Temporarily skip questions you aren't sure of. Be clear and concise. A poorly written response will not get full credit.

1. (15 pts) A non-conducting rod lies between \((0, 0, 0)\) and \((0, b, 0)\), where \(b\) is a constant. It has charge per unit length \(\lambda[y] = 6\alpha y^2\), where \(\alpha\) is a constant. (a) Find the units of \(b\) and \(\alpha\). (b) If \(\alpha = 2\) in appropriate SI units, find \(\lambda[0.4m]\), and estimate the charge \(\Delta Q\) that lies between \(y = 0.395\) and \(y = 0.405\). (c) In terms of \(\alpha\) and \(b\) (with \(b > 0.5\ m\)), find the total charge \(Q\) on the rod, and the average charge per unit length \(\bar{\lambda}\).

\[
\begin{aligned}
(a) & \quad \text{[b] = m.} \quad \text{[\(\alpha\)] = C/m, to make [\(\lambda\)] = C/m.} \\
(b) & \quad \lambda (0.4\ m) = 6(2) y^2 \ (0.4\ m)^2 = 1.92 \ \text{C/m}, \quad \Delta Q = \lambda \Delta y = (1.92 \times 2)(0.01) = 0.0192 \ C \\
(c) & \quad Q = \int_0^2 \lambda \, dy = \int_0^2 6 \alpha y^2 \, dy = 6 \alpha \left[ \frac{1}{3} y^3 \right]_0^2 = 2 \alpha b^3 \\
& \quad \bar{\lambda} = \frac{Q}{b} = \frac{2 \alpha b^3}{b} = 2 \alpha b^2 
\end{aligned}
\]

2. (10 pts) For the benefit of Bart Simpson's sixth grade teacher, concisely explain the physics of the "amber effect" (between a charged and a neutral object) and why it is attractive. Use a simple figure, with a negative source charge.

![Diagram]

1. Charge positive; particle with negative nearer positive source.
2. The negative is attracted to positive and is repelled.
3. Force falls off with distance.
4. The force between the source and the closer negative charge wins.

3. Assume that the charged conducting sheets in the figure are infinite in extent. The one on the top has total charge per unit area \(4\sigma_0\), and the one on the bottom has a total charge per unit area \(-2\sigma_0\), where \(\sigma_0 > 0\).

a. (5 pts) Find the total electric field (magnitude and direction) between the plates.

\[
E = E_1 + E_2 = 2\pi \kappa \sigma_0 + 2\pi \kappa (2\sigma_0) = 12\pi \kappa \sigma_0
\]

b. (5 pts) Find the charge density on the bottom surface of the top sheet.

\[
\hat{n} = \hat{j} \quad E_{out} = 12\pi \kappa \sigma_0 \hat{j}
\]

\[
S = \frac{E_{out} \cdot \hat{n}}{\sigma_0} = \frac{12\pi \kappa \sigma_0 \hat{j} \cdot \hat{j}}{4\pi \kappa} = 3\sigma_0
\]
4. (35 pts) Point charge $Q_1 = -6.0 \times 10^{-9}$ C is on the negative $y$-axis at $r_1 = 2$ cm from the origin. Point charge $Q_2 = -9.0 \times 10^{-9}$ C makes a counterclockwise angle $\theta = 60^\circ$ to the positive $x$-axis, at $r_2 = 3$ cm from the origin. A point charge $Q = 4.0 \times 10^{-9}$ C is placed at the origin. $Q_1$ and $Q_2$ act on $Q$ with forces $\mathbf{F}_1$ and $\mathbf{F}_2$.

\[ \mathbf{F} = \frac{\kappa Q_1 Q_2}{r^2} \mathbf{r} \]

\[
|\mathbf{F}_1| = \left( 9.0 \times 10^{-9} \text{ C} \right) \left( 7.0 \times 10^{-9} \text{ C} \right) \left( \frac{\text{N} \cdot \text{m}^2}{\text{C}^2} \right) = 5.4 \times 10^{-4} \text{ N}
\]

\[
|\mathbf{F}_2| = \left( 9.0 \times 10^{-9} \text{ C} \right) \left( 3.0 \times 10^{-9} \text{ C} \right) \left( \frac{\text{N} \cdot \text{m}^2}{\text{C}^2} \right) = 3.0 \times 10^{-4} \text{ N}
\]

b. On the figure, draw $\mathbf{F}_1$ and $\mathbf{F}_2$ with their tails on $Q$, and in relative proportion.

c. Find $F_x$, the $x$-component of the total force $\mathbf{F}$ on $Q$.

\[
F_x = F_1 \cos \theta - F_2 \cos \theta = -5.4 \times 10^{-4} \text{ N} + 3.0 \times 10^{-4} \text{ N} \cos 60^\circ
\]

\[
F_x = -3.60 \times 10^{-4} \text{ N}
\]

d. Find $F_y$, the $y$-component of $\mathbf{F}$.

