Example 1 (\(\alpha = \text{constant}\))

(1) What is the angular speed of rotation of the Earth?

1 rotation (\(2\pi\,\text{rad}\)) per 24 hours (1440 min)

(2) An old 33 1/3 rpm record player starts from rest and reaches operating speed in 2.00 seconds. Through what angle did it turn in those 2.00 seconds?

\[ t = 2.00\,\text{s}, \quad \omega_0 = 0, \quad \omega = 33\,\frac{1}{3}\,\text{rpm} \]

(3) A computer hard drive rotates at 5400 rpm. What angular acceleration will get it up to speed in just 150 revolutions starting from rest? Use Eq. (3) with \(\theta - \theta_0 = 150\,\text{rev.} = ?\,\text{rad}\)

Example 1 (cont’d)

(4) A hard drive reaches 5400 rpm in 3.20 seconds. What was the average angular speed assuming constant angular acceleration?

(5) A dentist’s drill accelerates to 1800 rpm in 2.50 seconds. what is its angular acceleration?

(6) The angular velocity changes from 47.0 rad/s to –47.0 rad/s in 2.00 seconds. What is the angular acceleration?
Example 2

A grinding wheel turns at a constant angular acceleration of \( 60.0 \text{ rad/s}^2 \) from \( 24.0 \text{ rad/s} \) for \( 2.00 \text{ sec} \). Then, a circuit breaker trips. It turns through 432 rad as it coasts to a stop at a constant angular acceleration.

Find:
(a) the total angle between \( t = 0 \) and the time it stopped;
(b) the time it stopped;
(c) the angular acceleration as it slowed down.

Also sketch \( \theta - t \) graph.

Practice Problem 1

Problem 3: (25 points)
A grinding wheel turns at a varying angular acceleration of \( \alpha(t) = [30.0 \text{ rad/s}^3] t \) for \( 2.00 \text{ sec} \). Assume the initial angular speed of \( 20.0 \text{ rad/s} \). Then, a circuit breaker trips. It turns through 400 rad as it coasts to a stop at a constant angular acceleration.

a. (5 pts) Find the total angle (\( \theta_{\text{total}} \)) between \( t = 0 \) and the time it stopped.
b. (10 pts) Find the time (\( t_{\text{final}} \)) it stopped. Find the angular acceleration as it slowed down.
c. (10 pts) Sketch \( \theta - t \), \( \omega - t \), \( \alpha - t \) graphs.
Practice Problem 2

A solid cylinder (radius \( R = 2 \text{ m} \) and height \( H = 5 \text{ m} \)) turns at a constant angular acceleration of 60.0 rad/s\(^2\) from 24.0 rad/s for 2.00 sec. Then, a circuit breaker trips. It turns through 432 rad as it coasts to a stop at a constant angular acceleration.

(a) Find the total angle between \( t = 0 \) and the time it stopped.
(b) Find the time it stopped.
(c) Find the angular acceleration as it slowed down.
(d) Find the speed (\( v \)) of point \( P \) at \( t = 2.00 \text{ sec} \).
(e) Sketch the motion of point \( P \) in \( v-t \) graph. Also sketch \( \theta t \) graph.

Torque due to Gravity?

\[ \vec{\tau} = \vec{r} \times \vec{F} = ? \]

We assume:

Center of mass (CM or cm)
= Center of gravity
(if uniform gravity)
**Torque due to Gravity?**

\[ \vec{\tau} = \vec{r} \times \vec{F} = ? \]

**Net Torque due to Gravity?**

One extra sphere!  
Do we need a new concept?  
No!

**Solid sphere \((M, r)\)**

\[ \vec{\tau}_{\text{net}} = \vec{\tau}_{\text{rod}} + \vec{\tau}_{\text{sphere}} \]
Example 1

Calculate the torque on the 2.00-m long beam due to a 50.0 N force (top) about
(a) point C (= c.m.)
(b) point P

Calculate the torque on the 2.00-m long beam due to a 60.0 N force about
(a) point C (= c.m.)
(b) point P

Calculate the torque on the 2.00-m long beam due to a 50.0 N force (bottom) about
(a) point C (= c.m.)
(b) point P

Example 1 (cont’d)

