Chap. 14: Oscillations, Periodic Motion, Simple Harmonic Motion

Characterized by:
Period \((T)\) and Frequency \((f)\)

Preparation for:
Mechanical wave motion
Electromagnetic wave

Dynamics: \(F = ma, \quad \tau = I \alpha\)

Equation of Motion:

General solution:
Looking Back to Chap. 7

(a) To describe oscillations in terms of amplitude, period, frequency and angular frequency

(b) To do calculations with simple harmonic motion (SHM); To analyze simple harmonic motion using energy

To apply the ideas of simple harmonic motion to different physical situations

To analyze the motion of a simple pendulum, followed by a physical pendulum

To explore how oscillations die out
To learn how a driving force can cause resonance
What is S.H.M.?

Periodic Motion
- Horizontal oscillation
- Vertical oscillation
- Vibration

Common characteristics
→ Simplified Model

Simple Harmonic Motion

Oscillations
S.H.M.
Useful Math and Physics

Trig. functions:
\[
\begin{align*}
\sin(\theta + \pi/2) &= \cos(\theta) \\
\sin(\theta + \pi) &= -\sin(\theta) \\
\cos(\theta - \pi/2) &= \sin(\theta)
\end{align*}
\]

S.H.O.
1) Spring plus block
   Horizontal
   Vertical

2) Pendulum
   Simple pendulum
   Physical pendulum

Derivative and integral
Trig. functions

Approximation:
\[
\sin \theta \sim \theta
\]
Circular Motion to SHM

Oscillations

Light shines on apparatus, casting shadow of ball on screen

Starts here
Analyzing Spring+Block System

Displacement to the left, restoring force to the right

Zero displacement, zero restoring force

Displacement to the right, restoring force to the left

(a)
S.H.M. : Spring+Block system and Simple Pendulum system

Spring+block System

Simple Pendulum
Physical Pendulum – S.H.M.?
Physical Pendulum – S.H.M.? 

- Simple Pendulum:
  \[ \alpha = \frac{\tau}{I} = \frac{F_{\text{tan}} \cdot d}{I} = \left(\frac{-mg \sin \theta}{I}\right) \cdot d \approx \left(\frac{-mgd}{I}\right) \theta \]

- Physical Pendulum:
  \[ F_{\text{tan}} = -mg \sin \theta \]

Axis of rotation
S.H.M. : Spring+Block system and Simple Pendulum system

Spring+block System

Simple Pendulum

\[ a = \frac{F}{m} = -\left(\frac{k}{m}\right)x \]

The acceleration is proportional to the displacement from equilibrium.

\[ \alpha = \frac{\tau}{I} = \frac{F_{\text{ang}}}{I} = -\frac{m\sin(\theta)}{I} \]

The angular acceleration is proportional to the angular displacement from equilibrium. \( \theta \): small

Simple Pendulum

Physical Pendulum – S.H.M.?

Physical Pendulum

Simple Pendulum

\[ \alpha = \frac{d^2 \theta}{dt^2} = \frac{F_{\text{ang}}}{I} = -\frac{(mg \sin(\theta))d}{I} \]

\[ \alpha = \frac{d^2 \theta}{dt^2} = -\left(\frac{mg d}{I}\right)\theta \]
S.H.M. – Dynamics (Summary)

Math about $C$

\[ \frac{d^2 x}{dt^2} = -C \cdot x \quad \text{or} \quad \frac{d^2 \theta}{dt^2} = -C \cdot \theta \]

If $x(t) = A \cos(\sqrt{C} \cdot t + D)$, then:

\[ \frac{dx}{dt} = -\sqrt{C} \cdot A \sin(\sqrt{C} \cdot t + D) \]

\[ \frac{d^2 x}{dt^2} = -\sqrt{C} \cdot \sqrt{C} \cdot A \cos(\sqrt{C} \cdot t + D) \]

\[ = -C \left( A \cos(\sqrt{C} \cdot t + D) \right) \]

\[ = -C \cdot x \]

\[ \therefore \quad \frac{d^2 x}{dt^2} = -C \cdot x \]

