Don’t waste time on problems you aren’t sure of. Be clear and concise. A cluttered response will not get full credit.

1. A parallel plate capacitor has one plate at -4.0 V and the other at 12.0 V, relative to the ground wire in the lab. They carry equal and opposite charges of ±8.0μ C.
   a. (5 pts) Find its capacitance. Indicate which plate is positively charged.

\[
C = \frac{Q}{ΔV} = \frac{8 \times 10^{-6} C}{16 V} = 0.5 \times 10^{-6} F = 0.5 μF
\]

\[
\left(\frac{ΔV}{C} = F\right)
\]

b. (5 pts) Find the electrical energy stored by the capacitor.

\[
U = \frac{1}{2} C (ΔV)^2 = \frac{1}{2} \left(0.5 μF\right) (16 V)^2 = 64 μJ
\]

\[
\left(F - V^2 = J\right)
\]

c. (5 pts) 100 electrons are moved from the positive plate to the negative plate. Find the change in energy stored by the capacitor. (A positive sign means the energy has gone up.)

\[
\text{Make negative plate more negative, so } \Delta U > 0 \text{ (energy rises)}
\]

\[
dU = dQ (ΔV) \Rightarrow \Delta U = \int (dQ) (ΔV) = (100 \times 1.6 \times 10^{-19} C) (16 V)
\]

\[
= 2.56 \times 10^{-16} J
\]

2. Consider two concentric spherical conducting shells of radii \(a\) and \(b\) with \(a < b\). Let a charge \(Q\) be on the inner (outer) shell. What follows is a difficult problem.

a. (10 pts) Starting from Gauss’s Law relating flux and charge, find the electric field in the region between the shells; explain your reasoning.

\[
\Phi_E - \int \vec{E} \cdot d\vec{A} = \frac{Q_{\text{enc}}}{\epsilon_0} \quad \text{Gauss's Law}
\]

For a concentric spherical Gaussian surface, as in Figure, \(\hat{n} = \hat{r}\).

For a spherical charge distribution, \(\vec{E} = \vec{E}_r \hat{r} \) and \(\vec{E}_r\) depends only on \(r\).

Thus \(\oint \vec{E} \cdot d\vec{A} = \oint \vec{E}_r \hat{r} \cdot d\hat{A} = \int \vec{E}_r \cdot d\vec{A} = \int \vec{E}_r \cdot (d\vec{A} = E_r d\vec{A} = E_r 4\pi r^2 d\vec{A})\)

Thus \(E_r (4\pi a^2) = 4\pi \frac{k \Phi_{\text{enc}}}{r^2}\), so \(E_r = \frac{kQ_{\text{enc}}}{r^2}\).

For \(a < r < b\), \(Q_{\text{enc}} = Q\), so \(E_r = \frac{kQ}{r^2}\).

b. (6 pts) Find \(V(r) - V(a)\) for \(a < r < b\).

\[
\begin{align*}
V(r) - V(a) &= -\int_a^r \vec{E}_r \cdot d\vec{r} = -\int_a^r E_r \hat{r} \cdot (\hat{r} dr) = -\int_a^r E_r dr \\
&= -\int_a^b \frac{kQ}{r^2} dr = kQ \left[ 1 \right]_a^b = kQ \left( \frac{1}{r} - \frac{1}{a} \right)
\end{align*}
\]

c. (4 pts) Find the capacitance of this system.

\[
C \equiv \frac{Q}{ΔV} = \frac{Q}{kQ \left( \frac{1}{b} - \frac{1}{a} \right)} = \frac{1}{k \left( \frac{1}{b} - \frac{1}{a} \right)} = \frac{1}{k \left( \frac{b}{b-a} \right)}
\]

\[
V_a V_\text{b} (>0)
\]
3. Consider three capacitors. \( C_1 = 12 \, \mu F \) and \( C_2 = 6 \, \mu F \) are in parallel, and \( C_3 = 9 \, \mu F \) is in series with them. \( V_a = 8 \, V \) and \( V_d = -4 \, V \).

a. (12 pts) Find the charge and voltage difference for each capacitor. Find \( V_b \).

