You have 60 minutes to complete the exam.
• Formulae are provided on a separate colored sheet. You may NOT use any other formula sheet.
• You may use a calculator.
• When calculating numerical values, be sure to keep track of units. Results must include proper units.
• Be alert to the number of significant figures in the information given. Results must have the correct number of significant figures.
• If you are unable to solve a problem whose solution is needed in another problem, then assign a symbol for the solution of the first problem and use that symbol in solving the second problem.
• If you need additional space to answer a problem, use the back of the sheet it is written on.
• Mark your answers clearly by drawing boxes around them.
• Please write clearly. You may gain marks for a partially correct calculation if your work can be deciphered.
Problem 1: (25 points)

The equation of motion for an object is given by:

\[ x(t) = b t^3 + c(t - e)^2 + f \]

where \( x \) is a position which is given in meters, \( t \) is the time which is given in seconds and \( b, c, e \) and \( f \) are constants. Find the following:

a. (5 pts) \( \frac{dx}{dt} \)

Use \( \frac{dx}{dt} = nAt^{n-1} \) for \( x = At^n \). \( \therefore \frac{dx}{dt} = 3b t^2 + 2c(t - e) \)

b. (10 pts) Express the \( x \) component of the velocity as a function of \( t \) using the following formula:

\[
v_x = \lim_{\Delta t \to 0} \frac{x(t + \Delta t) - x(t)}{\Delta t}
\]

\[
x(t + \Delta t) - x(t) = [b(t + \Delta t)^3 + c(t + \Delta t - e)^2 + f] - [bt^3 + c(t - e)^2 + f]
\]

\[
= [b \{t^3 + 3t^2(\Delta t) + 3t(\Delta t)^2 + (\Delta t)^3\} + c\{(t - e)^2 - 2(t - e)(\Delta t) + (\Delta t)^2\} + f]
\]

\[
- [bt^3 + c(t - e)^2 + f]
\]

\[
= 3bt^2(\Delta t) + 3bt(\Delta t)^2 + b(\Delta t)^3 + 2c(t - e)(\Delta t) + c(\Delta t)^2
\]

\[
= (\Delta t)[3bt^2 + 3bt(\Delta t) + b(\Delta t)^2 + 2c(t - e) + c(\Delta t)]
\]

\[
\therefore \frac{x(t + \Delta t) - x(t)}{\Delta t} = 3bt^2 + 3bt(\Delta t) + b(\Delta t)^2 + 2c(t - e) + c(\Delta t)
\]

\[
\therefore \lim_{\Delta t \to 0} \frac{x(t + \Delta t) - x(t)}{\Delta t} = 3bt^2 + 2c(t - e)
\]

c. (2 pts) The units of \( b \) \( \text{m/s}^3 \)

d. (2 pts) The units of \( c \) \( \text{m/s}^2 \)

e. (2 pts) The units of \( e \) \( \text{s} \)

f. (4 pts) Is this a motion with constant acceleration? Why?

No, because of the \( bt^3 \) term. \([a_x = 6bt + 2c \Rightarrow \text{Not constant}]\)
Problem 2a: (20 points) Reading graph…

An object moves in a straight line as described by the position ($x$) versus time ($t$) graph in the figure. There are two types of time intervals: AB, CD and DE are parabola; BC is straight. Sketch $v$-$t$ and $a$-$t$ graphs for the object’s motion.
Problem 2b: (5 points) Reading graph...

A person initially at point $P$ in the illustration stays there a moment and then moves along the axis to $Q$ and stays there a moment. She then runs quickly to $R$, stays there a moment, and then strolls slowly back to $P$. Which of the position vs. time graphs below correctly represents this motion?
Problem 3: (25 points)

Consider the street pattern as shown in the figure below. Each intersection has a traffic signal and the speed limit is $V_{\text{limit}} = 50.0$ km/h (13.9 m/s). Suppose you are coming from the West at the speed limit and when you are $d = 10.0$ m from the first intersection, all the lights turn green. The lights are green for $T_{\text{green}} = 13.0$ s each before turning to red. Ignore yellow lights. Ignore the length of your car and your reaction time. Ignore air friction.

a. (10 pts) Can you make it through all the lights without stopping? Explain why.

b. (15 pts) Another car was stopped at the first light when all the lights turned green. It can accelerate at the rate of 2.00 m/s$^2$ to the speed limit. Can the second car make it through all the lights without stopping? Explain why.