Physics about $C$

**Dimensional Analysis**

\[ \left[ \frac{k}{m} \right] = \frac{N}{m} = \frac{(kg)(m/s^2)}{kg} = \frac{1}{s^2} \]

\[ \left[ \frac{g}{l} \right] = \frac{m/s^2}{m} = \frac{1}{s^2} \]

\[ \sqrt{\text{Constant}} = \frac{1}{s} \]

We define “angular frequency” $\omega$:

\[ \cdot \quad \omega = \sqrt{\text{Constant}} \]

\[ \cdot \quad f = \frac{\omega}{2\pi} \]

\[ \cdot \quad T = \frac{1}{f} \]

$x(t) = A \cos(\omega t + \phi)$
Oscillations

Circular Motion to SHM

\[ x = R_0 \cos \theta \]

where \( \theta = \omega t \)

\[ x(t) = R_0 \cos (\omega t) \]

Kin. Equation of SHM

Starts here

\[ x(t) = R_0 \cos (\omega t) \]
S.H.M.

Acceleration is proportional to displacement.

\[ \frac{d^2 X}{d t^2} = -C \cdot X \]
What is $\phi$?

$x(t) = A \sin(\omega t)$

$x(t) = A \cos(\omega t)$
Oscillations

What is $\phi$?

S.H.M.
Math and Physics

Trig. functions:
- $\sin(\theta + \pi/2) = \cos(\theta)$
- $\sin(\theta + \pi) = -\sin(\theta)$
- $\cos(\theta - \pi/2) = \sin(\theta)$

S.H.O.
- 1) Spring plus block
- 2) Pendulum
  - Horizontal
  - Vertical
  - Simple pendulum
  - Physical pendulum

Derivative and integral
- Trig. functions
- Approximation:
  - $\sin \theta \sim \theta$

$x(t) = A \sin(\omega t)$

$= A \cos[\omega t + (-\pi/2)]$

$= A \cos[\omega (t - \pi/2\omega)]$

$= A \cos[\omega (t - 2\pi/4\omega)]$

$(2\pi/4) \omega = (2\pi/\omega)/4 = T/4$
S.H.M.

3 independent variables:
- Angular frequency
- Amplitude
- Phase angle

\( x(t) = A \cos(\omega t + \phi) \)
\( \theta(t) = A \cos(\omega t + \phi) \)

Period:
\[ T = \frac{1}{f} \rightarrow T = \frac{2\pi}{\omega} \]

Example of \( x(t) \), where \( \phi = 0 \)

Quick Check:
How do \( v(t) \) and \( a(t) \) look like?

Note: Angular frequency (\( \omega \)) is NOT the angular velocity.
Graphs
Oscillations

Technical Steps

3 independent variables:
- Angular frequency
- Amplitude
- Phase angle

S.H.M.

\[ x(t) = A \cos(\omega t + \phi) \]
\[ \theta(t) = A \cos(\omega t + \phi) \]

Period:
\[ T = \frac{1}{f} \rightarrow T = \frac{2\pi}{\omega} \]

Example of \( x(t) \), where \( \phi = 0 \)

Quick Check:
How do \( v(t) \) and \( a(t) \) look like?

Note: Angular frequency (\( \omega \)) is NOT the angular velocity.
3. Consider the particle whose motion is represented by the $x$-versus-$t$ graph below.

a. Is this periodic motion?  

b. Is this motion SHM?  

c. What is the period?  

d. What is the frequency?  

e. You learned in Chapter 2 to relate velocity graphs to position graphs. Use that knowledge to draw the particle’s velocity-versus-time graph on the axes provided.
4. Shown below is the velocity-versus-time graph of a particle.

   a. What is the period of the motion?  
   
   b. Draw the particle’s position-versus-time graph, starting from $x = 0$ at $t = 0$ s.

