1. Initially, a sample of radioactive nuclei of type A contains four times as many nuclei as a sample of radioactive nuclei of type B. Two days later the two samples contain the same number of nuclei.
   a) Which type of nucleus has the longer half-life?
   b) Determine the half-life of type B nuclei if the half-life of type A nuclei is known to be 0.500 d.

   \[ N_{10} = \text{initial number of type A nuclei} \]

   \[ N_{B0} = \text{initial number of type B nuclei} \]

   Type B nucleus decays slower since A must decay faster to reach B after two days.

   \[ N_A = N_{A0} e^{-0.693 \frac{t}{T_{1/2,A}}} \]

   \[ N_B = N_{B0} e^{-0.693 \frac{t}{T_{1/2,B}}} \]

   \[ \frac{N_A}{N_{B0}} = \frac{N_{A0} e^{-0.693 \frac{t}{T_{1/2,A}}}}{N_{B0}} \]

   After 2 days \( N_A = N_B + N_{A0} = 4 N_{B0} \)

   \[ 1 = 4 e^{-0.693 \times 2 \left[ \frac{1}{T_{1/2,B}} - \frac{1}{T_{1/2,B}} \right]} \]

   \[ 1 = \left[ 2 - \frac{1}{T_{1/2,B}} \right] \]

   \[ \frac{1}{T_{1/2,B}} = 1 \text{ d} \]
2. An α particle fired head-on at a stationary nickel ($^{58.7}_{28}$Ni) nucleus approaches to a radius of 12 fm before being turned around.
   a) What is the maximum coulomb force exerted on the α particle?
   b) What is the electric potential energy of the α particle at the point of closest approach?
   c) Find the initial kinetic energy of the α particle.

Note: \( k = 8.99 \times 10^9 \text{ N-m}^2/\text{C}^2 \); \( 1 \text{ u} = 1.66 \times 10^{-27} \text{ kg} \); \( e^- = 1.6 \times 10^{-19} \text{ C} \)

a) For Nickel \( Z = 28 \), for α \( (4 \text{He}) \) \( Z = 2 \)

\[
F_c = \frac{q_1 q_2}{r^2} = 8.99 \times 10^9 \frac{e^2}{(28 \times 2)^2} (12 \times 10^{-15} \text{ m})^{-2}
\]

\[ F_c = 89.5 \text{ N} \]

b) At point of closest approach

\[ E_{PE} = \frac{q_1 q_2}{r_{min}} \]

\[ = \frac{8.99 \times 10^9 \times 28 \times 2 \times e^2}{12 \times 10^{-15} \text{ m}} = 1.02 \times 10^{-12} \text{ J} \]

C) By conservation of energy

\[ KE_{before} = E_{PE} \text{ at closest approach since } KE = 0 \]

\[ KE_{before} = 1.07 \times 10^{-12} \text{ J} \]
3 A thin coating of thickness $t = 340.0$ nm and refractive index $n = 1.480$ is placed on a glass slide of refractive index $n = 1.35$. Which visible wavelengths $400 < \lambda < 700$ nm will be absent in the reflected beam?

we went destructive interference
Path difference must be $n \lambda_{med}$ \( n = 1.35 \)
Since phase shift at boundary puts wave already out of phase.

\[ 2t = n \lambda_{med} \]
\[ n = 1, 2, 3, \ldots \]

\[ 2 \times 340 \text{ nm} = \frac{n \lambda_{med}}{1.48} \]
\[ \lambda_{med} = \frac{1.006 \text{ nm}}{n} \]

\[ n = 1 \quad \lambda = 1006 \text{ nm} \]
\[ n = 2 \quad \lambda = 503.2 \text{ nm} \]
\[ n = 3 \quad \lambda = 335.3 \text{ nm} \]
4. A metal bar of mass $M$ and length $L$ is suspended from two conducting wires as shown below. A uniform magnetic field of magnitude $B$ points vertically downward. Find the angle $\theta$ the suspending wires make with the vertical when the bar carries a current $I$. Express your answer in terms of $B$, $M$, $I$, $L$, and $g$ (acceleration of gravity).

\[
F_m = \text{Magnetic force} = BIL
\]

\[
T \sin \theta = F_m = BIL
\]

\[
T \cos \theta = Mg
\]

\[
\tan \theta = \frac{BIL}{mg}
\]

\[
\theta = \tan^{-1} \left( \frac{BIL}{mg} \right)
\]
5. An object is located 16.0 cm in front of a converging lens of focal length 12.0 cm. To the right of the converging lens, separated by a distance of 20.0 cm, is a diverging lens of focal length -10.0 cm.

