MP Chap. 15
\[ a_y(x, t) = \frac{\partial^2 y(x, t)}{\partial t^2} = -\omega^2 A \cos(kx - \omega t) = -\omega^2 y(x, t) \quad \text{(15.10)} \]

The acceleration of a particle equals \(-\omega^2\) times its displacement, which is the result we obtained in Section 14.2 for simple harmonic motion.

We can also compute partial derivatives of \(y(x, t)\) with respect to \(x\), holding \(t\) constant. The first derivative \(\frac{\partial y(x, t)}{\partial x}\) is the slope of the string at point \(x\) and at time \(t\). The second partial derivative with respect to \(x\) is the curvature of the string:

\[
\frac{\partial^2 y(x, t)}{\partial x^2} = -k^2 A \cos(kx - \omega t) = -k^2 y(x, t) \quad \text{(15.11)}
\]

From Eqs. (15.10) and (15.11) and the relationship \(\omega = \nu k\) we see that:

\[
\frac{\partial^2 y(x, t)/\partial t^2}{\partial^2 y(x, t)/\partial x^2} = \frac{\omega^2}{k^2} = \nu^2 \quad \text{and}
\]

\[
\frac{\partial^2 y(x, t)}{\partial x^2} = \frac{1}{\nu^2} \frac{\partial^2 y(x, t)}{\partial t^2} \quad \text{(wave equation)} \quad \text{(15.12)}
\]

We will consider the motion of strings traveling in the positive \(x\)-direction. You
Exercise 15.4

**Description:** Ultrasound is the name given to frequencies above the human range of hearing, which is about 20000 Hz. Waves above this frequency can be used to penetrate the body and to produce images by reflecting from surfaces. In a typical ultrasound scan, the...

Ultrasound is the name given to frequencies above the human range of hearing, which is about 20000 Hz. Waves above this frequency can be used to penetrate the body and to produce images by reflecting from surfaces. In a typical ultrasound scan, the waves travel with a speed of 1500 m/s. For a good detailed image, the wavelength should be no more than 1.0 mm.
Exercise 15.12

**Description:** The equation \( y(x, t) = A \cos 2\pi f \left( \frac{x}{v} - t \right) \) may be written as \( y(x, t) = A \cos \left( \frac{2\pi}{\lambda}(x - vt) \right) \). (a) Use the last expression for \( y(x, t) \) to find an expression for the transverse velocity \( v_y \) of a particle in the string on which the wave...

The equation \( y(x, t) = A \cos 2\pi f \left( \frac{x}{v} - t \right) \) may be written as \( y(x, t) = A \cos \left[ \frac{2\pi}{\lambda}(x - vt) \right] \).

15.11 • A sinusoidal wave is propagating along a stretched string that lies along the \( x \)-axis. The displacement of the string as a function of time is graphed in Fig. E15.11 for particles at \( x = 0 \) and at \( x = 0.0900 \, \text{m} \). (a) What is the amplitude of the wave? (b) What is the period of the wave? (c) You are told that the two points \( x = 0 \) and \( x = 0.0900 \, \text{m} \) are within one wavelength of each other. If the wave is moving in the \( +x \)-direction, determine the wavelength and the wave speed. (d) If instead the wave is moving in the \( -x \)-direction, determine the wavelength and the wave speed. (e) Would it be possible to determine definitively the wavelength in parts (c) and (d) if you were not told that the two points were within one wavelength of each other? Why or why not?

15.12 • **CALC** Speed of Propagation vs. Particle Speed.
(a) Show that Eq. (15.3) may be written as

\[ y(x, t) = A \cos \left( \frac{2\pi}{\lambda}(x - vt) \right) \]

(b) Use \( y(x, t) \) to find an expression for the transverse velocity \( v_y \) of a particle in the string on which the wave travels. (c) Find the maximum speed of a particle of the string. Under what circumstances is this equal to the propagation speed \( v \)? Less than \( v \)? Greater than \( v \)?
Standard Expression for a Traveling Wave

**Description:** Identify independant variables and parameters in the standard travelling wave; find phase, wavelength, period, and velocity of the wave using omega and k.

**Learning Goal:**
To understand the standard formula for a sinusoidal traveling wave.

One formula for a wave with a y displacement (e.g., of a string) traveling in the x direction is

\[ y(x, t) = A \sin(kx - \omega t). \]

All the questions in this problem refer to this formula and to the wave it describes.

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**15.10** - A water wave traveling in a straight line on a lake is described by the equation

\[ y(x, t) = (3.75 \text{ cm}) \cos(0.450 \text{ cm}^{-1} x + 5.40 \text{ s}^{-1} t) \]

where y is the displacement perpendicular to the undisturbed surface of the lake.

(a) How much time does it take for one complete wave pattern to go past a fisherman in a boat at anchor, and what horizontal distance does the wave crest travel in that time?

(b) What are the wave number and the number of waves per second that pass the fisherman?

(c) How fast does a wave crest travel past the fisherman, and what is the maximum speed of his cork floater as the wave causes it to bob up and down?
Video Tutor: Out-of-Phase Speakers

**Description:** Two face-to-face speakers play the same tone. What happens when the signal phase is reversed for one speaker?

First, [launch the video](#) below. You will be asked to use your knowledge of physics to predict the outcome of an experiment. Then, close the video window and answer the question on the right. You can watch the video again at any point.

interference
Exercise 15.20: Weighty Rope

**Description:** One end of a nylon rope is tied to a stationary support at the top of a vertical mine shaft of depth $h$. The rope is stretched taut by a box of mineral samples with mass $m_1$ attached at the lower end. The mass of the rope is $m_2$. The geologist at the... 

One end of a nylon rope is tied to a stationary support at the top of a vertical mine shaft of depth $79.0\,\text{m}$. The rope is stretched taut by a box of mineral samples with mass $25.0\,\text{kg}$ attached at the lower end. The mass of the rope is $1.70\,\text{kg}$. The geologist at the bottom of the mine signals to his colleague at the top by jerking the rope sideways. (Do *not* neglect the weight of the rope.)

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**Example 15.3**

**Eq. 15.19**

**P15.72**

**Eq. 15.6**

**Sec. 15.3 and 15.4**

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**15.18** • A 1.50-m string of weight $0.0125\,\text{N}$ is tied to the ceiling at its upper end, and the lower end supports a weight $W$. Neglect the very small variation in tension along the length of the string that is produced by the weight of the string. When you pluck the string slightly, the waves traveling up the string obey the equation

$$y(x, t) = (8.50\,\text{mm}) \cos(172\,\text{m}^{-1}x - 4830\,\text{s}^{-1}t)$$

Assume that the tension of the string is constant and equal to $W$.

(a) How much time does it take a pulse to travel the full length of the string? (b) What is the weight $W$? (c) How many wavelengths are on the string at any instant of time? (d) What is the equation for waves traveling *down* the string?

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**15.21** • A simple harmonic oscillator at the point $x = 0$ generates a wave on a rope. The oscillator operates at a frequency of $40.0\,\text{Hz}$ and with an amplitude of $3.00\,\text{cm}$. The rope has a linear mass density of $50.0\,\text{g/m}$ and is stretched with a tension of $5.00\,\text{N}$.

(a) Determine the speed of the wave. (b) Find the wavelength.

(c) Write the wave function $y(x, t)$ for the wave. Assume that the oscillator has its maximum upward displacement at time $t = 0$.

(d) Find the maximum transverse acceleration of points on the rope. (e) In the discussion of transverse waves in this chapter, the force of gravity was ignored. Is that a reasonable approximation for this wave? Explain.