5. The graph on the next page is the position-versus-time graph of an oscillating particle. It is constructed of parabolic segments that are joined at $x = 0$.

   a. Is this simple harmonic motion? Why or why not?

   b. Draw the corresponding velocity-versus-time graph.
      Hint: What is the derivative of a parabolic function?

   c. Draw the corresponding acceleration-versus-time graph.
5. The graph on the next page is the position-versus-time graph of an oscillating particle. It is constructed of parabolic segments that are joined at $x = 0$.
   a. Is this simple harmonic motion? Why or why not?

   b. Draw the corresponding velocity-versus-time graph.
      Hint: What is the derivative of a parabolic function?
   c. Draw the corresponding acceleration-versus-time graph.

   d. At what times is the position a maximum? __________________________
      At those times, is the velocity a maximum, a minimum, or zero? __________
      At those times, is the acceleration a maximum, a minimum, or zero? __________
   e. At what times is the position a minimum (most negative)? _________________
      At those times, is the velocity a maximum, a minimum, or zero? __________
      At those times, is the acceleration a maximum, a minimum, or zero? __________
   f. At what times is the velocity a maximum? __________________________
      At those times, where is the particle? ________________________________
   g. Can you find a simple relationship between the sign of the position and the sign of the acceleration at the same instant of time? If so, what is it?

6. The figure shows the position-versus-time graph of a particle in SHM.
   a. At what time or times is the particle moving to the right at maximum speed?
   b. At what time or times is the particle moving to the left at maximum speed?
   c. At what time or times is the particle instantaneously at rest?
An object on the end of a spring is oscillating in simple harmonic motion. If the amplitude of oscillation is doubled, how does this affect the oscillation period $T$ and the object’s maximum speed $v_{\text{max}}$?

A. $T$ and $v_{\text{max}}$ both double.

B. $T$ remains the same and $v_{\text{max}}$ doubles.

C. $T$ and $v_{\text{max}}$ both remain the same.

D. $T$ doubles and $v_{\text{max}}$ remains the same.

E. $T$ remains the same and $v_{\text{max}}$ increases by a factor of $\sqrt{2}$. 

Q14.1
Q14.2

This is an \( x-t \) graph for an object in simple harmonic motion.

At which of the following times does the object have the **most negative velocity** \( v_x \)?

A. \( t = \frac{T}{4} \)

B. \( t = \frac{T}{2} \)

C. \( t = 3 \frac{T}{4} \)

D. \( t = T \)
Q14.3

This is an $x$-$t$ graph for an object in simple harmonic motion. At which of the following times does the object have the most negative acceleration $a_x$?

A. $t = \frac{T}{4}$
B. $t = \frac{T}{2}$
C. $t = 3\frac{T}{4}$
D. $t = T$
Q14.4

This is an $a_x$-$t$ graph for an object in simple harmonic motion. At which of the following times does the object have the most negative displacement $x$?

A. $t = 0.10$ s
B. $t = 0.15$ s
C. $t = 0.20$ s
D. $t = 0.25$ s
This is an $a_x$-$t$ graph for an object in simple harmonic motion.

At which of the following times does the object have the most negative velocity $v_x$?

A. $t = 0.10$ s
B. $t = 0.15$ s
C. $t = 0.20$ s
D. $t = 0.25$ s
Q14.6

This is an $x$-$t$ graph for an object connected to a spring and moving in simple harmonic motion.

At which of the following times is the potential energy of the spring the greatest?

A. $t = T/8$
B. $t = T/4$
C. $t = 3T/8$
D. $t = T/2$
E. more than one of the above
Q14.7

This is an $x$-$t$ graph for an object connected to a spring and moving in simple harmonic motion.

At which of the following times is the kinetic energy of the object the greatest?

A. $t = T/8$

B. $t = T/4$

C. $t = 3T/8$

D. $t = T/2$

E. more than one of the above
To double the total energy of a mass-spring system oscillating in simple harmonic motion, the amplitude must increase by a factor of

A. 4.
B. $2\sqrt{2} = 2.828$.
C. 2.
D. $\sqrt{2} = 1.414$.
E. $\frac{1}{2}\sqrt{2} = 1.189$. 
A simple pendulum consists of a point mass suspended by a massless, unstretchable string.