a) Locate the image formed by the converging lens and compute its magnification.
b) Use this image as the object for the second lens and compute the image distance and magnification of the second lens.
c) Find the magnification of the final image.
d) Is the final image real or virtual?
e) Is the final image upright or inverted?

\[ \frac{1}{d_i} + \frac{1}{d_o} = \frac{1}{f} \]

\[ \frac{1}{16} + \frac{1}{d_i} = \frac{1}{12} \]

\[ d_i = 48 \text{ cm} \]

\[ M_1 = \frac{-d_i}{d_o} = -\frac{48}{16} = -3 \]

b) \[ \frac{1}{d_i} + \frac{1}{d_o} = \frac{1}{f} \] Note: 1st image is now a virtual object for second lens.

\[ \frac{1}{d_i} = \frac{1}{28} - \frac{1}{10} \]

\[ d_i = -15.6 \text{ cm} \]

\[ M_2 = \frac{-d_i}{d_o} = \frac{15.6}{-28} = -0.557 \]

\[ M_T = M_1, M_2 = 1.67 \]

d) Final image is virtual.

e) Final image is upright since \( M_T > 0 \).
6. A 40.0-mH inductor, with internal resistance of 30.0 Ω, is connected to an AC source 
v(t) = 286 sin(390t) volts where t is in seconds.

a) What is the impedance of the inductor in the circuit?
b) What are the peak and rms voltages across the inductor (including the internal resistance)?
c) What is the peak current in the circuit?
d) What is the average power dissipated in the circuit?

\[ V_L = \omega L = 390 \times 40 \times 10^{-3} \text{H} \]
\[ X_L = 390 \times 40 \times 10^{-3} \text{H} \]
\[ Z = \sqrt{(30)^2 + (15.6)^2} \]
\[ Z = 33.8 \Omega \]
\[ V_{\text{rms}} = 202 V \]
\[ V_{\text{peak}} = 286 V \]
\[ I_{\text{peak}} = \frac{V_{\text{peak}}}{Z} = 8.46 A \]
\[ P = \frac{V_{\text{rms}} \times I_{\text{rms}} \times \text{coeff}}{2} = \frac{R}{Z} \]
\[ P = \frac{286 \times 8.46 \times 0.888}{2} = \frac{30.0 \times 2}{33.8} \text{W} \]
\[ P = 1074 \text{W} \]
7. The circuit in Figure below has been connected for a long time.

a) Find the current through the 1.00 Ω and 8.00 Ω resistors
b) Find the potential difference across the capacitor
c) Find the charge on the capacitor
d) Find the energy stored in the capacitor

\[ \frac{1}{R_T} = \frac{1}{10 \, \Omega} + \frac{1}{5 \, \Omega} \]
\[ R_T = 10 \, \Omega \]
\[ V = I R_T \]
\[ 10\, V = I \cdot \frac{10 \, \Omega}{3} \]
\[ I = \frac{3}{2} \, \text{A} \]
\[ I = I_1 + I_2 \]
\[ 3 = 3 I_2 \]
\[ I_2 = 1 \, \text{A} \quad \text{and} \quad I_1 = 2 \, \text{A} \]
\[ b) \quad V_p - V_a = V_c \]
\[ V_p - V_a = 2 \, \text{V} \]
\[ V_p - V_b = 5 \, \text{V} \]
\[ V_c - V_a = -6 \, \text{V} \]
\[ C = \frac{Q}{V} \]
\[ Q = 1.0 \times 10^{-6} \, \text{C} \times 6 \, \text{V} \]
\[ Q = 6.0 \, \mu \text{C} \]
\[ d) \quad E = \frac{1}{2} \left( \frac{Q^2}{C} \right) \]
\[ E = \frac{1}{2} \left( \frac{36 \times 10^{-12}}{1 \times 10^{-6} \text{F}} \right) \]
\[ E = 18 \times 10^{-6} \, \text{J} \]
8. Compute the electrical potential energy of four identical charges of charge Q situated at the four corners of a square of side L. This is also the amount of work it takes to assemble the configuration. Express your answer in terms of Q, k (the constant in Coulomb’s law), and L.

\[
EPE = \text{sum of } EPE \text{ from all distinct pairs},
\]

\[
EPE = EPE_{12} + EPE_{13} + EPE_{14} + EPE_{23} + EPE_{24} + EPE_{34}
\]

\[
EPE_{12} = \frac{kQ^2}{L} = EPE_{23} = EPE_{34}
\]

\[
EPE_{13} = \frac{kQ^2}{\sqrt{2}L} = EPE_{24}
\]

\[
EPE = 4 \frac{kQ^2}{L} + 2 \frac{kQ^2}{\sqrt{2}L}
\]

\[
EPE = 5.41 \frac{kQ^2}{L}
\]