac{m_{violet}+\frac{1}{2}}{m_{red}+\frac{1}{2}} = \frac{2m_{violet}+1}{2m_{red}+1} = \frac{5}{3}$$

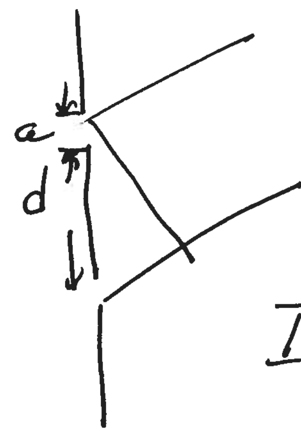
$$\therefore m_{violet} = 2$$

$$m_{red} = 1$$

$$\therefore 2t = \frac{3}{2}\frac{650}{1.5} =$$

$$\boxed{t = 325\,nm}$$

4. In a double slit experiment, the distance between the slits was forty wavelengths and this was five times the slit width. Compare (as a ratio) the intensity of the third order interference maximum with that of the zero-order maximum.



$$d = 40\lambda$$

$$5a = 40\lambda$$

$$a = 8\lambda$$

$$I = I_0 \cos^2\frac{\phi}{2}\left(\frac{\sin \beta/2}{\beta/2}\right)^2$$

$\phi = kd\sin\theta$, $\beta = ka\sin\theta$

for zero-order , $\phi = 0$, $\beta = 0$

$$I = I_0$$

for third-order , $\phi = 6\pi$ ($\frac{\phi}{2} = 3\pi$) $\therefore k\sin\theta = \frac{6\pi}{d}$

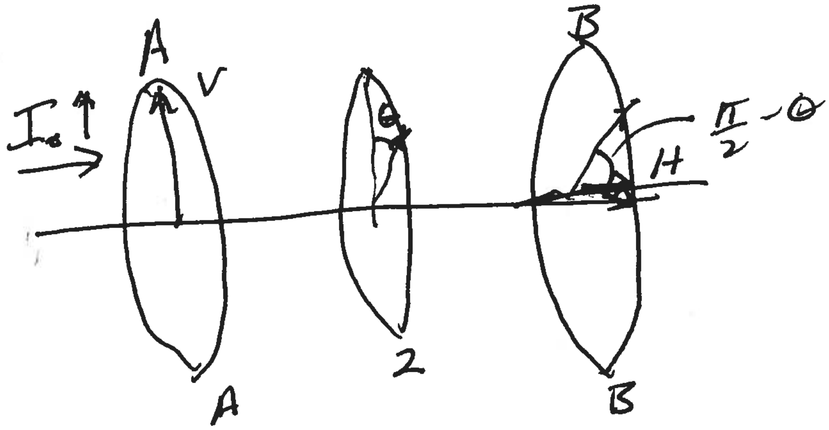
$$\beta = \frac{6\pi a}{d} = \frac{6\pi \times 8}{40} = \frac{6\pi}{5}$$

$$I = I_0\left(\frac{\sin\frac{3\pi}{5}}{\frac{3\pi}{5}}\right)^2 = 0.255$$

5. Two polarizers A and B are aligned so that their transmission axes are vertical and horizontal respectively. A third polarizer is placed between these two with its axis aligned at an angle θ with respect to the vertical. Assuming vertically polarized light of intensity I_0 is incident upon polarizer A,

a) Find an expression for the light intensity I transmitted through this three-polarizer sequence(Express your answer in terms of I_0 and θ).

b) Calculate dI/dθ then use it to find the angle that maximizes I.



Since I_0 vertically polarized no intensity loss through A.

$I_A = I_0$

Now going through ② $I_2 = I_0 \cos^2\theta$

Now going through B $I_B = I_2 \cos^2(\frac{\pi}{2} - \theta)$

$= I_2 \sin^2\theta$

$\therefore$ $I_B = I_0 \cos^2\theta \sin^2\theta = \frac{I_0}{4}(\sin 2\theta)^2$

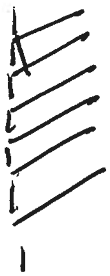
b) $\frac{dI_B}{d\theta} = \frac{I_0}{2} \sin 2\theta \cos 2\theta \times 2 = 0$

$\sin 4\theta = 0$

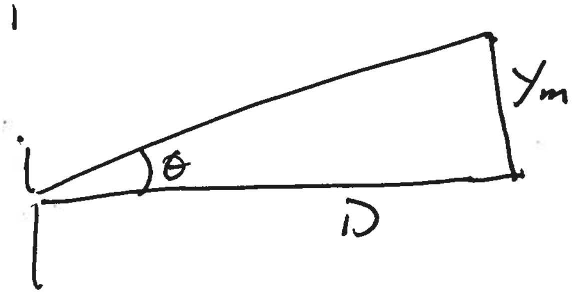
or $4\theta = \pi$

$\theta = \pi/4 = 45°$

6. When yellow sodium light, $\lambda = 589$ nm, falls on a diffraction grating, its first-order peak on a screen 66.0 cm away falls 3.32 cm from the central peak. Another source produces a first-order line 3.71 cm from the central peak.
a) What is its wavelength?
b) How many lines/cm are on the grating?



$d \sin\theta = m\lambda$ for Const Int.



$\tan\theta = \frac{y_m}{D} \approx \sin\theta \quad \sin\theta \ll 1$

$d \sin\theta = \lambda$

$\frac{d\, y_m}{D} = \lambda$

$\therefore \frac{d\, y^1_{m=1}}{D} = \lambda_1$

$\frac{d\, y^2_{m=1}}{D} = \lambda_2$

$\frac{y^1_{m=1}}{y^2_{m=1}} = \frac{\lambda_1}{\lambda_2} = \frac{3.32}{3.71}$

$\lambda_2 = \frac{3.71}{3.32} \times 589\,\text{nm}$

a) $\boxed{\lambda_2 = 658\ \text{nm}}$

b) ~~$D = \frac{d\, y_m}{\lambda}$~~ $d = \frac{D\lambda}{y_m} = \frac{66 \times 589 \times 10^{-9}\,\text{m}}{3.32}$

$d = 1.171 \times 10^{-5}\,\text{m}$

$\therefore$ $\boxed{854\ \text{lines/cm}}$