\[
\begin{align*}
C_{12} &= C_1 + C_2 = 18 \, \mu F \\
C_{12}^{-1} &= \frac{1}{C_1} + \frac{1}{C_2} = \frac{1}{12} + \frac{1}{6} = \frac{2}{18} = \frac{1}{9} \\
\Rightarrow \quad C &= 6 \, \mu F \\
\Delta V &= V_d - V_a = -8 - 4 = -12 \, V \\
\Delta V_3 &= V_b - V_d = \frac{Q_3}{C_3} = \frac{2Q_3}{9} = \frac{8V}{3} \quad \text{so} \quad V_b = V_d + \frac{8V}{3} = -4 + \frac{8V}{3} = \frac{4V}{3}
\end{align*}
\]

\[
\begin{align*}
\Delta V_1 - \Delta V_2 &= \Delta V - \Delta V_3 = 12 - 8 = 4 \, V \\
Q_1 &= C_1 \Delta V_1 = (6 \, \mu F)(4 \, V) = 24 \mu C \\
Q_2 &= C_2 \Delta V_2 = (6 \, \mu F)(4 \, V) = 24 \mu C \\
Q_3 &= C_3 \Delta V_3 = (6 \, \mu F)(4 \, V) = 24 \mu C
\end{align*}
\]

b. (8 pts) Using insulating gloves, \( C_3 \) is disconnected at \( b-c \) and then with connecting wires is placed in parallel with \( C_1 \) and \( C_2 \) (so \( b \) and \( d \) are in contact, and \( a \) and \( c \) are in contact). Find the new charge and voltage difference for each capacitor.

4. A parallel plate capacitor has electrical energy \( 3.6 \times 10^{-6} \) ergs when connected to a 6 V battery.

a. (4 pts) Find the charge on positive plate, and the voltage difference.

\[
Q = \frac{\Delta V}{\epsilon_0} = \frac{7.2 \times 10^{-6} \times 10^{-7} J}{6 \, V} = 1.2 \times 10^{-13} \, \epsilon_0
\]

b. (4 pts) The capacitor is now disconnected from the battery. A slab of dielectric constant \( \kappa = 4 \) and nearly the same thickness as the capacitor is slid into the capacitor. Find the charge on positive plate, and the voltage difference.

\[
\begin{align*}
Q' &= Q = 1.2 \times 10^{-13} \, \epsilon_0 \\
\Delta V' &= \frac{\Delta V}{\kappa} = 1.5 \, V
\end{align*}
\]

c. (7 pts) Was the dielectric attracted, repelled, or did it feel no force when it was part way in the capacitor? Give a physical reason why; no reason, no credit.

It was attracted by the amber effect (polarization in non-uniform electric field).
5. You are given a voltaic cell with internal resistance of 8 \Omega. When shorted, it briefly produces a current of 0.4 A.
   a. (5 pts) Find its emf and the rate at which energy is initially discharged.
      \[ \mathcal{E} = I r = (0.4 A)(8 \Omega) = 3.2 \text{ V} \]
      \[ P = I^2 r = (0.4)^2 (8) = 0.32 \text{ W} \]
   b. (5 pts) For what load resistance \( R \) does this voltaic cell provide maximum power to load? Find that maximum power.
      \[ \text{Impedance matching: } R = r = 8 \Omega \]
      \[ \frac{I} {r + R} = \frac{3.2} {16} = 0.2 A \]
      \[ P = I^2 R = (0.2)^2 (8) = 0.32 \text{ W} \]
   c. (5 pts) A coffee maker for home use produces 840 Watts. Find the current it uses, and its resistance.
      \[ \Delta V = 120 \text{ V. Then } P = I(\Delta V) \text{ and } R = \frac{\Delta V^2} {P} = \frac{120^2} {840} = 1.54 \text{ \Omega} \]