If the mass is doubled while the length of the string remains the same, the period of the pendulum

A. becomes 4 times greater.
B. becomes twice as great.
C. becomes greater by a factor of \( \sqrt{2} \).
D. remains unchanged.
E. decreases.
Practice Problems

Problem 8: (25 points)

A student wants to use a meter stick (mass $M = 0.500 \text{ kg}$ and length $L = 1.00 \text{ m}$) as a pendulum. She plans to drill a small hole through the meter stick and suspend it from a smooth pin attached to the wall (see the figure below). The center of mass (c.m.) of the stick is displaced a small angle ($\theta = 2.00^\circ$) from the vertical and released. Note that the gravitational acceleration near the Earth's surface is $g = 9.80 \text{ m/s}^2$.

a. (10 pts) Express the moment of inertia of the stick around the pin in terms of $x$.

b. (5 pts) Express the angular frequency ($\omega$) of small oscillations in terms of $M$, $L$, $g$, and $x$.

c. (5 pts) Find the numerical value of $x$ to obtain the shortest possible period for small oscillations?

d. (5 pts) If she takes the system to the Moon and perform the same experiment, does the period of the oscillation increase, decrease, or remain in the same? Explain your answer.

![Diagram of the meter stick with a small angle displacement and the pin labeled x]
Problem 5: (50 points) Rotational motion. Remember Exam 3!
You drill a small hole through a uniform plastic sphere (radius $R$, mass $M$) at distance $x$ from its edge. The sphere is hung from the wall by means of a metal pin through the hole. The sphere is held at rest horizontally (position A) and then released. Position B shows the sphere where $0 \leq \theta \leq 90^\circ$. Ignore friction between the pin and the hole, and air resistance. Note that the acceleration due to the Earth’s gravity is $g$.

a. (10 pts) Express the moment of inertia about the pin in terms of $x$, $M$, $R$, and/or $g$.

b. (10 pts) Express the torque (magnitude and direction) at position B due to gravity on the sphere about the pin in terms of $x$, $M$, $R$, $g$, and/or $\theta$.

c. (5 pts) Express the magnitude of the angular acceleration at position B in terms of $x$, $M$, $R$, $g$, and/or $\theta$.

d. (5 pts) The angular acceleration in part (c) is a function of the angle $\theta$. At what angle $\theta$ is the magnitude of angular acceleration half its maximum value? Note $0 \leq \theta \leq 90^\circ$.

e. (10 pts) Use the conservation of mechanical energy to find the magnitude ($\omega$) of angular velocity of the sphere when the center-of-mass (c.m.) point reaches the vertical line (position C).

f. (5 pts) Using the result of part (e), express the speed ($v$) of the c.m. point at position C.

g. (5 pts) If you used a uniform steel sphere of the same radius $R$ (this means the mass is heavier than $M$) and perform the same experiment, is the angular speed in part (e) increased, decreased, or same? Explain why.

Problem 6: (25 points) S.H.M.
You take the same plastic sphere (radius $R$ and mass $M$) in Problem 5 and use it as a physical pendulum. The sphere is held at rest at position B ($\theta_0 = 2^\circ$) and then released. Ignore friction between the pin and the hole, and air resistance. Thus, it undergoes a simple harmonic motion (S.H.M.). Note that the magnitude of acceleration due to the Earth’s gravity is $g$.

a. (15 pts) Express the period of the S.H.M. in terms of $x$, $M$, $R$, and/or $g$.

b. (10 pts) If you used a uniform steel sphere of the same radius $R$ (this means the mass is heavier than $M$) and perform the same experiment, is the period in part (a) increased, decreased, or same? Explain why.
2(c) A solid disk (mass \( M = 3.00 \) kg and radius \( R = 20.0 \) cm) is hung from the wall by means of a metal pin through the hole, and used as a pendulum. Calculate the moment of inertia of the disk about the pin (= the axis of the rotation).
Concepts of S.H.M.

Dynamics & Kinematics:
Spring+block
→ \( F = ma \) & \((x, v, a)\)

Pendulum
→ a part of circular motion
→ \( \tau = I \alpha \) & \((\theta, \omega, \alpha)\)

Force:
Conservative force
Restoring force

Conservation:
\[ K + U = \text{constant} \]

+ S.H.M. (\( \omega \) as angular frequency)

Problem 1: (25 points)
A plywood disk of radius \( R \) and mass \( M \) has a small hole drilled through it at distance \( x \) from its edge. The disk is hung from the wall by means of a metal pin through the hole, and is used as a pendulum. The disk is held at rest at \( \theta_0 = 30^\circ \) and then released. The figure shows the disk where \( \theta < \theta_0 \).

(a) (5 pts) Express the torque (magnitude and direction) due to gravity on a disk about the pin in terms of \( x, M, R, g, \) and \( \theta \).

