Problems from Ch. 10
15. (II) A small rubber wheel is used to drive a large pottery wheel, and they are mounted so that their circular edges touch. If the small wheel has a radius of 2.0 cm and accelerates at the rate of 7.2 rad/s², and it is in contact with the pottery wheel (radius 25.0 cm) without slipping, calculate (a) the angular acceleration of the pottery wheel, and (b) the time it takes the pottery wheel to reach its required speed of 65 rpm.

16. (II) The angle through which a rotating wheel has turned in time \( t \) is given by \( \theta = 6.0t - 8.0t^2 + 4.5t^4 \), where \( \theta \) is in radians and \( t \) in seconds. Determine an expression (a) for the instantaneous angular velocity \( \omega \) and (b) for the instantaneous angular acceleration \( \alpha \). (c) Evaluate \( \omega \) and \( \alpha \) at \( t = 3.0 \) s. (d) What is the average angular velocity, and (e) the average angular acceleration between \( t = 2.0 \) s and \( t = 3.0 \) s?

17. (II) The angular acceleration of a wheel, as a function of time, is \( \alpha = 5.0t^2 - 3.5t \), where \( \alpha \) is in rad/s² and \( t \) in seconds. If the wheel starts from rest (\( \theta = \omega = 0 \) at \( t = 0 \)), determine a formula for (a) the angular velocity \( \omega \) and (b) the angular position \( \theta \), both as a function of time. (c) Evaluate \( \omega \) and \( \theta \) at \( t = 2.0 \) s.

18. (II) The tires of a car make 85 revolutions as the car reduces its speed uniformly from 90.0 km/h to 60.0 km/h. The tires have a diameter of 0.90 m. (a) What was the angular acceleration? (b) If the car continues to decelerate at this rate, how much more time is required for it to stop?

Section 10–4

19. (II) The axle of a wheel is mounted on supports that rest on a rotating turntable as shown in Fig. 10–50. The wheel has angular velocity \( \omega_1 = 50.0 \) rad/s about its axle, and the turntable has angular velocity \( \omega_2 = 35.0 \) rad/s about a vertical axis. (Note arrows showing these motions in the figure.) (a) What are the directions of \( \omega_1 \) and \( \omega_2 \)? (b) What is the resultant angular velocity of the wheel, as seen by an outside observer, at the instant shown? Give the magnitude and direction. (c) What is the magnitude and direction of the angular acceleration of the wheel at the instant shown? Take the \( z \) axis vertically upward and the direction of the axle at the moment shown to be the \( x \) axis pointing to the right.

22. (II) A wheel of diameter 1.80 m lies in the \( xy \) plane, about the \( z \) axis as shown in Fig. 10–52. A force \( \textbf{F} = (-31.0\hat{i} + 25.0\hat{j}) \) N acts on the wheel that lies exactly in the \( xy \) plane. What is the torque about the \( z \) axis?

23. (II) Calculate the net torque about the \( z \) axis shown in Fig. 10–52 if a 0.30 m·N opposes the rotation of the \( y \) axis.

24. (II) Determine the radius \( r \) shown in Fig. 10–53 and (b) point \( P \) at one
Problem 3: (20 points)
The angular velocity of a wheel, as a function of time, is \( \omega(t) = b \ t^2 + c \ t \), where \( b \) and \( c \) are constants. Note that \( \omega \) is in rad/s and \( t \) in seconds.

a. (10 pts) What are the units for constants \( b \) and \( c \).
b. (10 pts) If the wheel starts from rest \( (\theta = 0 \) and \( \omega = 0 \) at \( t = 0 \)), determine a formula for the angular acceleration \( \alpha(t) \) and the angular position \( \theta(t) \), both as a function of time.
c. (5 pts-bonus!!!) What is the average angular acceleration between \( t = 1 \) s and \( t = 3 \) s?
26. (I) Determine the moment of inertia of a 12.0-kg solid sphere of radius 0.80 m when the axis of rotation is through its center.