6. A voltaic cell has internal resistance \( r = 0.05 \Omega \) and open circuit voltages across the left and right electrodes of 0.4 V and 1.9 V, for a net emf of \( \mathcal{E} = 1.5 \) V. It is in series with a resistor \( R = 0.25 \Omega \). Let \( V_a = 0.0 \) V. The connecting wires have zero resistance.
   a. (14 pts) Find the current, the voltage drops across the resistances, and sketch the voltage around the circuit.
   \[ \mathcal{E} = 1.5 = 1.9 - 0.4 \]
   \[ I = \frac{\mathcal{E}} {r + R} = \frac{1.5} {0.3} = 5 \text{ A} \]
   \[ \Delta V_R = I r = 5(0.05) = 0.25 \text{ V} \]
   \[ \Delta V = I R = 5(0.25) = 1.25 \text{ V} \]

b. (6 pts) If the voltaic cell discharges in 40 minutes, find its initial "charge" and its initial energy.
   \[ Q = \int I \, dT = (SA)(40 \times 60) = 12,000 \text{ C} \]
   \[ S = \frac{2} {3} \text{ hr} = \frac{10} {3} \text{ A} \cdot \text{hr} \]
   \[ U_{init} = \mathcal{E}(Q) = (1.5)(1.2 \times 10^4 \text{ C}) = 1.8 \times 10^7 \text{ J} \]
7. (15 pts) A 14 cm long rod of semiconductor with 5 mm-by-3 mm cross-section carries 2 mA when a voltage difference of 25 V is placed across its ends. Find the conductivity. Find the electric field within the rod. Estimate the drift velocity of the charge-carriers, taken to be of density \( n = 2.4 \times 10^{24} \text{m}^{-3} \).

\[
I = \frac{\Delta V}{R} \Rightarrow R = \frac{\Delta V}{I} = \frac{25 \text{V}}{2 \text{mA}} = 12.5 \text{ k}\Omega
\]

But \( R = \rho \frac{l}{A} = \frac{1}{\sigma} \frac{l}{A} \), so \( \sigma = \frac{1}{R} \frac{l}{A} = \frac{1}{12.5 \times 10^{3} \Omega} \times \frac{0.14 \text{m}}{15 \times 10^{-6} \text{m}^{2}} = 0.74 \text{ S/m} \)

\[
E = \frac{\Delta V}{l} = \frac{25 \text{V}}{0.14 \text{m}} = 178.6 \text{ V/m}
\]

\[ J = ne \nu, \text{ so } \nu = \frac{J}{ne} = \frac{I}{Ane} = \frac{2 \times 10^{-3} \text{A}}{(15 \times 10^{-6} \text{m}^{2})(2 \times 10^{-3} \text{m}^{3}/\text{s})} = 0.347 \text{ m/s} \]

8. Let \( V_{A} = 6 \text{V}, I = 8 \text{A}, \text{ and } \epsilon = 16 \text{V}, \) with the resistances being 4 ohms and 2 ohms.

\[
\Sigma_{4} + \Sigma_{2} = I = 8
\]

\[ \Delta V = V_{A} - V_{C}, \quad \Sigma_{4} = \frac{\Delta V}{4}, \quad \Sigma_{2} = \frac{\epsilon + \Delta V}{20} \]

\[ \frac{\Delta V}{4} + \frac{I_{4} + \Delta V}{20} = 8 \Rightarrow \Delta V \left( \frac{6}{5} \right) = 7 \frac{1}{5} = \frac{36}{5} \]

\[ \Rightarrow \Delta V = 24 \text{V} \]

\[ \Sigma_{2} = 8 - \Sigma_{4} = 2A \]

a. (10 pts) Find \( \Sigma_{4} \) and \( \Sigma_{2} \).

\[ \Sigma_{4} = \frac{\Delta V}{4} = \frac{24}{4} = 6 \text{A}, \quad \Sigma_{2} = I - \Sigma_{4} = 8 - 6 = 2 \text{A} \]

\[
\text{or } \Sigma_{2} = \frac{16 + 24}{20} = 2 \text{A}
\]

b. (5 pts) Find \( V_{B} \) and \( V_{C} \).

\[ V_{A} - V_{B} = \frac{\Delta V}{2} - 2(2) = 40, \Rightarrow V_{B} = V_{A} - 40 = 6 - 40 = -34 \text{V} \]

\[ V_{A} - V_{C} = \Sigma_{4} = 6, \Rightarrow V_{C} = V_{A} - 24 = 6 - 24 = -18 \text{V} \]

9. (15 pts) Find the unknown currents and the unknown resistance for the circuit in the figure.

