1. (10 pts) The figure below shows a valve separating a reservoir from a water tank. If this valve is opened, what is the maximum height above point B attained by the water stream coming out of the right side of the tank? ASSUME: \( h = 10.0 \text{ m} \), \( L = 2.00 \text{ m} \), and \( \theta = 30.0^\circ \), and assume the cross sectional area \( A \) is very large compared with that at \( B \).

Let's compute the velocity of the output stream.

Applying Bernoulli's Principle:

\[
P_A + \rho g h + \frac{1}{2} \rho V_A^2 = P_B + \rho g L \sin \theta + \frac{1}{2} \rho V_B^2
\]

\[
P_A = P_B = P_0
\]

\[
V_A < V_B
\]

\[
\rho g h = \rho g L \sin \theta + \frac{1}{2} \rho V_B^2
\]

\[
\frac{V_B}{2} = g (h - L \sin \theta)
\]

\[
V_B = 2g (h - L \sin \theta)
\]

\[
V_B = \sqrt{2 \times 9.8 \times (10 - 2 \times \frac{2}{2})} = 13.3 \text{ m/s}
\]

\[
h) \quad V_y = V_{i,y} - 2g y_{max}
\]

\[
y_{max} = \frac{V_{i,y}^2}{2g} = \frac{(V_B \sin \theta)^2}{2g} = \frac{(13.3)^2}{2g} = \frac{2.25 \text{ m}}{2g}
\]
2. (12 pts) The weight of a rectangular block of low density material is 15.0 N. With a thin string, the center of the horizontal bottom face of the block is tied to the bottom of a beaker partly filled with water. When 25% of the block’s volume is submerged the tension in the string is 10.0 N.

a) Find the buoyant force on the block

b) Oil of density 800 kg/m³ is now steadily added to the beaker, forming a layer above the water and surrounding the block. What happens to the string tension as the oil is added?

c) The string breaks when its tension reaches 60.0 N. At this moment, 25% of the block’s volume is still below the water line. What additional fraction of the block’s volume is below the top surface of the oil?

\[ T + W - F_B = 0 \]
\[ F_B = T + W \]

\( F_B = 10 + 15 \)
\( F_B = 25 \text{ N} \)

b) As oil is added this increases the pressure at the base of the block which increases the buoyant force which increases the tension in the string.

\[ \frac{F_o}{F_B} = \frac{50N}{25N} = 2 = \frac{P_o \Delta V_o g}{P_{H_2O} V g} \]
\[ 2 = \frac{P_o \Delta V_o}{P_{H_2O} V} \]
\[ \frac{\Delta V_o}{V} = \frac{P_{H_2O}}{2P_o} = \frac{1000}{1600} = \frac{5}{8} = 0.625 \]

\[ \text{Additional volume covered} = 62.5\% \]
3. (10 pts) A particle of mass m moves along a straight line with constant velocity \( \vec{v}_0 \) in the x direction, a distance b from the axis.

a) What is the angular momentum (magnitude and direction) of the particle about the origin O?

b) Show that Kepler's second law is satisfied by showing that the two shaded triangles in the figure have the same area when \( t_D - t_C = t_B - t_A \)

\[ \vec{L} = |\vec{v} \times \vec{p}| = m \vec{v}_0 \cdot \vec{b} \]

Direction is into paper.

b) Consider triangle \( O O'B \)

its area = \( \frac{V_0 \cdot t_B \cdot b}{2} \)

Area of triangle \( O O'A = \frac{V_0 \cdot t_A \cdot b}{2} \)

Area of triangle \( OAB = \frac{V_0 \cdot b \cdot (t_B - t_A)}{2} \)

Similarly, Area of triangle \( OCD = \frac{V_0 \cdot b \cdot (t_D - t_C)}{2} \)

i.e. if time intervals equal; name i.e.