27. (I) A 1.4-kg grindstone in the shape of a uniform cylinder of radius 0.20 m acquires a rotational rate of 1800 rev/s from rest over a 6.0-s interval at constant angular acceleration. Calculate the torque delivered by the motor.

28. (I) An oxygen molecule consists of two oxygen atoms whose total mass is $5.3 \times 10^{-26}$ kg and whose moment of inertia about an axis perpendicular to the line joining the two atoms, midway between them, is $1.9 \times 10^{-46}$ kg·m². Estimate, from these data, the effective distance between the atoms.

29. (II) A 2.4-kg ball on the end of a thin, light rod is rotated in a horizontal circle of radius 1.2 m. Calculate (a) the moment of inertia of the ball about the center of the circle, and (b) the torque needed to keep the ball rotating at constant angular velocity if air resistance exerts a force of 0.020 N on the ball. Ignore the rod's moment of inertia and air resistance.

30. (II) Calculate the moment of inertia of the array of point objects shown in Fig. 10–55 about (a) the vertical axis, and (b) the horizontal axis. Assume the objects are connected by very light rigid wires. About which axis would it be harder to accelerate this array? In Fig. 10–55, $m = 1.8$ kg and $M = 3.1$ kg. The array is rectangular and it is split through the middle by the horizontal axis.

![Figure 10–55](image)

**FIGURE 10–55** Problem 30.

33. (II) A merry-go-round accelerates at 24 s. Assuming the merry-go-round radius 7.0 m and mass 3. required to accelerate it.

34. (II) Four equal masses along a horizontal straight. The system is to be rotated through the mass at the le lar to it. (a) What is the about this axis? (b) What is the direction of this force? the mass, will impart an

35. (II) Suppose the force $F_T = 3.00t - 0.20t^2$ (newt) wheel starts from rest, what is its rim 8.0 s later?

36. (II) A centrifuge rotor and is eventually brought to 1.00 m·N. If the mass of the rotor is considered a solid cylinder: many revolutions will the rotor make and how much time will the rotor take to make these revolutions?

37. (II) Two blocks are connected by a pulley of radius 0.25 m. The blocks move to the right on inclines with friction. (a) Draw free-body diagram for each block. (b) Determine the tension in the string. (c) Calculate the acceleration of the system.
**Problem 4: (20 points)**
Consider an array of four point objects \((M > m)\) as shown in the figure. The objects are connected by very light rigid wires. The array is a rectangular and it is split through the middle by the horizontal \((x)\) axis.

a. (10 pts) Calculate the moment of inertia of the array about:
   1. the vertical \((y)\) axis;
   2. the horizontal \((x)\) axis.

b. (10 pts) About which axis would it be harder to accelerate this array? Why?
1. (25 points) Finding the moment of inertia.

1(a) Express the moment of inertia of the array of point objects about the y-axis in terms of $m$ and $M$.

1(b) Consider a helicopter rotor blade as a long thin rod. If each of the three blades is 3.75 m long and has a mass of 160 kg, calculate the moment of inertia of the three blades about the axis of the rotation.
1(c) A ball (solid sphere) of mass $M$ and radius $R$ on the end of a thin rod (mass $m$ and length $l$). Express the moment of inertia of the system of the rod and the ball about the A-B Axis in terms of $M$, $R$, $m$, and $l$.

![Diagram of a ball on a thin rod]

Quiz (Moment of Inertia)

1(d) A door (solid rectangular thin plate) of mass $M = 15.0$ kg is free to rotate on about hinge line. Calculate the moment of inertia of the door about the hinge line.

![Diagram of a door with dimensions]

Quiz (Moment of Inertia)
6. A wagon wheel is constructed as shown. The radius of the wheel is 0.300 m, and the rim has a mass of 1.60 kg. Each of the eight spokes, which lie along a diameter and are 0.300 m long, has a mass of 0.320 kg. What is the moment of inertia of the wheel through its center and perpendicular to the plane of the wheel?
1. (25 points) Finding the moment of inertia.

1(a) Express the moment of inertia of the array of point objects about the $y$-axis in terms of $m$ and $M$.