\[ \Delta V_{R} = (2A)(6 \Omega) = 12 \text{V}, \quad \bar{I}_{R} = \frac{\Delta V_{R}}{R} = \frac{12 \text{V}}{6 \Omega} = 2 \text{A} \]

\[ \text{No current through } 2 \Omega \text{ or } 5 \Omega \text{ resistors on left N} \]

9A through 7Ω resistor \( ( \Sigma_{7} = 9 \text{A}) \)

\[ \frac{1}{R'} = \frac{1}{6} + \frac{1}{3} + \frac{1}{2} = \frac{1 + 2 + 6}{6} = \frac{9}{6} = \frac{1}{3} \]

\[ R' = \frac{9}{3}, \Delta V' = \bar{I} R' = 9 \times \frac{9}{3} = 27 \text{V} \]

\[ I_{1} = \frac{\Delta V'}{1} = 27 \text{A} \]

\[ I_{2} = \frac{6}{3} = 2 \text{A} \]

\[ I_{6} = \frac{6}{6} = 1 \text{A} \]
10. (20 pts) For the circuit below, take $E_1 = 8 \text{ V}$, $E_2 = 6 \text{ V}$, $r_1 = 0.04 \text{ } \Omega$, $r_2 = 0.01 \text{ } \Omega$, $R = 0.02 \text{ } \Omega$. (1) Indicate and label the directions of positive currents and indicate the positive side of the voltage $\Delta V$ across $R$. (2) Analyze the circuit using Kirchhoff's rules. (3) Solve for the voltage across $R$. (4) Find the current through $R$ and the currents provided by each of the batteries.

\[ \Delta V_R = \Delta V \]
\[ I_1 + I_2 = I \]
\[ \frac{E_1 - \Delta V}{r_1} + \frac{E_2 - \Delta V}{r_2} = \frac{\Delta V}{R} \Rightarrow \frac{E_1}{r_1} + \frac{E_2}{r_2} = \Delta V \left( \frac{1}{r_1} + \frac{1}{r_2} \right) \]

\[ \Rightarrow \frac{8}{.04} + \frac{6}{.01} = \Delta V \left( \frac{1}{.02} + \frac{1}{.04} + \frac{1}{.01} \right) \Rightarrow 200 + 600 = \Delta V \left( 50 + 25 + 100 \right) \]
\[ \Rightarrow \Delta V = \frac{800}{115} = 6.97 \text{ } \text{V} \]
\[ I_1 = \frac{E_1 - \Delta V}{r_1} = \frac{8 - 6.97}{.04} = \frac{2.03}{.04} = 50.75 \text{ } \text{A} \]
\[ I_2 = \frac{E_2 - \Delta V}{r_2} = \frac{6 - 6.97}{.01} = \frac{-.97}{.01} = 97 \text{ } \text{A} \]
\[ I = \frac{\Delta V}{R} = \frac{6.97}{.02} = 348.5 = I_1 + I_2 = 228.6 \text{ } \text{A} \]

11. The capacitor is uncharged initially. The switch is then closed at $t = 0$. Let $E=12\text{ V}$, $r = 3\text{ } \Omega$, $R = 6\text{ } \Omega$, $C = 4\mu\text{F}$.

a. (10 pts) Find $I_r$, $Q$, $I$, and $I_R$ just after the switch is closed.

In general, $I_t = I_R + I$. \\
At $t=0$, have $Q=0$, so $\Delta V_R = \Delta V_R = \frac{Q}{C} = I_R R = 0$. \\
Thus $I_R = 0$ at $t=0$. \\
Thus $I_t = I$, and
\[ I_t = I = \frac{E}{r} = \frac{12}{3} = 4 \text{ A} \]

b. (10 pts) Find $I_t$, $Q$, $I$, and $I_R$ a long time after the switch is closed.

After a long time, $I_t \rightarrow 0$.
Then
\[ I_t = I_R = \frac{E}{r+R} = \frac{12}{3+6} = \frac{4}{9} = 1.333 \text{ A} \]
Also, $\Delta V_R = I_R R = \frac{4}{3} \times 6 = 8 \text{ V} = \Delta V \frac{Q}{C}$, so $Q = C \Delta V \frac{R}{R} = 4 \mu \text{F} \times (8 \text{ V}) = 32 \mu \text{C}$

c. (5 pts) Sketch $I$ as a function of time.