Problem 1 (25 points)

(Step 1) F.B.D.
(Step 2) $W_i$ by each force
(Step 3) $W_{net} = \Sigma W_i$
(Step 4) W-E Theorem to find $v_2$:

\[ W_{net} = K_2 - K_1 \]

where $K_1 = 0$

(b) 5 pts

\[
\begin{align*}
W_P &= F_P d \cos \theta \\
W_N &= F_N d \cos(90^\circ) = 0 \\
W_f &= F_f d \cos(180^\circ) = -\mu_k F_N d \\
W_G &= F_G d \cos(\theta + 90^\circ) = -F_G d \sin \theta
\end{align*}
\]

(a) 5 pts

\[
\begin{align*}
v &= 0 \\
d &= 5.00 \text{ m}
\end{align*}
\]

(c-1) 5 pts

\[
\begin{align*}
W_{net} &= K_2 - K_1 \\
\text{where } K_1 &= 0
\end{align*}
\]

(c-2) 5 pts

\[
\begin{align*}
W_{net} &= F_P d \cos \theta - \mu_k [F_G \cos \theta + F_P \sin \theta] d \\
&\quad - F_G d \sin \theta \\
&\quad = F_P d (\cos \theta - \mu_k \sin \theta) \\
&\quad - M g d (\sin \theta + \mu_k \cos \theta) = (1/2) M v_2^2
\end{align*}
\]

(c-3) 5 pts

\[
v_2 = \sqrt{\frac{2 [F_P d (\cos \theta - \mu_k \sin \theta) - M g d (\sin \theta + \mu_k \cos \theta)]}{M}}
\]
**Problem 2 (25 points)**

(a) (5 points)

(b) (10 points) Momentum conservation:

\[ M\nu_0 = (2/3)M\nu_{2x} + (1/3)M(2\nu_0)\cos\theta \quad \ldots \quad (3 \text{ pts}) \]

\[ 0 = (1/3)M(2\nu_0)\sin\theta + (2/3)M\nu_{2y} \quad \ldots \quad (3 \text{ pts}) \]

Then:

\[ \nu_{2x} = (3/2)\nu_0 \left[ 1 - (2/3)\cos\theta \right] = \nu_0 \]

\[ \nu_{2y} = -\nu_0\sin\theta = -\left[ \sqrt{3}/2 \right] \nu_0 = -0.866 \nu_0 \]

Therefore:

\[ \nu_2 = [\nu_0] \hat{i} + [-\nu_0\sin\theta] \hat{j} \quad \text{or any other equivalent answer} \quad \ldots \quad (4 \text{ pts}) \]

**cf.) magnitude:** \( \nu_2 = \sqrt{7/4} \nu_0 = 1.32 \nu_0 \)

**direction:** \( \phi = \tan^{-1}(0.866/1.000) = 40.9^\circ \)

(c) (10 points) “Inelastic” (4 pts),

because \( \Delta K(=K_f - K_i) \) (3 pts) is nonzero (3 pts):

\[ K_f - K_i = [(1/2)(M/3)(\nu_1)^2 + (1/2)(2M/3)(\nu_2)^2] - (1/2)M(\nu_0)^2 \]

\[ = 0.75 \, M \, (\nu_0)^2 \quad \ldots \quad \text{Mechanical energy is not conserved.} \]
Problem 3 (25 points)
(a) (4 points)

\[ [b] = \text{rad/s}^2 < 2 \text{ pts} > ; \ [c] = \text{rad/s}^2 < 2 \text{ pts} > \]

(b) (4 points)

\[ \alpha(t) = \frac{d\omega(t)}{dt} = 2bt + c \]

\[ \theta(t) = \int \omega(t) dt = \frac{1}{3}bt^3 + \frac{1}{2}ct^2 + d \text{ (where } d = 0, \text{ because } \theta = 0 \text{ at } t = 0) \]

\[ = \frac{1}{3}bt^3 + \frac{1}{2}ct^2 \]

(c) (2 points)

\[ \alpha_{\text{ave}}(t) = \frac{\Delta \omega(t)}{\Delta t} = \frac{\omega(3) - \omega(1)}{3 - 1} = \frac{(9b + 3c) - (b + c)}{2} = 4b + c \]

(d) (10 points) \( M = 160 \text{ kg}, L = 3.75 \text{ m}, x = 0.5 \text{ m.} \) Use parallel-axis theorem.