\( t_B - t_A = t_D - t_C \)

then \( OAB = OCD \) in equal intervals of time; hence verifying Kepler's 2nd Law!!
4. (10 pts) A puck of mass \( m_1 = 80.0 \text{ g} \) and radius \( r_1 = 4.00 \text{ cm} \) glides across an air table at a speed \( v = 1.50 \text{ m/s} \) as shown below. It makes a glancing collision with a second puck of radius \( r_2 = 6.00 \text{ cm} \) and mass \( m_2 = 120 \text{ g} \) (initially at rest) such that their rims just touch. Since their rims are coated with instant glue, the pucks stick together and rotate after the collision.

a) What is the angular momentum of the system relative to the center of mass?

b) What is the angular speed about the center of mass after sticking together?

\[ R_{cm} = \frac{m_1 v_1 + m_2 v_2}{m_1 + m_2} \]

\[ R_{cm} = \frac{80 \text{g} (10 \text{ cm})}{14 \text{ cm}} = 4 \text{ cm} \]

\[ L_{cm} = m_1 r_1 \omega \times 0.06 \]

\[ e = 0.08 \text{ kg} \times 1.50 \text{ m} \times 0.06 \text{ m} \]

\[ L_{cm} = 7.2 \times 10^{-3} \text{ kg m}^2/\text{s} \]

b) After

\[ L_{cm} = I_1 \omega + I_2 \omega \]

\[ 7.2 \times 10^{-3} = \left( I_1 + I_2 \right) \omega \]

\[ I_1 = m_1 r_1^2 + m_1 d_1^2 \]

\[ I_2 = m_2 r_2^2 + m_2 d_2^2 \]

\[ 7.2 \times 10^{-3} = \left[ 0.08 \left( \frac{0.04^2}{2} + 0.06^2 \right) \right] + \left[ 0.12 \left( \frac{0.06^2}{2} + 0.04^2 \right) \right] \omega \]

\[ = 3.52 \times 10^{-4} + 0.08 \times 10^{-4} \omega \]

\[ \omega = 9.47 \text{ rad/ s} \]
5. (12 pts) A block of mass $M$ hangs from a rubber cord. The block is supported so that the cord is not stretched. The unstretched length of the cord is $L_0$ and its mass is $m \ll M$. The "spring constant" for the cord is $k$. The block is released and momentarily stops at the lowest point.

a) Determine the tension in the string when the block is at its lowest point.

b) What is the length of the cord in this "stretched" position?

c) If the block is held in this lowest position, find the speed of a transverse wave in the cord.

\[
\begin{align*}
\text{a) Cons. of energy} & & \frac{mg}{2} y^2 \\
\text{\hspace{1cm}} & & y = \frac{3gM}{k} \\
\text{\hspace{1cm}} & & T = ky = 2.9M \\
\text{b) At lowest point.} & & y = 0 \\
\text{c) } U = \sqrt{\frac{T}{m}} & = \sqrt{\frac{2.9M}{m}} \frac{k}{k} y \\
\text{\hspace{1cm}} & & U = \sqrt{\frac{2.9M(L_0+y)}{m}} = \sqrt{\frac{2.9M(L_0+2mg)}{k}}
\end{align*}
\]
6. (12 pts) A small sphere of radius \( r \) starts from rest at the top of a larger sphere of radius \( R \) where \( r \ll R \). It rolls without slipping down the larger sphere until it reaches a point where it leaves the larger sphere.

a) Draw a free body diagram of the small sphere showing all the forces acting on it at some point \( \theta \) in its motion.

b) Find the angle \( \theta \) where the small sphere leaves the large sphere.

HINT: This point occurs when the normal force vanishes!

\[

\text{2nd Law b) } mg \cos \theta - N = \frac{m^2 \omega^2}{R} \quad \text{Note } R \ll r
\]

\[
\text{Cons. of energy } \quad mg R = mgR(1 - \cos \theta) + \frac{1}{2} m \omega^2 + \frac{1}{2} I \omega^2
\]

\[
mg R(1 - \cos \theta) = \frac{1}{2} m U_{cm}^2 + \frac{1}{2} \frac{2}{5} m r^2 \omega^2
\]