(b) (5 pts) Express the moment of inertia about the pin in terms of \( x, M \) and \( R \).

(c) (5 pts) Express the angular acceleration in terms of \( x, M, R, g, \) and/or \( \theta \).

(d) (5 pts) The angular acceleration in part (c) is a function of the angle \( \theta \). Find the angle \( \theta_{\text{min}} \) which minimize the magnitude of angular acceleration. Note \( 0 \leq \theta_{\text{min}} \leq 30^\circ \). If necessary, use \( \frac{d}{d\theta} (A\theta) = A\cos(A\theta) \) and \( \frac{d}{d\theta} (A\cos(A\theta)) = -A\sin(A\theta) \) where \( A \) is constant.

(e) (5 pts) Use the conservation of mechanical energy to find the magnitude of angular velocity of the disk when the center-of-mass point reaches the vertical line.
Problem 5: (50 points) Rotational motion. Remember Exam 3!
You drill a small hole through a uniform plastic sphere (radius $R$, mass $M$) at distance $x$ from its edge. The sphere is hung from the wall by means of a metal pin through the hole. The sphere is held at rest horizontally (position A) and then released. Position B shows the sphere where $0 \leq \theta \leq 90^\circ$. Ignore friction between the pin and the hole, and air resistance. Note that the acceleration due to the Earth's gravity is $g$.

a. (10 pts) Express the moment of inertia about the pin in terms of $x$, $M$, $R$, and/or $g$.

b. (10 pts) Express the torque (magnitude and direction) at position B due to gravity on the sphere about the pin in terms of $x$, $M$, $R$, $g$, and/or $\theta$.

c. (5 pts) Express the magnitude of the angular acceleration at position B in terms of $x$, $M$, $R$, $g$, and/or $\dot{\theta}$.

d. (5 pts) The angular acceleration in part (c) is a function of the angle $\theta$. At what angle $\theta$ is the magnitude of angular acceleration half its maximum value? Note $0 \leq \theta \leq 90^\circ$.

e. (10 pts) Use the conservation of mechanical energy to find the magnitude ($\alpha$) of angular velocity of the sphere when the center-of-mass (c.m.) point reaches the vertical line (position C).

f. (5 pts) Using the result of part (e), express the speed ($v$) of the c.m. point at position C.

g. (5 pts) If you used a uniform steel sphere of the same radius $R$ (this means the mass is heavier than $M$) and perform the same experiment, is the angular speed in part (e) increased, decreased, or same? Explain why.
Oscillations
Periodic Motion
Simple Harmonic Motion (S.H.M.)

Equation of Motion:
\[
\frac{d^2x}{dt^2} + \omega^2 x = 0 \quad \text{or} \quad \frac{d^2\theta}{dt^2} + \omega^2 \theta = 0
\]

General solution:
\[
x(t) = A \cos(\omega t + \phi)
\]
\[
\theta(t) = A \cos(\omega t + \phi)
\]

→ Position \((x \text{ or } \theta)\) as a function of time.
Example 7: Angular Velocity?

\[ \omega = \sqrt{\frac{mgd}{I}} \]

\[ T = \frac{2\pi}{\omega} \]

If the pendulum is placed on the Moon,

\[ \omega_{\text{Moon}} < \omega_{\text{Earth}} \]

\[ T_{\text{Moon}} > T_{\text{Earth}} \]

**NOTE:**

\[ \frac{1}{2}I\omega^2 + 0 = 0 + mgh \]

Angular frequency

Angular velocity
How to find $T$?

Want to find $T$
Need to know $\omega$
Then, find $I$

Find $I$
Calculate $\omega$
Then, find $T$
Example 6: Physical Pendulums (1)

Exercise 6.1: Express $\omega$

$\omega = \sqrt{\frac{mgd_{cm}}{I}}$

= [Find $I$]
Example 6: Physical Pendulums (2)

Exercise 6.2: Express $\alpha$

$= [\text{Find } I \text{ and } d]$
Example 6: Physical Pendulums (3)

Exercise 6.3: Express $\omega$

$$\alpha = -\frac{mgd}{I} \quad \theta \rightarrow \omega = \sqrt{\frac{mgd}{I}}$$