Fig.1 : \( I_1 = 3 \left( \frac{1}{12}ML^2 + M \left( \frac{L}{2} - x \right)^2 \right) = \frac{1}{4}ML^2 + 3M \left( \frac{L}{2} - x \right)^2 = 1470 \text{ kg} \cdot \text{m}^2 < 5 \text{ pts} > \)

Fig.2 : \( I_2 = 3 \left( \frac{1}{12}ML^2 + M \left( \frac{L}{2} \right)^2 \right) = ML^2 = 2256 \text{ kg} \cdot \text{m}^2 < 5 \text{ pts} > \)

[Note : either "formula" or numerical answer is accepted.]

(e) (5 points)

The moment of inertia is an indicator of how mass is distributed from the rotational axis. Smaller (greater) moment of inertia means easier (harder) to start rotating. Using \( \tau = I\alpha \), we have the following relation between two cases if the rotors are rotating at the same angular acceleration:

\[ \frac{\tau_1}{I_1} = \frac{\tau_2}{I_2} \]

Thus, the Fig. 2 design needs greater torque. In other words, this is harder to accelerate...
Problem 4 (30 points)

(a) (5 pts) Draw F.B.D. here

(b) (10 pts) Testing your knowledge of Newton's 2nd laws.

Step 1 (9 pts): Write down Newton's 2nd laws. 3 points for each equation.

\[ \ddot{F} = m \ddot{a} \]
\[ \ddot{\tau} = I \dddot{\alpha} \]

\[ [x] \quad Mg \sin \theta - F_f = M \dot{a}_x \quad \cdots(1) \]
\[ [y] \quad F_f \cdot R_0 = MR_0^2 \left( \frac{a_x}{R_0} \right) \Rightarrow F_f = Ma_x \quad \cdots(3) \]

Note that 3 equations and 3 unknowns \( a_x, F_f, \) and \( F_g \)

Step 2 (1 pt): Solve for \( a_x \).

\[ (3) \Rightarrow (1): Mg \sin \theta - Ma_x = Ma_x \quad \therefore Ma_x = \frac{1}{2} Mg \sin \theta \quad \therefore a_x = \frac{1}{2} g \sin \theta \quad \cdots(4) \]

(c) (5 pts) Testing your knowledge of kinematics.

Eq. (4) \( \Rightarrow a_x \) is constant. So \( v_x^2 - v_{0x}^2 = 2a_x(x - x_0) \) \( \langle 3 \text{ pts} \rangle \)

\[ \therefore v_x^2 - 0 = 2a_x \frac{H}{\sin \theta} \quad \cdots(5) \]
\[ \therefore v_x^2 = 2 \left( \frac{1}{2} g \sin \theta \right) \left( \frac{H}{\sin \theta} \right) = gH \quad \therefore v_x = \sqrt{gH} \quad \langle 1 \text{ pt} \rangle \]

(d) (5 pts) Testing your knowledge of conservation of mechanical energy in rolling motion

Step 1 (1 pt) \( K_f + U_i = K_f + U_r \), where \( U = \) gravitational potential energy

Step 2 (4 pts) \( 0 + MgH = \left( \frac{1}{2} M v_x^2 + \frac{1}{2} I_{cm} \omega^2 \right) + 0, \) where \( I_{cm} = MR_0^2 \) and \( \omega = \frac{v_x}{R_0} \)

\[ K_f = K_{trans} + K_{rotation} \quad \langle 1 \text{ pt} \rangle \]

\[ \therefore MgH = \frac{1}{2} M v_x^2 + \frac{1}{2} MR_0^2 \left( \frac{v_x}{R_0} \right)^2 \quad \therefore MgH = \frac{1}{2} M v_x^2 + \frac{1}{2} M v_x^2 = M v_x^2 \quad \therefore v_x = \sqrt{gH} < 1 \text{ pt} > \]

(e) (5 pts) Testing your knowledge of rolling motion of different rigid body.

Acceleration (3 pts): Use \( I_{cm} = \frac{2}{5} MR_0^2 \) in Eq. (3).

\[ \therefore a_x (\text{solid sphere}) = \frac{5}{7} g \sin \theta \quad \therefore a_x (\text{solid sphere}) > a_x (\text{thin hollow pipe}) \quad \therefore \text{increase} \]

Speed (2 pts): Look back at Eq. (5). \( \therefore v_x = \sqrt{\frac{10}{7} gH} \quad \therefore v_x \text{ increases when } a_x \text{ increases.} \]