1. The reel shown in the figure has radius $R$ and moment of inertia $I$. One end of the block of mass $m$ is connected to a spring of force constant $k$ and the other end is fastened to a cord which is wrapped around the reel. The reel axle and incline are frictionless. The reel is turned counterclockwise so the spring stretches a distance $d$ from its unstretched position and the reel is then released from rest. Find the angular speed when the spring is again unstretched.

\[ \text{Cons. of energy!} \]

\[
y = 0 \quad \text{TME Before release: } T = mgd \sin \theta + \frac{1}{2} kd^2
\]

\[
T = \frac{1}{2} mv^2 + \frac{1}{2} I \omega^2
\]

\[
mgd \sin \theta + \frac{1}{2} kd^2 = \frac{1}{2} mv^2 + \frac{1}{2} I \omega^2
\]

\[
\text{but } V = R \omega = \omega \left[ \frac{mR^2}{2} + \frac{I}{2} \right]
\]

\[
\omega = \left( \frac{mgd \sin \theta + \frac{1}{2} kd^2}{mR^2 + I} \right)^{1/2}
\]

\[
\omega = \left[ \frac{2mgd \sin \theta + kd^2}{mR^2 + I} \right]^{1/2}
\]
2. A spool of wire of mass $M$ and radius $R$ is unwound under a constant force $\vec{F}$. Assuming the spool is a uniform solid cylinder that doesn’t slip ($I_{CM} = MR^2/2$):

a) Find the acceleration of the center of mass in terms of $F$ and $M$. (HINT: The friction force acts to the right in this problem)

b) Find the magnitude of the force of friction.

c) If the cylinder starts from rest and rolls without slipping, what is the speed of its center of mass after it has rolled through a distance $d$.

\[
F + f = MA_{cm} \quad (1)
\]

\[
(F - f)R = I\alpha = \frac{MR^2}{2}
\]

\[
F - f = \frac{MR\alpha}{2} \quad \text{but} \quad Rd = A_{cm}
\]

\[
\Rightarrow \frac{M\alpha}{2} = \frac{MA_{cm}}{2} \quad (2)
\]

Adding (1) and (2):

\[
2F = \frac{3MA_{cm}}{2}
\]

\[
A_{cm} = \frac{4F}{3M}
\]

b) $f = F - \frac{M A_{cm}}{2} = F - \frac{M}{2} \cdot \frac{4F}{3M} = F - \frac{2F}{3}
$

\[
f = \frac{F}{3}
\]

c) $V_s^2 = V_{ic}^2 + 2A_{cm}d
$

\[
V_{icm} = \sqrt{\frac{4Fd}{3M}}
\]
3. Rachel Jagielski and Bill Linz, have decided to become astronauts and have traveled into space. Their space suits have been adjusted so they have the same mass $M$ and they are connected by a rope of length $d$ having negligible mass. They are isolated in space, orbiting their center of mass at speeds $v$. Treating these new astronauts as particles, calculate:

a) the magnitude of the angular momentum of the two astronaut system

b) the rotational energy of the system

Now by pulling on the rope Rachel shortens the distance between them to $d/2$

c) what is the new angular momentum of the system?

d) what are the astronauts new speeds?

e) what is the new rotational energy of the system?

\[ L = \frac{mv_d + mv_d}{2} \]

\[ L = mvd \]

\[ RKE = \frac{1}{2}mv^2 + \frac{1}{2}mv^2 = mv^2 \]

Also

\[ RKE = \frac{1}{2}I\omega^2 + \frac{1}{2}I\omega^2 \]

\[ = I\omega^2 = \frac{md^2}{4} \omega^2 \]

\[ \omega = \frac{d}{2} \omega = mv^2 \]

\[ \frac{d}{2} \]

\[ C) \text{ Ang. mom. conserved } = mvd \]

\[ d) \quad L = mv'd' = mvd \]

\[ = \frac{mv'd'}{2} = mvd \]

\[ V' = 2V \]

\[ C) \quad RKE' = mv' = 4mv^2 \]
4. A rigid, massless rod has three particles with equal masses attached to it as shown in the Fig. below.
   The rod is free to rotate in a vertical plane about a frictionless axle perpendicular to the rod through the
   point P and is released from rest in the horizontal position at \( t = 0 \). Assuming \( m \) and \( d \) are known, find:
   a) The moment of inertia of the system of three particles about the pivot.
   b) The torque acting on the system at \( t = 0 \).
   c) The angular acceleration of the system at \( t = 0 \).
   d) The linear acceleration of the particle labeled 3 at \( t = 0 \).
   e) The maximum kinetic energy of the system
   f) The maximum angular speed reached by the rod
   g) The maximum angular momentum of the system

\[ \begin{align*}
   2) \quad I_p &= m \left( \frac{4}{9} d^2 + \frac{md^2}{3} + \frac{m(2d)^2}{9} \right) \\
   &= \frac{2}{9} md^2 \\
   3) \quad I_p &= \frac{7}{3} md^2 \\
   -2) \quad \tau &= -mg \left( \frac{2d}{3} + \frac{mgd}{3} + \frac{mgd}{3} \right) \\
   \tau &= +mgd \cdot \frac{1}{2}
\end{align*} \]

\[ \begin{align*}
   C) \quad \tau &= I \alpha = \frac{7}{3} m d^2 \alpha = \frac{2g}{7d} \alpha = \text{counter clockwise} \\
   D) \quad \omega^2 &= \frac{2mgd}{3} = \frac{2mgd}{7d} = \frac{6g}{7d} \\
   \omega &= \sqrt{\frac{6g}{7d}} \\
   = md^2 \sqrt{\frac{14g}{3}}
\end{align*} \]
5. Two identical hard spheres, each of mass \( m \) and radius \( r \), are released from rest in otherwise empty space with their centers separated by the distance \( R \). They are allowed to collide under their gravitational attraction.

a) Find the velocity of each sphere just before they make contact
b) Find the momentum of each sphere just before they make contact

NOTE: Express your answers in terms of \( m \), \( G \), \( r \), and \( R \)

2) Let's work this from energy conservation.

Before: \( T + U = -\frac{Gmm}{R} \)

After: \( T + U = -\frac{Gmm}{R} + \frac{mv^2}{2m} \)

\[ \frac{mv^2}{2m} - \frac{Gmm}{2m} = -\frac{Gmm}{R} \]

\[ v^2 = Gm \left[ \frac{1}{2m} - \frac{1}{R} \right] \]

\[ v = \sqrt{\frac{Gm}{2m} \left( \frac{1}{2m} - \frac{1}{R} \right)} \]

\[ \rho = m v = m \sqrt{\frac{Gm}{2m} \left( \frac{1}{2m} - \frac{1}{R} \right)} \]

or \[ \sqrt{\frac{Gm^3}{2m} \left( \frac{1}{2m} - \frac{1}{R} \right)} \]