Physics 218 Fall 1997 Exam #3 (Sections 517–520)

Name (Last, First): ___________________________  ID #: __________________
Section #: _______________________
Row-Seat #: _______________________

- You have 50 minutes to complete the exam.
- Formulae are provided below. You may not use a separate formulae sheet.
- You may use a calculator.
- It is sufficient to leave the solutions in their analytic expressions (you do not need to calculate the numerical values of the solutions) unless otherwise stated.
- It is expected that your analytic solutions are in a simplified form.
- If you are calculating the numerical values, be sure to keep track of units.
- If you find yourself unable to solve a problem whose solution is needed in another problem, then assign a symbol for the solution of the first problem and use that symbol in solving the second problem.
- Mark your answers clearly by drawing boxes around them.
- There may be additional clarifications and corrections which will be given on the blackboard during the exam period.

Constants:

\[
g = 9.8 \text{ m/s}^2 \text{ (grav. acceleration on Earth)} \quad R_E = 6.38 \times 10^6 \text{ m (radius of Earth)}
\]

\[
G = 6.67 \times 10^{-11} \text{ N} \cdot \text{m}^2/\text{kg}^2 \text{ (grav. constant)} \quad M_E = 5.98 \times 10^{24} \text{ kg (mass of Earth)}
\]

\[
K_S = 2.97 \times 10^{-19} \text{ s}^2/\text{m}^3 \text{ (constant in Kepler’s 3rd law)} \quad R_M = 1.74 \times 10^6 \text{ m (radius of Moon)}
\]

\[
R_S = 6.96 \times 10^8 \text{ m (radius of Sun)} \quad M_M = 7.35 \times 10^{22} \text{ kg (mass of Moon)}
\]

\[
M_S = 1.991 \times 10^{30} \text{ kg (mass of Sun)}
\]

Formulae:

Description of motion:

Newton’s 2nd law:
1. **(25 points)** A uniform solid cylinder of mass \( M_c = 2.0 \) kg and radius \( R = 0.3 \) m rests on a horizontal table top. A light string is attached by a light yoke to a frictionless axle through the center of the cylinder, so that the cylinder can rotate about the axle. The string runs over a frictionless pulley. A block of mass \( M_b = 6.0 \) kg is suspended from the free end of the string. The cylinder rolls without slipping on the table top. The system is released from the rest and the block has a motion with a constant acceleration. The acceleration due to the earth's gravity is \( g = 9.8 \) m/s\(^2\).

(a) (4 pts) Draw the free body diagram for the cylinder and label carefully.

(b) (2 pts) Draw the free body diagram for the block and label carefully.

(c) (7 pts) Write the equations of translational and rotational motions for the cylinder.

(d) (3 pts) Write the equations of translational motion for the block.

(e) (9 pts) Solve the equations in (c) and (d), and find the numerical values:

(i) the magnitude of the downward acceleration of the block;

(ii) the friction force between the cylinder and the table;

(iii) the tension force on the string.
A uniform beam (length \( l = 5.0 \text{ m} \), mass \( m = 20.0 \text{ kg} \)) is inclined at an angle \( \theta \) (unknown) to the horizontal floor with its upper end supported by a rope and its lower end resting on the floor. The rope is tied to the floor at an angle \( \phi = 30^\circ \) to the floor. The maximum mass of a block that can be suspended from the top end is \( M = 33.1 \text{ kg} \). See the figure. The coefficient of static friction between beam and floor is \( \mu_s = 0.5 \). Note: \( \sin(\alpha \pm \beta) = \sin \alpha \cos \beta \pm \cos \alpha \sin \beta \) and \( \cos(\alpha \pm \beta) = \cos \alpha \cos \beta \mp \sin \alpha \sin \beta \),

(a) (5 pts) Draw the free body diagram for the uniform beam and label carefully.
(b) (10 pts) Write two equations by applying the first condition for equilibrium (no translational movement).
(c) (5 pts) Write an equation by applying the second condition for equilibrium (no rotational movement) about the lower end of the beam.
(d) (5 pts) Determine the angle \( \theta \).
3. **(25 points)** A 300-kg lunar lander is initially in a circular orbit with radius $r_1 = 3000$ km.

(a) **(10 pts)** Find the speed of the lander when it is in the circular orbit.

(b) **(10 pts)** The lander engines are turned on briefly at point $P_1$ and the lander coasts until it reaches an outer orbit with radius $r_2 = 4000$ km (point $P_2$). See the figure. Find the work done by the lander engines to move this lander to the outer orbit.

(c) **(5 pts)** Find the work done by the lander engines to make this lander escape the moon completely.

Assume the moon to be a uniform sphere. Neglect the gravitational pull from any other planet.
4. **(25 points)** On the surface of the earth, you hang an ideal spring (unstretched length of \( \ell_0 = 1.0 \text{ m} \)) of negligible mass and suspend from it a block with a mass of \( m = 5.0 \text{ kg} \), as shown in the figure. The length of the spring is \( \ell = 1.5 \text{ m} \) when the block is at rest (= equilibrium position). The height of the block at the equilibrium position is \( h = 2.0 \text{ m} \) above the floor. The block is displaced downward by \( d = 0.3 \text{ m} \) from the equilibrium position and released at \( t = 0 \), so that it oscillates vertically (i.e., simple harmonic oscillation). Ignore air resistance.

(a) (5 pts) Find the spring constant.

(b) (5 pts) At what point in the motion of the block is the kinetic energy greatest?

(c) (5 pts) Find the value for the kinetic energy of the block at the position in part (b).

(d) (5 pts) Find the elapsed time from when the block first passes through the equilibrium point to the second time it passes through that point.

(e) (5 pts) If you take the system to the moon and perform the same experiment, does the elapsed time in part (d) increase, decrease, or remain the same? Explain your answer. Note that the acceleration due to the moon’s gravity is \( g_{\text{moon}} = 1.6 \text{ m/s}^2 \).