PHYSICS 218 EXAM III
FALL SEMESTER 2001
(60 minutes)

***** INSTRUCTIONS *****

Before you begin:

1. Enter your LAST and FIRST names
2. Enter your student ID number (LAST 5 digits)
3. Enter your "registered" SECTION number.
4. DO NOT OPEN UNTIL INSTRUCTED TO DO SO.

Name (Last, First): ____________________________

Last Five Digit ID #: _________________________

Section #: ____________________________

Row-Seat #: ____________________________

- Formulae are provided. You may NOT use a separate formulae sheet.
- You may use a calculator.
**Problem 1: (20 points)**

In the coordinate system shown in the figure, a ball of mass $m$ is held and then dropped from a height $h$ above an uncompressed ideal spring. The ball falls and compresses the spring, and eventually comes to stop at some at a height $Y$ below the $y$ axis. Note that the gravitational acceleration near the Earth’s surface is $g$.

a. (5 pts) At the top, height $h$, what is the total energy of the ball?

b. (5 pts) At $y = 0$, what is the total energy of the ball?

c. (5 pts) At $y = 0$, what is the speed of the ball?

d. (5 pts) What is the spring constant, $k$, of the spring?
Problem 2: (20 points)
A rocket of mass $M$ in outer space, traveling with speed $v_0$ along the $x$ axis, suddenly shoots out one-third its mass parallel to the $y$ axis (as seen by an observer at rest) with speed $2v_0$. Ignore the influence of gravity.

a. (5 pts) Sketch a diagram describing the motion.

b. (10 pts) Express the velocity of the remainder of the object in terms of unit vectors $\mathbf{i}$, $\mathbf{j}$, and $\mathbf{k}$.

c. (5 pts) Is this “separation” elastic or inelastic? Why?
Problem 3: (20 points)
The angular velocity of a wheel, as a function of time, is $\omega(t) = b t^2 + c t$, where $b$ and $c$ are constants. Note that $\omega$ is in rad/s and $t$ in seconds.

a. (10 pts) What are the units for constants $b$ and $c$.
b. (10 pts) If the wheel starts from rest ($\theta = 0$ and $\omega = 0$ at $t = 0$), determine a formula for the angular acceleration $\alpha(t)$ and the angular position $\theta(t)$, both as a function of time.
c. (5 pts-bonus!!!) What is the average angular acceleration between $t = 1$ s and $t = 3$ s?
Problem 4: (20 points)
Consider an array of four point objects \( M > m \) as shown in the figure. The objects are connected by very light rigid wires. The array is a rectangular and it is split through the middle by the horizontal \((x)\) axis.

a. (10 pts) Calculate the moment of inertia of the array about:
   1. the vertical \((y)\) axis;
   2. the horizontal \((x)\) axis.

b. (10 pts) About which axis would it be harder to accelerate this array? Why?
Problem 5: (20+ points)
Consider a solid sphere of uniform composition rolling down an incline. Assume the sphere has mass $M$ and radius $R$. The moment of inertia for the sphere about a rotational axis through its center is $\mathbf{I} = (2/5)MR^2$. The sphere is placed at rest at the top of an incline at height $H$ above the ground. The incline is at an angle $\theta$ as shown in the figure. It then rolls down the incline without slipping. The free-body diagram for the sphere is also provided in the figure. Note that the gravitational acceleration near the Earth's surface is $g$. Ignore energy losses due to dissipative forces. [$F_f$ is due to static friction and we cannot assume $F_f = \mu_s F_N$. Only $F_f \leq \mu_s F_N$.]

a. (10 pts) Using conservation of energy, express the final speed of the sphere in terms of the above parameters when it reaches the bottom of the incline.

b. (10 pts) Express the linear acceleration of the rolling sphere in terms of the above parameters. Using this acceleration, express the final speed of the sphere. [Compare your result to that in part a. Does it make sense?]

c. (5 pts–bonus!!!) Explain why $F_f$ must be static friction.