Calculate the net torque on the 2.00-m long beam about
(a) point C (= c.m.)
(b) point P
Practice Problem 2

Determine the net torque (magnitude and direction) on the 2.00-m-long beam about:

a. (15 pts) point C (the CM position);
b. (10 pts) point P at one end.

\[
\vec{\tau}_i = \vec{r}_i \times \vec{F}_i = ? \\
\vec{\tau}_{\text{net}} = \sum_i \vec{\tau}_i
\]

Practice Problem 2

Determine the net torque (magnitude and direction) on the disk about pin.

\[
\vec{\tau}_i = \vec{r}_i \times \vec{F}_i = ? \\
\vec{\tau}_{\text{net}} = \sum_i \vec{\tau}_i
\]
2(a) Express the moment of inertia of the array of point objects about the y-axis in terms of $m, M, X_1, X_2,$ and/or $Y$.

$$\text{Moment of Inertia} = m \times 0.50 \text{ m} + M \times 1.50 \text{ m}.$$ 

2(b) Consider a helicopter rotor blade as a long thin rod. If each of the three blades is 3.75 m long and has a mass of 160 kg, calculate the moment of inertia of the three blades about the axis of the rotation.

$$\text{Moment of Inertia} = 3 \times m \times (3.75 \text{ m})^2.$$
2(c) A meter stick (mass \( M = 0.500 \text{ kg} \) and length \( L = 1.00 \text{ m} \)) is hung from the wall by means of a metal pin through the hole, and used as a pendulum. Express the moment of inertia of the stick about the pin (= the axis of the rotation) in terms of \( M, L, \) and \( x \).

2(d) A door (solid rectangular thin plate) of mass \( M = 15.0 \text{ kg} \) is free to rotate on about hinge line. Calculate the moment of inertia of the door about the hinge line.
2(e) A solid disk (mass $M = 3.00$ kg and radius $R = 20.0$ cm) is hung from the wall by means of a metal pin through the hole, and used as a pendulum. Calculate the moment of inertia of the disk about the pin (= the axis of the rotation).

2(f) A ball (solid sphere) of mass $M$ and radius $R$ on the end of a thin rod (mass $m$ and length $l$). Express the moment of inertia of the system of the rod and the ball about the A-B axis (thin rod; mass $m$ and length $l$) in terms of $M, R, m,$ and $l$. 

**Hint**

Uniform sphere of radius $r_0$
Practice Problem 4

A uniform disk turns at 7.00 rev/s around a frictionless spindle. A nonrotating rod, of the same mass \( m \) as the disk and length \( l \) equal to the disk’s diameter, is dropped onto the disk. They then both turnaround the spindle with their centers superposed. There is no slipping between the rod and the disk. What is the moment of inertia of the disk+rod system about the axis?

Example 3

What will be the speed of each of the following objects when it reaches the bottom of an incline if it starts from rest at a vertical height \( H \) and rolls without slipping? Find the speed using the conservation of mechanical energy.
(i) Solid sphere \((M, R_0)\)
(ii) Solid cylinder \((M, R_{op}, l)\)
(iii) Thin hoop \((M, R_{op}, l)\)
Example 4

Analyze the rolling sphere in terms of forces and torques: find the magnitudes of the velocity $v$ and the friction force $F_{fr}$.

1. F.B.D.?

2. Is $F_{fr}$ static or kinetic friction?
   - Static Friction
   - No work
   - No energy loss
   - $K+U=$constant

Example 5

A sting is wrapped around a uniform solid cylinder of mass $M$ ($= 0.500$ kg) and radius $R$ ($= 0.200$ m), and the cylinder starts falling from rest. Draw the free-body diagram for the cylinder while it descends. Also find (a) its acceleration, (b) the tension in the string, and (c) the speed after the cylinder has descended $h = 0.800$ m.
Example 5 - Challenge

A string is wrapped around a hollow cylinder of mass $M$ (= 0.500 kg), inner radius $R_1$ (= 0.100 m), outer radius $R_2$ (= 0.200 m). The cylinder starts falling from rest.

(a) Draw the free-body diagram for the cylinder while it descends.

Also find:
(b) its acceleration;
(c) the tension in the string;
(d) the speed after the cylinder has descended $h = 0.800$ m.