1(b) Consider a helicopter rotor blade as a long thin rod. If each of the three blades is 3.75 m long and has a mass of 160 kg, calculate the moment of inertia of the three blades about the axis of the rotation.

1(c) A ball (solid sphere) of mass $M$ and radius $R$ on the end of a thin rod (mass $m$ and length $l$). Express the moment of inertia of the system of the rod and the ball about the A-B axis (thin rod; mass $m$ and length $l$) in terms of $M$, $R$, $m$, and $l$. 
1(d) A door (solid rectangular thin plate) of mass $M = 15.0$ kg is free to rotate on about hinge line. Calculate the moment of inertia of the door about the hinge line.

![Door Diagram]

1(e) A solid disk (mass $M = 3.00$ kg and radius $R = 20.0$ cm) is hung from the wall by means of a metal pin through the hole, and used as a pendulum. Calculate the moment of inertia of the disk about the pin (= the axis of the rotation).

![Disk Diagram]

1(f) A meter stick (mass $M = 0.500$ kg and length $L = 1.00$ m) is hung from the wall by means of a metal pin through the hole, and used as a pendulum. Express the moment of inertia of the stick about the pin (= the axis of the rotation) in terms of $M$, $L$, and $x$.

![Stick Diagram]
5. (20 points) A string passing over a pulley has a mass $m_1 = 4\text{ kg}$ hanging from one end and another mass $m_2 = 3\text{ kg}$ hanging from the other end, as illustrated below. The pulley is a uniform solid cylinder of radius $R = 0.1\text{ m}$ and mass $m_3 = 8\text{ kg}$. It has moment of inertia $I = \frac{1}{2}m_3R^2$.

(a) Carefully indicate all of the forces acting on the system. Also indicate the direction of acceleration or angular acceleration of each part of the system.

(b) What is the acceleration of the two masses? $a =$
\(\text{(25 points)}\) A constant horizontal force (magnitude \(F\)) is applied to a lawn roller in the form of a uniform solid cylinder of radius \(R\) and mass \(M\). The roller rolls without slipping on the horizontal surface. The acceleration due to the Earth's gravity is \(g\).

(a) (5 pts) Draw a free-body diagram for the roller.

(b) (10 pts) Determine an expression for the magnitude of the acceleration of the center of mass in terms of the above parameters.

(c) (10 pts) Determine an expression for the minimum coefficient of friction necessary to prevent slipping in terms of the above parameters.
2. Mass Rolling Down an Incline (35 points)

Consider an object with mass $m$, radius $R$, and moment of Inertia $I = mR^2$. The object is placed at rest at the top of an incline a height $h$ above the ground. The incline is at an angle $\theta$ as shown in the figure. It then rolls down the incline without slipping.

a. (5 pts) Draw the force diagram for the system

b. (15 pts) Using the fact that the object rolls without slipping, find the linear acceleration of the object. Hint: Assume constant friction produces a torque.

c. (5 pts) Using this acceleration, find the final velocity of the object when it reaches the ground.

d. (10 pts) As a check, using conservation of energy, find the final velocity
Problem 5: (20+ points)
Consider a solid sphere of uniform composition rolling down an incline. Assume the sphere has mass $M$ and radius $R$. The moment of inertia for the sphere about a rotational axis through its center is $I = (2/5)MR^2$. The sphere is placed at rest at the top of an incline at height $H$ above the ground. The incline is at an angle $\theta$ as shown in the figure. It then rolls down the incline without slipping. The free-body diagram for the sphere is also provided in the figure. Note that the gravitational acceleration near the Earth’s surface is $g$. Ignore energy losses due to dissipative forces. [$F_f$ is due to static friction and we cannot assume $F_f = \mu_r F_N$. Only $F_f \leq \mu_s F_N$.]

a. (10 pts) Using conservation of energy, express the final speed of the sphere in terms of the above parameters when it reaches the bottom of the incline.

b. (10 pts) Express the linear acceleration of the rolling sphere in terms of the above parameters. Using this acceleration, express the final speed of the sphere. [Compare your result to that in part a. Does it make sense?]

c. (5 pts–bonus!!!) Explain why $F_f$ must be static friction.
4. (25 points) A uniform solid cylinder of mass $M$ and radius $R$ rests on a horizontal table top. A light string is attached by a light yoke to a frictionless axle through the center of the cylinder, so that the cylinder can rotate about the axle. The string runs over a frictionless pulley. A block of mass $3M$ is suspended from the free end of the string. The cylinder rolls without slipping on the table top. The system is released from the rest. The acceleration due to the Earth’s gravity is $g$.