Pure rolling \( \Rightarrow \) \( R \omega_{cm} = U_{cm} \)

\[
mg R(1 - \cos \theta) = \frac{1}{2} m U_{cm}^2 + \frac{1}{5} m U_{cm}^2
\]

\[
g R(1 - \cos \theta) = \frac{7}{10} \frac{m U_{cm}^2}{R}
\]

When \( N = 0 \) \( mg \cos \theta = \frac{U_{cm}^2}{R} \)

\[
g = g \cos \theta = \frac{7}{10} g \cos \theta
\]

\[
1 = \frac{17}{10} \cos \theta \quad \cos \theta = \frac{10}{17}
\]

\[
\theta = \arccos \left( \frac{10}{17} \right) \approx 54.0^\circ
\]
7. (10 pts) A small block of mass $m_1 = 0.500$ kg is released from rest at the top of a frictionless, curve shaped wedge of mass $m_2 = 3.00$ kg which sits on a frictionless horizontal surface as shown below. When the block leaves the wedge its velocity is measured to be 4.00 m/s to the right as shown.
a) What is the velocity of the wedge after the block reaches the horizontal surface.
b) What is the height of the wedge?

b) Cons. of energy
\[ \frac{1}{2}m_1v_1^2 + \frac{1}{2}m_2v_2^2 = m_1gh \]
\[ \frac{1}{2}[0.5x(4)^2 + 3(0.667)^2] = 0.5x9.8 \times 9.8 \]
\[ 9.33 = 9.8 \]
\[ h \approx 0.952 \text{ m} \]
8. (12 pts) The launching mechanism of a popgun consists of a trigger-released spring. The spring is compressed to a position \( y_A \), and is fired. The projectile of mass \( m \) rises to a position \( y_C \) above the position it leaves the spring indicated as position \( y_B = 0 \). Consider a firing of the gun for which \( m = 35.0 \) g, \( y_A = -0.120 \) m, and \( y_C = 20.0 \) m.

a) Calculate the spring constant assuming no friction forces anywhere.
b) Now suppose that everything remains the same but now there is a friction force of magnitude 2.00 N acting on the projectile as it rubs on the barrel. The vertical height from point A to the end of the barrel is 0.600 m. After the spring is compressed and the popgun fired, to what height does the projectile rise above point B?

\[
L = \frac{1}{2} k (0.120 m)^2 - 0.035 kg \times 9.8 \times 0.12 m
\]

\[
TME \text{ at } C = mg \cdot y_C
\]

\[
\therefore \quad \frac{1}{2} k (0.0144) = mg(y_C + 0.12 m)
\]

\[
= 0.035 \times 9.8 \times (20.12 m)
\]

\[
= 6.90
\]

\[
k = 958.8 \text{ N/m}
\]

b) Use Work-Energy Theorem

\[
W_{\text{thrust}} \quad \Delta TME = TME_f - TME_i
\]

\[
\therefore \quad W_{\text{thrust}} = -2.0N \times 0.600m = mg \cdot y - \left[ \frac{1}{2} k (0.0144) - mg \cdot (0.12) \right]
\]

\[
-1.2 + \frac{1}{2} k (0.0144) - mg \cdot (0.12) = mg \cdot y
\]

\[
-1.2 + 6.9 - 0.0411 = 0.035 \times 9.8 \cdot y
\]

\[
5.66 = 0.35 \times 9.8 \cdot y
\]

\[
y = 16.5 \text{ m}
\]
9. (12 pts) A car accelerates down a hill, going from rest to 30.0 m/s in 6.00 s. A toy inside the car hangs by a string from the car's ceiling. The ball in the figure represents the toy, of mass 0.100 kg. The acceleration is such that the string remains perpendicular to the ceiling.
a) Determine the angle $\theta$.
b) Determine the tension in the string.

\[ \sin \theta = 0.1 / g \]

\[ \sin \theta = \frac{5 \text{ m/s}^2}{4.8 \text{ m/s}^2} = 0.810 \]

\[ \theta = 30.7^\circ \]

\[ T = 0.110 g \times 4.8 \cos(30.7^\circ) \]

\[ T = 0.843 N \]