**Example 2: Analysis 1**

A roller coaster sliding without friction along a circular vertical loop (radius $R$) is to remain on the track at all times. Find the minimum release height $h$.

![Diagram](Image)

1. Energy Conservation
2. Rotation
3. F.B.D.

**Example 2: Analysis 2**

A roller coaster sliding without friction along a circular vertical loop (radius $R$) is to remain on the track at all times. Find the minimum release height $h$.

![Diagram](Image)

1. $U_A = U_C + K_C$
2. $F = m(t^2/R)$

**Example 2: Analysis 3**

A roller coaster sliding without friction along a circular vertical loop (radius $R$) is to remain on the track at all times. Find the minimum release height $h$.

![Diagram](Image)

1. $mgh = mg(2R) + \frac{1}{2}mv^2 \cdots (1)$
2. $g = \frac{v^2}{R} \cdots (2)$

(2) $\rightarrow v^2 = gR \cdots (2)'$

(2)' $\rightarrow$ (1):

$$mgh = mg(2R) = \frac{1}{2}m(gR)$$

$$h = \frac{5}{2}R (= 2.5R)$$

[Reference: 8-82]
Problem 2: (20 points) - Solution

**Analysis - Visualization**

![Diagram showing Momentum Conservation](image)

(a) **Momentum conservation:**

\[
\begin{align*}
[p_x] M v_0 &= M_1 v_1 \cos \theta \\
[p_y] 0 &= -M_1 v_1 \sin \theta + M_2 v_2
\end{align*}
\]

\[\tan \theta = \frac{M_2 v_2}{M v_0} = \frac{(900 \text{ kg})(1.00 \times 10^3 \text{ m/s})}{(1000 \text{ kg})(5.00 \times 10^3 \text{ m/s})} = 0.18 \Rightarrow \theta = 10.2^\circ
\]

\[\therefore v_1 = \frac{M v_0}{M_1 \cos \theta} = \frac{(1000 \text{ kg})(5.00 \times 10^3 \text{ m/s})}{(100 \text{ kg}) \cos(10.2^\circ)} = 5.08 \times 10^4 \text{ m/s}
\]

(b) **\(\Delta K = K_f - K_i\)**

\[
\Delta K = \left(\frac{1}{2} M_1 v_1^2 + \frac{1}{2} M_2 v_2^2\right) - \left(\frac{1}{2} M v_0^2\right) = 1.17 \times 10^{11} [\text{J}]
\]
Problem 3: (25 points) - Solution

**Visualization via Graph**

(1) Two motions: before and after the breaker trips.
(2) ... with constant angular acceleration.
(3) Use the kinematic equations:

\[ \theta = \theta_0 + \omega_0 t + \frac{1}{2} \alpha t^2 \]
\[ \omega = \omega_0 + \alpha t \]
\[ \omega^2 = \omega_0^2 + 2\alpha(\theta - \theta_0) \]

**Example 2(a) - Solution**

\[ \theta_1 = 0 + (24.0 \text{ rad/s})t + \frac{1}{2} (60.0 \text{ rad/s}^2)t^2 = 168 \text{ rad} \]
\[ \omega_1 = (24.0 \text{ rad/s}) + (60.0 \text{ rad/s}^2)t = 144 \text{ rad/s} \]

(a) \[ \theta_{\text{total}} = 168 + 432 = 600 \text{ rad} \]

**Example 2(b/c) - Solution**

432 rad = 0 + \omega_1 t + \frac{1}{2} \alpha T^2

\[ 0 = \omega_1 + \alpha T \]

(b) & (c)

\[ T = 6.00 \text{ s} \]
\[ t_{\text{total}} = 8.00 \text{ s} \]
\[ \alpha = -24.0 \text{ rad/s}^2 \]

(d) \[ v_p = R \omega_1 = 288 \text{ m/s} \]

(e) \[ 288 \text{ m/s} \]

\[ 48 \text{ m/s} \]

\[ 0 \quad 2 \text{ s} \quad 8 \text{ s} \quad t \]
Problem 4: (20 points) - Solution

Consider an array of four point objects \( M > m \) as shown in the figure. The objects are connected by very light rigid wires. The array is a rectangular and it is split through the middle by the horizontal \((x)\) axis. Ignore any effect due to the Earth’s gravity.

(a) (10 pts) Find the moment of inertia of the array about:

(i) the vertical \((y)\) axis;
(ii) the horizontal \((x)\) axis.

(b) (5 pts) About which axis would it be harder to accelerate this array? Why?

(c) (5 pts - Bonus) Find the moment of inertia of the array about the vertical \((y)\) axis if the mass of each rigid wire is \(M\).

\[
\begin{align*}
\text{a. } \mathbf{I_y} &= \sum I_i = m (0.50)^2 + m (1.00)^2 + M (0.50)^2 + M (1.00)^2 \\
&= 1.25 (m + M) \\
\mathbf{I_x} &= \sum I_i = m (0.25)^2 \times 2 + M (0.25)^2 \times 2 \\
&= 0.125 (m + M)
\end{align*}
\]

\[
\begin{align*}
\text{b. } \mathbf{I_y} > \mathbf{I_x} \rightarrow \text{It is harder to accelerate the array about the } y \text{ axis.}
\end{align*}
\]

\[
\begin{align*}
\text{c. } \mathbf{I_y} &= \sum I_i = \mathbf{I_y} \text{ (from a)} \\
&= \mathbf{I_y} + M (0.50)^2 + M (1.00)^2 \quad \leftarrow \text{two vertical wires} \\
&= \mathbf{I_y} + [(1/12) M (1.50)^2 + M d^2 ] \times 2 \quad \leftarrow \text{two horizontal wires}
\end{align*}
\]

where \(d = 0.25\). So,

\[
\begin{align*}
\mathbf{I_y} &= 1.25 (m + M) + 0.5 M (= 1.25 m + 1.75 M)
\end{align*}
\]
Problem 5: (20 points) - Solution

(a) (5 pts) Draw the free-body diagram for each of the two blocks.
(b) (15 pts) Determine the tensions in the two parts of the string and acceleration of the blocks.

(b) Newton’s laws of motion ($\vec{F} = m\vec{a}, \vec{\tau} = I\vec{\alpha}$)

\[
\begin{align*}
F_{T1} - m_1g \sin 30^\circ &= m_1a \cdots (1) \\
m_2g \sin 50^\circ - F_{T2} &= m_2a \cdots (2) \\
F_{T2}R - F_{T1}R &= \left(\frac{1}{2}MR^2\right)\left(\frac{a}{R}\right) \Rightarrow F_{T2} - F_{T1} = \frac{1}{2}Ma \cdots (3)
\end{align*}
\]

(1), (2) $\rightarrow$ (3) to eliminate $F_{T1}$ and $F_{T2}$

\[
\begin{align*}
(m_2g \sin 50^\circ - m_2a) - (m_1g \sin 30^\circ + m_1a) &= \frac{1}{2}Ma \\
\therefore a &= \frac{m_2g \sin 50^\circ - m_1g \sin 30^\circ}{m_1 + m_2 + \frac{1}{2}M} \
&= 1.89 \text{ m/s}^2 \cdots (4)
\end{align*}
\]

\[
\begin{align*}
(4) \rightarrow (1) & : F_{T1} = 54.3 \text{ N} \\
(4) \rightarrow (2) & : F_{T2} = 56.2 \text{ N}
\end{align*}
\]