(a) (10 pts) Draw a free-body diagram for the cylinder.
(b) (15 pts) Determine an expression for each of the following quantities in terms of the above parameters:
   
   (i) the magnitude of the downward acceleration of the block:
   
   (ii) the friction force between the cylinder and the table:
   
   (iii) the tension force on the string.
3. **(25 points)** A uniform solid cylinder of mass \( M_c = 5.00 \text{ kg} \) and radius \( R = 0.50 \text{ m} \) is placed on a surface inclined \( \theta = 30.0^\circ \) to the horizontal. A light string is attached by a light yoke to a frictionless axle through the center of the cylinder, so that the cylinder can rotate about the axle. The string runs over a frictionless pulley. A block of mass \( M_b = 15.0 \text{ kg} \) is suspended from the free end of the string. The cylinder rolls without slipping on the surface. The system is released from the rest. The acceleration due to the Earth’s gravity is \( g \).

(a) **(10 pts)** Draw a free-body diagram for the cylinder.

(b) **(5 pts)** Draw a free-body diagram for the block.

(c) **(10 pts)** Find the magnitude of the downward acceleration of the block.
3. **(25 points)** Two blocks ($M_1 = 8.00$ kg, $M_2 = 10.0$ kg) are connected by a light string passing over a pulley (solid disk) of radius $r = 0.25$ m and mass $m = 2.00$ kg. The blocks move to the right with a constant acceleration on inclines ($\theta_1 = 30.0^\circ$, $\theta_2 = 50.0^\circ$) with frictionless surfaces.

   a. **(10 pts)** Draw free-body diagrams for each of the two blocks and the pulley.

   b. **(15 pts)** Determine the tensions in the two parts of the string and the acceleration of the blocks.
A rigid rod of mass $M$ and length $l$ rotates in a vertical plane about a frictionless pivot through its center. Particles of masses $m_1, m_2$ are attached at the ends of the rod.

a. Determine the angular momentum when the angular velocity is $\omega$.

b. Determine the angular acceleration of the system when the rod makes an angle $\theta$ with the horizontal.
4. (25 points) A person stands, hands at the side, on a platform that is rotating at a rate of 1.60 rev/s. If the person now raises his arms to a horizontal position, the speed of rotation decreases to 0.800 rev/s.

a. (15 pts) Why does this occur? Explain using the following key words: external torque and angular momentum.

b. (10 pts) By what factor has the moment of inertia of the person changed?
4. Ice Skater (15 points)

An ice skater with moment of Inertia I is spinning with angular velocity \( \omega \). She pulls in her arms and reduces her moment of inertia to \( I/5 \).

a. (5 pts) What is her final angular momentum?

b. (5 pts) What is her initial and final angular kinetic energy?

c. (5 pts) What is the change in kinetic energy? Where did that energy come from?
4. (25 points) A block of mass $M_b$, attached to a massless string, moves in a circle of constant radius $R$ about a small hole in a frictionless, horizontal table. The end of the string is connected to a stone (mass $M_s$) through the hole as shown in the figure. The block may be treated as a particle.

(a) (5 pts) What is the magnitude of the centripetal force on the block?
(b) (5 pts) Find the speed ($v_1$) of the block in terms of $g$, $R$, $M_b$ and $M_s$.
(c) (40 pts) A man pulls the stone down by $\frac{1}{2}R$. Find the resultant speed ($v_2$) of the rotation in terms of $v_1$ in part (b).
(d) (5 pts) How much work was done in pulling the string?