Steps:
1. What is asked?
   $\omega$ (rigid body) $\rightarrow I$ and $d$

2. How to find $I$ and $d$ for a system of two rigid bodies?
   $$I = I_1 + I_2$$

3. How to find $I_1$ (or $I_2$)?
   Where is the c.m. position? Use parallel-axis theorem

4. How to find $d$?
   Where is the c.m. position of the system?
Example 6: Physical Pendulums (3)

\[ d = 0.13 \text{ m} \]
\[ I = I_{\text{rod}} + I_{\text{sphere}} = \frac{1}{3} m_1 l^2 + \frac{2}{5} m_2 r^2 + m_2 (l+r)^2 \]
\[ = 0.0487 \text{ kg m}^2 \]
\[ m = m_1 + m_2 = 2.5 \text{ kg} \]
\[ \omega = \sqrt{\frac{mgd}{I}} = 8.09 \text{ rad/s} \]
Problem 4: (25 points)
Determine the net torque (magnitude and direction) due to gravity on the system about the pin, shown in the figure below. A beam has mass $M$ and length $l$; a big solid sphere has mass $M$ and radius $R$; a small sphere has mass $M/2$ and radius $R/2$. Assume $l > R$ and $l > 2x$. Also determine the moment of inertia of the system about the pin. [Hint: use Parallel-axis theorem.]

But, this can also be a Chap.14 problem, If I ask you to find $\omega$ (angular frequency) in S.H.M. of a physical pendulum.
Work on (a)

Problem 6: (25 points) S.H.M.
You take the same plastic sphere (radius $R$ and mass $M$) in Problem 5 and use it as a physical pendulum. The sphere is held at rest at position B ($\theta_0 = 2^\circ$) and then released. Ignore friction between the pin and the hole, and air resistance. Thus, it undergoes a simple harmonic motion (S.H.M.). Note that the magnitude of acceleration due to the Earth’s gravity is $g$.

a. (15 pts) Express the period of the S.H.M. in terms of $x$, $M$, $R$, and/or $g$.

b. (10 pts) If you used a uniform steel sphere of the same radius $R$ (this means the mass is heavier than $M$) and perform the same experiment, is the period in part (a) increased, decreased, or same? Explain why.

Repeat (a) without and with a particle (mass $m$)
Example 8: Physical Pendulums

Example of a physical pendulum with a point mass and a radius of 20.0 cm.

- Mass: $M = 3.00 \, \text{kg}$
- Radius: $R = 20.0 \, \text{cm}$
- Distance from pin: $2.00 \, \text{cm}$

Diagram showing the pendulum in motion with a point labeled $C$ and a radius $R_0$. The pendulum oscillates around the vertical axis.
Oscillations

Block & spring

\[ F = -kx \]

\[ F = ma \]

\[ \tau = I \alpha \]

\[ a = \frac{F}{m} \]

\[ \alpha = \frac{\tau}{I} \]

\[ \omega = \sqrt{\frac{k}{m}} \]

Motion

\[ x = A_x \cos(\omega t + \varphi) \]
\[ \theta = A_\theta \cos(\omega t + \varphi) \]
\[ \theta = A_\theta \cos(\omega t + \varphi) \]

Period

\[ T = \frac{2\pi}{\omega} \]

\[ v = -A_x \omega \sin(\omega t + \varphi) \]

\[ \omega_a = -A_\theta \omega \sin(\omega t + \varphi) \]

Don't be confused!
You take the same plastic sphere (radius $R$ and mass $M$) in Problem 5 and use it as a physical pendulum. The sphere is held at rest at position B ($\theta_0 = 2^\circ$) and then released. Ignore friction between the pin and the hole, and air resistance. Thus, it undergoes a simple harmonic motion (S.H.M.). Note that the magnitude of acceleration due to the Earth’s gravity is $g$.

a. (15 pts) Express the period of the S.H.M. in terms of $x$, $M$, $R$, and/or $g$.

b. (10 pts) If you used a uniform steel sphere of the same radius $R$ (this means the mass is heavier than $M$) and perform the same experiment, is the period in part (a) increased, decreased, or same? Explain why.
Periodic Motion

\[ T^2 = \left(\frac{4\pi^2}{GM}\right) s^3 \]

\[ T = 2\pi \sqrt{\frac{L \cos \theta}{g}} \]
Example 1: (a) $F_T = \ ?$ (b) $T = \ ?$