(a) This is a motion with constant velocity. Obtain the time needed for your car to pass through all lights before the lights turn red.

Use $x = v_0 t$, where $v_0 = V_{\text{limit}} = 13.9$ m/s and $x = \text{total distance} = d + D = 175$ m.

Thus $t = \frac{d + D}{V_{\text{limit}}} = 12.6$ s. This is before 13.0 s.

Answer: Yes. You can pass through all the lights before the lights are green.

(b) [Case 1] Treat this as a motion with constant acceleration.

Obtain the time needed for the second car to pass through all lights before the lights turn red and check the speed.

Use $x = \frac{1}{2} a_x t^2$, where $x = D = 165$ m and $a_x = 2.00$ m/s$^2$.

Thus $t = \sqrt{\frac{2(165 \text{ m})}{2.00 \text{ m/s}^2}} = 12.8$ s. This is before 13.0 s.

At $t = 12.8$ s, the car’s velocity is:

$v_x = a_x t = (2.00 \text{ m/s}^2)(12.8 \text{ s}) = 25.6$ m/s, which is over the speed limit.

Case 2] First, the car accelerates to the speed limit (13.9 m/s).

It reaches to the speed limit in 6.95 s [$V_{\text{limit}} = (2.00 \text{ m/s}^2) t$] and travels 48.3 m.

After that, the car keeps the speed limit (a motion with a constant velocity).

It takes another 8.40 s to pass the remaining lights [$t = (165 \text{ m} - 48.3 \text{ m}) / (13.9 \text{ m/s})$].

Thus, the total elapsed time is $6.95 \text{ s} + 8.40 \text{ s} = 15.4$ s.

The second car can not pass all the lights in 13 s.

Answer: No, without the speeding ticket!
Problem 4: (25 points)

A projectile is launched from ground level to the top of a cliff which is \( R = 195 \text{ m} \) away and \( H = 155 \text{ m} \) high. The projectile lands on top of the cliff \( T = 7.60 \text{ s} \) after it is fired. Use \( 2\sin \theta \cos \theta = \sin 2\theta \) if necessary. The acceleration due to gravity is \( g = 9.80 \text{ m/s}^2 \) pointing down. Ignore air friction. Define your coordinate (including the origin) and find a formula of \( \tan \theta \) in terms of \( g, R, H \) and \( T \).

**Solutions**

Kinematic eqs:

\[
\begin{align*}
R &= v_0 \cos \theta \ T \quad \text{(1)} \\
H &= v_0 \sin \theta \ T - \frac{1}{2} g T^2 \quad \text{(2)}
\end{align*}
\]

\( v_0 \) and \( \theta \) are unknown.

\( v_0 \cos \theta = \frac{R}{T} \quad \text{(3)} \)

\( v_0 \sin \theta = \frac{H}{T} + \frac{1}{2} g T \quad \text{(4)} \)

\( \tan \theta = \frac{H + \frac{1}{2} g T^2}{R} \quad \text{(5)} \)

\[
\begin{align*}
\frac{3^2 + 4^2}{\text{v}_0^2} &= \left(\frac{R}{T}\right)^2 + \left(\frac{H}{T} + \frac{1}{2} g T\right)^2 \quad \text{(6)}
\end{align*}
\]

\( \tan \theta = \frac{155 \text{ m}}{195 \text{ m}} + \frac{(9.80 \text{ m/s}^2)(7.60 \text{ s})^2}{2(195 \text{ m})} = 2.246 \ldots \)

\( \theta = 66.0^\circ \) or \( 1.15 \text{ rad} \)

\( v_0 = \sqrt{\left(\frac{195 \text{ m}}{7.60 \text{ s}}\right)^2 + \left(\frac{155 \text{ m}}{7.60 \text{ s}} + \frac{(9.80 \text{ m/s}^2)(7.60 \text{ s})}{2}\right)^2} = 63.1 \text{ m/s} \)

15 pts for two correct eqs. plus the coordinates

10 pts for solving 2 eqs.

Not for grading.