\[
F_y = 3.0 \times 10^{-4} \text{ N} \sin 60^\circ
\]

\[
F_y = 3.12 \times 10^{-4} \text{ N}
\]

e. Find the angle $\phi$ makes with respect to the $x$-axis. On the figure sketch its direction.

\[
\phi = \tan^{-1} \left( \frac{3.12 \times 10^{-4} \text{ N}}{-3.60 \times 10^{-4} \text{ N}} \right) + 180^\circ = -40.9^\circ + 180^\circ = 139^\circ
\]

f. Find $|\mathbf{F}|$.

\[
|\mathbf{F}| = \sqrt{(-3.60 \times 10^{-4} \text{ N})^2 + (3.12 \times 10^{-4} \text{ N})^2}
\]

\[
|\mathbf{F}| = 4.76 \times 10^{-4} \text{ N}
\]

g. $Q_1$ and $Q_2$ are rotated clockwise by 60 degrees about the origin. Now find $F_x$.

\[
F_x = |\mathbf{F}| \cos (139^\circ - 60^\circ) = 4.76 \times 10^{-4} \text{ N} \cos 79^\circ = 0.969 \times 10^{-4} \text{ N}
\]
5. Consider the 2D situation in the figure; the three side walls are conductors.
   a. (7 pts) Sketch the field lines.

   ![Field lines sketch]

   b. (4 pts) For the lower plate, indicate the sign of the charge on the right and left. Explain briefly. Field lines originate on positive charge.

   c. (4 pts) Indicate the locations of the weakest and strongest electric fields. Explain briefly.

6. Two line charges are normal to the page. A, with charge density $4\lambda$, passes through the origin.
   B, with charge density $2\lambda$, passes through $(4a, 0, 0)$. See part c for figure.
   a. (8 pts) Find the position $(a, 0, 0)$ where the electric field is zero. Field of a line charge

   $$E_{\text{ext}} = \frac{1}{2\pi\epsilon_0} \left[ \frac{4\lambda}{x} - \frac{2\lambda}{4a-x} \right] = 0 \Rightarrow \frac{4\lambda}{x} = \frac{2\lambda}{4a-x} \Rightarrow 2a - x = 4a - 2x \Rightarrow x = \frac{3}{2}a$$

   b. (7 pts) Represent $\lambda$ by four field lines. Find the angle between field lines as they originate from A. Repeat for B. Repeat for the angle between field lines as viewed from a distance.

   16 lines leave A at $\frac{3}{8} \pi = 22.5^\circ$ to each other.
   8 lines leave B at $\frac{4}{9} \pi = 45^\circ$.
   2 lines at A+B at $\frac{2}{9} \pi = 15^\circ$.

   c. (10 pts) Taking one field line from A to go directly to the left, and one field line from B to go directly to the right, sketch the field lines for this geometry.

   ![Field lines sketch]
7. (15 pts) A charge $q > 0$ is uniformly distributed on the $x$-axis from $(-b, 0, 0)$ to $(0, 0, 0)$. Compute $E_x$ on the $x$-axis for $x > 0$.

$$dE_x = \frac{\kappa}{(x-v)^2} \quad \Rightarrow \quad d\phi = \frac{\kappa}{x} dx$$

$$E_x = \int_{-b}^{0} \frac{\kappa}{b(x-x')^2} dx' = \frac{-\kappa}{a(x-x')^2} = \frac{kq}{a} \left( \frac{1}{x} - \frac{1}{x-b} \right)$$

$$E_x = \frac{kq}{a} \left( \frac{1}{x} - \frac{1}{x-b} \right)$$

8. For a positively charged conductor, a surface element of area $dA = 2.7 \times 10^{-6}$ m$^2$ has its outward normal $\hat{n}$ along $(3, 5, -4)$. Just outside this surface, $|\vec{E}| = 470$ V/m.

a. (5 pts) Find $\hat{n}$.

$$\hat{n} = \frac{3 \hat{i} + 5 \hat{j} - 4 \hat{k}}{\sqrt{3^2 + 5^2 + 4^2}} = \frac{1}{10} \left( 3 \hat{i} + 5 \hat{j} - 4 \hat{k} \right)$$

$$\hat{n} = (0.3, 0.5, -0.4)$$

b. (5 pts) Find the direction of $\vec{E}$, called $\hat{E}$.

$$\vec{E} = \hat{n} \text{ out of positively charged conductor.}$$

c. (5 pts) Find the flux $d\Phi_E$ through $dA$.

$$d \Phi_E = \vec{E} \cdot \hat{n} dA = 1E1^2 A dA = (470 \text{ V/m})(2.7 \times 10^{-6} \text{ m}^2)$$

$$d \Phi_E = 1.27 \times 10^{-3} \text{ V} \cdot \text{m}$$

d. (5 pts) Find the surface charge $dQ_S$.