**Be Critical Thinker**

$$F_T = mg / \cos \theta$$

$$T = 2\pi \sqrt{\frac{L \cos \theta}{g}}$$

$$F_T = mg \cos \theta$$

$$T = 2\pi \sqrt{\frac{l}{g}}$$
S.H.M. : \( T = \frac{2\pi}{\omega} \)

Oscillations
Example 2(A)

Oscillations

Equilibrium Positions

(a) $F$  $x = 0$  $x = A$  $v = 0$

(b) $F = 0$  $x = 0$  $v = -v_{max}$

(c) $F$  $v = 0$  $x = -A$  $x = 0$

(d) $F = 0$  $x = 0$  $v = +v_{max}$

(e) $F$  $v = 0$  $x = 0$  $x = A$

$0  T/4  3T/4  T/2  T$
Example 2(B)

\[ E_a = E_b = E_c = E_d \]

(a) \[ E = \frac{1}{2} kA^2 \]

(b) \[ E = \frac{1}{2} mv_{\text{max}}^2 \]

(c) \[ E = \frac{1}{2} kA^2 \]

(d) \[ E = \frac{1}{2} mv^2 + \frac{1}{2} kx^2 \]
Example 4: Momentum conservation!

What is the speed of the bullet?
Express the speed in terms of $m$, $M$, $k$, and $d$. 
Recap from the Previous Lecture

S.H.M. – Dynamics (I)

Simple Pendulum

\[ a = \frac{F}{m} = -\left(\frac{A}{m}\right) x \]

The acceleration is proportional to the displacement from equilibrium.

\[ a_0 = \frac{F}{m} = -\left(\frac{A}{m}\right) x \]

The angular acceleration is proportional to the angular displacement from equilibrium.

\[ a = -\omega^2 x \]

\[ \theta = \theta(0) + \theta(t) \]

\[ \theta(t) = A \cos(\omega t + \phi) \]

Angular Frequency

Equation of Motion:

\[ \frac{d^2 x}{dt^2} + \omega^2 x = 0 \quad \text{or} \quad \frac{d^2 \theta}{dt^2} + \omega^2 \theta = 0 \]

General solution:

\[ x(t) = A \cos(\omega t + \phi) \quad \text{Amplitude} \]

\[ \theta(t) = A \cos(\omega t + \phi) \]

\[ \rightarrow \text{Position (x or } \theta) \text{ as a function of time.} \]

S.H.M. – Dynamics (III)

Dimensional Analysis

\[ \left[ \frac{A}{m} \right] = \frac{N/m}{m^2} = \frac{(kg) (m/s^2)/m}{m^2} = \frac{1}{s^2} \]

\[ \left[ \frac{g}{t} \right] = \frac{m/s^2}{m} = \frac{1}{s} \]

We define "angular frequency" \( \omega \):

\[ \omega = \frac{\sqrt{g}}{L} \]

\[ T = \frac{1}{f} \]

Note!
Example 5

You hold the block at \(x = A\) (= 0.030 m) by applying 6.0 N. Then, the block was released.

The motion of the block undergoes SHM. Can you show that \(a = -(k/m) x\)?

Also find:

(a) \(k\)
(b) \(\omega\)
(c) \(T\)
(d) \(v_{\text{max}}\) (where?, when?)
(e) \(x, v\) and \(a\) at \(t = 2\) sec
Example 9: Vertical S.H.M.

\[ F = -kx_0 \]

\( x \) now measured from here

1.00 m

Oscillations
Physical Pendulum (I)

Small oscillation

$\theta = small$

$L_{\parallel} = L \cos \theta \sim L$

$1.00 \text{ m}$
Physical Pendulum (II)

Equilibrium ($\Sigma \tau_i = 0$)

$\Rightarrow \quad mg(L/2) - kx_0(L) = 0$
Equation of motion ($\sum \tau_i = I \alpha$)

$\sum \tau_i = mg(L/2) - kx(L) = kx_0(L) - kx(L)$

$\therefore -k(x-x_0)(L) = I_{rod(P)} \frac{d^2 \theta}{dt^2}$