$$d \Phi_E = 4 \pi \kappa \text{ d}Q_S \Rightarrow dQ_S = \frac{d \Phi_E}{4 \pi \kappa} = \frac{1.27 \times 10^{-3} \text{ V} \cdot \text{m}}{4 \pi (9 \times 10^9 \text{ C/m}^2)}$$

$$dQ_S = 1.12 \times 10^{-14} \text{ C}$$

9. Consider a conductor with a cavity. The total charge on the material of the conductor is $4q$.

If we associate 4 field lines with each unit of charge, then when viewed from afar a net of 8 field lines points to the conductor, which is in equilibrium.

a. (5 pts) How much charge is within the cavity?

**4.9 = 16 field lines leaving, 3q, but 8 field lines entering to the conductor, so the conductor's cavity must produce 24 lines entering, ⇒ $q_{\text{cavity}} = -6q$**

b. (10 pts) How much total charge is on the inner surface of the conductor? On the outer surface? In the bulk?

In the bulk:

$$q_{\text{bulk}} = q_{\text{cavity}} = 0$$

In the inner surface:

$$q_{\text{inner}} = -q_{\text{cavity}} = 6q$$

For the outer surface:

$$q_{\text{outer}} = 4q - q_{\text{inner}}$$

$$q_{\text{outer}} = 4q - 6q = -2q$$

Total charge:

$$\sum q = q_{\text{inner}} + q_{\text{outer}} = q_{\text{net}} = 4q - 2q = 2q$$
10. Answer the following questions about voltage.

a. (5 pts) Equipotentials A and B have $V_A = 5.8 \text{ V}$ and $V_B = 6.0 \text{ V}$, and a separation of 5 mm. For a point C midway between them, estimate the electric field magnitude and, with an arrow whose tail is on C, indicate its direction.

$$E = \frac{dV}{dx} = \frac{0.2 \text{ V}}{5 \times 10^{-3} \text{ m}} = 40 \text{ V/m}$$

b. (5 pts) Let $V(y) = -4y^3$, where $V$ is in volts and $y$ in in meters. From the voltages at $y=0.9 \text{ m}$ and $y=1.1 \text{ m}$, estimate the electric field at $y=1.0 \text{ m}$, including magnitude and direction.

$$V(0.9\text{ m}) = -4(0.9)^3 = -2.92 \text{ V} \quad V(1.1\text{ m}) = -4(1.1)^3 = -5.32 \text{ V}$$

$$E = \frac{dV}{dy} = \frac{-2.92 \text{ V}}{0.2 \text{ m}} = 120 \text{ V/m}$$

$$E \text{ points from 0.9 m to 1.1 or up.}$$

11. An electron is held at rest at the origin, where $E = (3.0 \times 10^4 \text{ N/C})$. The voltages are 4, 0, and $-2 \text{ V}$ for $y = -1, 0, 1 \text{ cm}$. In general, the field is not uniform.

a. (5 pts) Find the non-electric force needed to hold the electron in place.

$$F_{\text{net}} = -F_{\text{elec}} = -\frac{e}{c \beta^2} \frac{\vec{E}}{E} = -\frac{e}{c \beta^2} \left( 3.0 \times 10^4 \text{ N/C} \right) \beta^2$$

$$= 4.8 \times 10^{-15} \text{ N}$$

b. (5 pts) If the non-electric force is removed, indicate to which of $y = \pm 1 \text{ cm}$ the electron will move to, and determine its speed at that point.

$$\text{It moves to } -1 \text{ cm.} \quad \frac{1}{2} m \frac{v_y^2}{2} + (-e) V_y = \frac{1}{2} m v^2 + (-e) V$$

$$v_y^2 = \frac{2 e V_y}{m} = \frac{2(1.6 \times 10^{-19} \text{ C})(-2 \text{ V})}{9 \times 10^4 \text{ N/C}}$$

$$v = 8.39 \times 10^4 \text{ m/s}$$

12. A quarter and a penny each sits on an insulator. The quarter has charge $-27 \times 10^{-9} \text{ C}$, and the penny has charge $6 \times 10^{-9} \text{ C}$. They are now connected by a thin insulated wire, which is then removed.

a. (5 pts) The penny now is found to have a charge of $-9 \times 10^{-9} \text{ C}$. What is the charge on the quarter?

$$Q_{\text{penny}} = (27 - 9) \times 10^{-9} \text{ C} = 18 \times 10^{-9} \text{ C}$$

b. (5 pts) The voltage of the penny is found to be $+2.7 \text{ V}$ relative to a copper doorknob.

What is the voltage of the quarter?

$$V_{\text{quarter}} = V_{\text{penny}} = +2.7 \text{ V}$$